MILNE'S PROGRESSIVE ARITHMETICS

COMPLETE BOOK

PROGRESSIVE ARITHMETIC

COMPLETE

BY

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NEW YORK ·:· CINCINNATI ·:· CHICAGO
AMERICAN BOOK COMPANY

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PREFACE

The Progressive Complete Arithmetic is designed to cover in a single volume the work in arithmetic usually given to pupils during the last four years of their school course, thus forming with the First Book of the author's Progressive Series a complete course in elementary arithmetic.

The book is divided into four parts, each containing the work intended for a year. It begins with a brief but comprehensive review of the work of the first four years, preparing the pupils to enter upon the study of the new topics with intelligence and ease. Reviews in various forms are, indeed, a feature of the book. Each part, like the first, begins with a thorough review of everything that has preceded, and at various places throughout the book are series of practical problems designed to apply in the most varied form the principles that have been taught. In addition to these general reviews, each advanced step in any topic is preceded by a brief résumé of the concepts already acquired, so that the pupil may attain and retain a comprehensive grasp of the whole.

Great care has been expended on the method of presenting the various principles and on the solutions, with the aim of making them in the highest degree clear, concise, accurate, and practical. The author holds to his idea of properly developing definitions, principles, and all new concepts. This has been a marked feature of all the books he has ever prepared and the main cause of their great success.

The exercises, both oral and written, have been carefully graded, making the advance from one step to the next easy and natural. They are so numerous and varied that the pupils can not fail to become thoroughly familiar with numbers and with the practical processes of computation.

Analysis of problems has received adequate attention, and algebraic methods of solution have been introduced for the purpose of simplifying the processes in certain classes of problems.

The problems themselves have been framed with the greatest care. They are not numerical puzzles, nor are they based on unreal conditions. They have been made both rational and practical, and they relate to a wide range of subjects drawn from modern life and industries. The information they embody has been gathered from reliable sources. The prime object of the book, however, is not to convey information on extraneous topics, but to teach the processes of arithmetic in the best way possible.

WILLIAM J. MILNE.

CONTENTS

PART I

PAGE

Preliminary Review . . . 9
Notation and Numeration . . . 18
- **Arabic System** — integers and decimals . . . 18
- **United States Money** . . . 23
- **Roman System** . . . 24

Addition . . . 25
Subtraction . . . 32
Multiplication . . . 37
Division . . . 50
Combination of Processes . . . 58
Factors and Divisors . . . 59
- **Tests of Divisibility** . . . 59
- **Factoring** . . . 61
- **Cancellation** . . . 62

Fractions . . . 64
- **Reduction** — to higher or lower terms, integers or mixed numbers to fractions, improper fractions to integers or mixed numbers, least common denominator . . . 66
- **Addition and Subtraction** . . . 72
- **Multiplication** — integers by fractions, fractions by integers . . . 76
- **Division** — finding what part one number is of another, dividing fractions by integers, integers by fractions, finding the whole from a fractional part . . . 83

Decimal Fractions — to three places . . . 96
- **Reduction** . . . 96
- **Addition and Subtraction** . . . 98
- **Multiplication** — decimals by integers, by 10, 100, 1000, etc. . . . 101
- **Division** — decimals by integers, by 10, 100, 1000, etc. . . . 104

Useful Parts of a Dollar — finding cost and quantity . . . 108
Bills and Accounts . . . 113
Measurements — review of measures of length, surface, and volume; liquid and dry measures; measures of time . . . 118

PAGE
Counting 120
Stationers' Measures 122
Angles and Arcs 123
Surfaces — rectangles, parallelograms, triangles 125
Rectangular Solids 131
FRACTIONS 134
Review 134
Multiplication — fractions by fractions 138
Division — fractions by fractions, simplifying complex fractions, finding what part one number is of another, comparison . 141
DECIMAL FRACTIONS — to more than three places 150
Notation and Numeration 150
Reduction 152
Addition and Subtraction 154
Multiplication — decimals by decimals 156
Division — decimals by decimals, cancellation 159
REVIEW PROBLEMS IN INDUSTRIES 165

PART II

PRELIMINARY REVIEW 169
DENOMINATE NUMBERS 177
Reduction 177
Addition and Subtraction 181
Multiplication 184
Division 185
PERCENTAGE — finding per cent equivalents, a per cent of a number, what per cent one number is of another, a number when a per cent of it is given. Summary 188
Commercial Discount — one discount 203
REVIEW PROBLEMS IN INDUSTRIES 206
MEASURES AND EQUIVALENTS 213
Weight — avoirdupois and troy 213
Volume and Capacity — useful equivalents 216
Temperature 220
Lumber 222
Plastering, Painting, and Kalsomining 225
Roofing 226
Papering and Carpeting 228

PAGE

PERCENTAGE (AND APPLICATIONS) — increase and decrease, finding a number when the number increased or decreased by some per cent of itself is given 233
Profit and Loss 245
Commission and Brokerage 249
Commercial Discount — more than one discount 253
Marking Goods 257
REVIEW PROBLEMS IN INDUSTRIES 259

PART III

PRELIMINARY REVIEW 275
ALIQUOT PARTS — multiplication and division by 283
ACCOUNTS 285
METHODS OF SOLVING PROBLEMS 288
Oral Analysis 288
Written Analysis 293
Analysis by Equations — solution of equations in x and in x and y 296
DENOMINATE NUMBERS 315
Review — practice in reduction and operations 315
Foreign Money 318
Longitude and Time — including Standard Time 322
METRIC SYSTEM 330
Measures of Length 330
Measures of Surface — including land measures 332
Measures of Volume — including wood measures 334
Measures of Capacity 336
Measures of Weight 337
Equivalents 338
PRACTICAL MEASUREMENTS — miscellaneous in review 342
REVIEW PROBLEMS IN INDUSTRIES 348
PERCENTAGE (AND APPLICATIONS) 354
Review 354
Government Revenue — taxes and duties 358
Insurance — property and personal 364
REVIEW PROBLEMS IN INDUSTRIES 369
SIMPLE INTEREST 376
Aliquot Parts Method 376
Six Per Cent Method 379

PAGE

Cancellation Method . . . 383
Interest Table Method . . . 384
Accurate Interest . . . 387
Problems in Interest . . . 388
Present Worth and True Discount . . . 391
ANNUAL INTEREST . . . 393
COMPOUND INTEREST . . . 394

PART IV

PRELIMINARY REVIEW . . . 397
PROMISSORY NOTES — forms, indorsements, etc. . . . 404
Partial Payments — United States and Mercantile Rules . . . 409
BANKING — in general . . . 412
Bank Discount . . . 416
Savings Banks . . . 419
EXCHANGE . . . 422
Domestic — including all kinds . . . 422
Foreign . . . 429
STOCKS AND BONDS . . . 433
RATIO AND PROPORTION . . . 441
Ratio . . . 441
Simple Proportion . . . 443
Partitive Proportion . . . 447
Partnership . . . 448
POWERS AND ROOTS . . . 451
Raising to Powers . . . 451
Extracting Roots — by factoring, square root . . . 454
MENSURATION . . . 462
Plane Figures — review; also trapezoids, regular polygons, circles . 462
Solids — prisms, cylinders, pyramids, cones, spheres . . . 471
Similar Surfaces . . . 479
Similar Solids . . . 481
GENERAL REVIEW . . . 483
APPENDIX . . . 495
Greatest Common Divisor . . . 495
Least Common Multiple . . . 497
Cube Root . . . 499
Tables . . . 506

PROGRESSIVE ARITHMETIC

COMPLETE

PART I

PRELIMINARY REVIEW

Addition and Subtraction

1 **1.** Read $25 + 7 = 32$; $3 + 6 + 8 = 17$.

2. Tell what the sign $+$ indicates; the sign $=$.

3. When two or more numbers are united into one number by addition, what is the result called?

4. Read $10 - 9 = 1$. What does the sign $-$ indicate?

5. How much greater is 175 than 60? How do you find the answer? What is the answer called?

6. Subtract 486 from 500. Which number is the minuend? Which is the subtrahend? How may the answer be tested?

7. Add 23 to 45. Tell how the units and tens of the sum are obtained.

8. Add 46 to 28. Add 879 to 1668. Explain the process.

9. Subtract 44 from 95; test the result; explain.

10. Subtract 236 from 362, test, and explain.

11. If you sold a person something for 85 cents and received \$2 in payment, how should you count out the change?

Add and test:

12. $98 + 7588 + 5009 + 777 + 4275 + 6235 + 4900$

13. $\$7.59 + \$8167 + \$7526 + \$324.50 + \$.75 + \$7350.50$

14. $56{,}300 + 4 + 99 + 999 + 9999 + 99{,}999 + 999{,}999 + 99{,}999$

Subtract from a dollar:

15. 44¢	**17.** 36¢	**19.** 29¢	**21.** 17¢
16. 51¢	**18.** 85¢	**20.** 74¢	**22.** 43¢

Subtract from two dollars; test each result:

23. $1.25	**25.** $1.45	**27.** $0.82	**29.** $1.17
24. $1.50	**26.** $0.12	**28.** $1.64	**30.** $0.85

31. Tell how to count the change out of $2 for each of the amounts mentioned in exercises 23–30.

Multiplication

2. 1. Find the value of $67 + 67 + 67 + 67 + 67 + 67$ by addition; by multiplication.

2. Draw an oblong 4 inches long and 3 inches wide, and divide it into inch squares. Use the oblong to show why 4 times 3 is equal to 3 times 4.

3. Show with dots or marks that 6 times 8 = 8 times 6.

4. Multiply 48 by 23. Point out the multiplicand, the multiplier, and the product. Test your answer.

5. Multiply 347 by 265. Point out the units' partial product; the tens' partial product; the hundreds' partial product.

Under what figure of the multiplicand is the right-hand figure of each partial product written?

6. Multiply 274 by 204. Which partial product is not written?

7. Multiply 96 by 2004. When there are 0's in the multiplier, how is the process of multiplication shortened?

8. Multiply 45 by 10; 36 by 100; 725 by 1000.

How are the products found?

How may a whole number (called also *integer*, or *integral number*) be multiplied by 10, 100, or 1000?

Division

3. **1.** Divide 1000 by 77. Point out the dividend, the divisor, the quotient, and the remainder.

2. How may the answer in exercise 1 be tested?

3. Read the following, using the words "divided by":

$$66 \div 3 = 22 \qquad 5\overline{)75} \atop \quad 15 \qquad \frac{42}{21} = 2$$

Point out the dividend, the divisor, and the quotient in each case. Test each result.

4. Tell how $\frac{1}{5}$ of 365 days is found.
Indicate the process, using the signs $\div$ and $=$.

5. How many times does 1000 bushels contain 8 bushels?
Indicate the process in three different ways.

6. With what divisors is short division used?

7. Divide 8000 by 20; 8411 by 40.
How is an integer divided by another integer ending in 0?
What is done in such a case if the dividend does not end in 0?

8. Without writing any figures, divide 5280 by 10; by 100; by 1000. Tell how you perform these divisions.

9. Divide 3843 by 200.
Write the result as quotient and remainder.
Write it in the form of a whole number and a fraction, or in the form of a *mixed number*.

10. If 4 yards of silk cost \$9, how many dollars will 1 yard cost? Test your answer.

11. I have \$9. How many 4-dollar hats can I buy and how much money shall I have left?

12. Edward divided 72 by 13, obtaining 5 for a quotient and 6 for a remainder. Test his answer.

13. Divide 16,854 by 36.

Use your work to illustrate the answers to the following questions:

14. If the quotient figure should be taken too large, how would the error be discovered? if it should be taken too small?

15. When the proper quotient figure is taken, how is the fact shown by the remainder?

16. Divide, and test each result by multiplication:

$$2\overline{)608} \qquad 5\overline{)2005} \qquad 12\overline{)1296} \qquad 12\overline{)6108} \qquad 8\overline{)241648}$$

When any remainder with the next figure of the dividend annexed does not contain the divisor, what figure is written in the quotient?

Common and Decimal Fractions

4. **1.** Read the following fractions and name the numerator and the denominator of each:

$$\frac{3}{4} \qquad \frac{5}{6} \qquad \frac{3}{8} \qquad \frac{5}{8} \qquad \frac{5}{4} \qquad \frac{7}{16} \qquad \frac{10}{10} \qquad \frac{9}{2}$$

2. What does each fraction in exercise 1 denote?

3. Which fractions are *proper* and which are *improper?*

4. Add $\frac{3}{4}$ and $\frac{5}{8}$. Explain your work.

5. Which is the greater, $\frac{1}{2}$ or $\frac{3}{4}$? How do you know? Illustrate with a drawing.

6. If two fractions have different denominators, what must be done before the fractions can be united into a single fraction by addition or subtraction? Illustrate with examples of your own.

7. Which of the following fractions are in their lowest terms?

$$\frac{5}{7} \qquad \frac{8}{16} \qquad \frac{14}{32} \qquad \frac{9}{6} \qquad \frac{10}{12} \qquad \frac{8}{15} \qquad \frac{9}{24} \qquad \frac{32}{18}$$

Express them all in their lowest terms.

8. Add $18\frac{3}{8}$ to $25\frac{3}{4}$. Explain your work.

9. Subtract $16\frac{2}{3}$ from 100. Explain.

10. Reduce $4\frac{5}{8}$ to an improper fraction. Explain.

11. Reduce $\frac{33}{10}$ to a mixed number. Explain.

12. Find $\frac{4}{5}$ of 73,245.

Tell how to find $\frac{4}{5}$ of any number; $\frac{3}{8}$ of any number.

13. Multiply 5280 by $2\frac{5}{11}$.

Tell how to multiply an integer by a mixed number.

14. A decimal fraction is one that expresses tenths, or hundredths, or thousandths, etc. All other fractions are common fractions.

Which of the following are decimal fractions and which are common fractions?

$\frac{1}{2}$ .5 $\frac{5}{10}$ $\frac{3}{5}$ .72 $\frac{9}{100}$ .325 $\frac{7}{8}$

15. Reduce to decimal fractions:

$\frac{1}{2}$ $\frac{1}{5}$ $\frac{1}{4}$ $\frac{2}{5}$ $\frac{4}{5}$ $\frac{1}{20}$ $\frac{3}{4}$ $\frac{3}{5}$

16. Reduce to common fractions:

.6 .25 .75 .16 .8 .40 .64 .125

17. Add $\frac{1}{2}$ and $\frac{3}{10}$; $\frac{3}{4}$ and .2; $\frac{1}{5}$ and .25.

18. How much greater is $\frac{3}{8}$ than $\frac{3}{10}$? $\frac{1}{2}$ than .45? $\frac{3}{4}$ than .6? .8 than .55? $\frac{4}{10}$ than $\frac{7}{25}$?

Measurements and Comparisons

5. **1.** What measures are used in buying and selling milk? peanuts? grain? hay? coal? kerosene?

2. How are long distances measured? short distances?

3. What measures are used in measuring land?

4. Give the table of measures of time.

5. Name the months and tell the number of days in each.

6. Tell why a square foot contains 144 square inches. How is the number of cubic inches in a cubic foot found?

7. Compare 1 yard with 1 foot in two ways:

1 yd. is —— ft. longer than 1 ft.;
1 yd. is —— times 1 ft.

8. Compare 6 with 8 in two ways:

6 is —— less than 8; 6 is —— of 8.

Compare in two ways:

9. 1 ft. with 2 in.
10. 1 gal. with 2 qt.
11. 1 gal. with 1 pt.
12. 1 lb. with 4 oz.
13. 2 bu. with 1 pk.
14. 1 hr. with 12 min.
15. 24 with 6.
16. 6 with 24.
17. 25 with 100.
18. 75 with 100.
19. 60 with 100.
20. 120 with 100.

Which is the greater and how much:

21. 6×8 or 7×7?
22. 7×9 or 8×8?
23. 3×15 or 4×12?
24. 4×20 or 3×25?
25. $81 \div 9$ or $42 \div 7$?
26. $45 \div 3$ or $32 \div 2$?
27. $\frac{1}{3}$ of 66 or $\frac{2}{3}$ of 36?
28. $\frac{1}{2}$ of 100 or $\frac{2}{3}$ of 60?

Miscellaneous Problems

SUGGESTION. — When the problems are related to one another, the answer to each should be kept until the series is completed.

6. 1. Edith invited 11 friends of her own age to a party to celebrate her eleventh birthday. What were the combined ages of her guests?

2. She bought invitation cards with envelopes at 2¢ each, and used 2-cent stamps. Find the cost of sending invitations.

3. Edith bought 2 loaves of bread @ 10¢ and cut from

each 16 slices. Each slice made one sandwich. How much did the bread for the sandwiches cost? for each sandwich?

4. A chicken costing 64¢ and $\frac{3}{8}$ lb. butter @ 32¢ were used for the sandwiches. What was the total cost of each sandwich?

5. Edith bought 4 dozen fancy cakes at 6 for 5¢ and a birthday cake for $1.25. Find her change out of $5.

6. She made 7 quarts of lemonade. How many glasses, $\frac{1}{2}$ pt. to a glass, did this allow for each child, and how many over?

7. One lemon was used for every pint, and one was left over. Find the cost of the lemons purchased, at 3 for 5¢.

8. If a quart of ice cream will serve 8 children, how many quarts were needed, allowing enough for 8 second helpings?

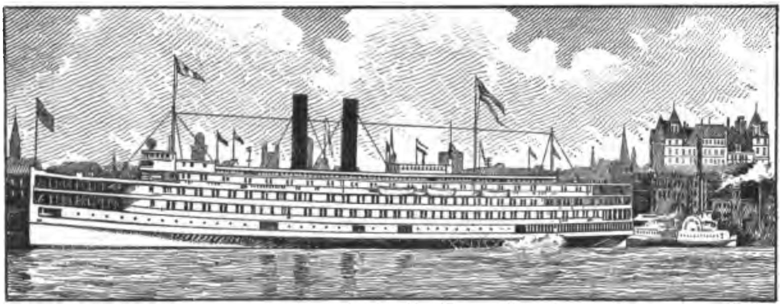
9. At 40¢ a quart, what was the cost of the $2\frac{1}{2}$ quarts of ice cream? How much did all the refreshments cost?

10. The children played at tossing rings on pegs. A ring on the center peg counted 20, but on any corner peg only 5. What was Albert's score, if he threw the ring 6 times on the center peg, and 15 times on the corner pegs?

11. Marjorie arrived at 3:15 P.M. and left at 5:42 P.M. How long was she at the party?

12. Robert and his uncle went from Albany to New York on the night boat. Robert bought a half-fare ticket, 75¢, and his uncle a full-fare ticket. They bought a stateroom ticket for $2. How much did they pay for tickets?

13. Other expenses of the trip were: dinner, $2.25; magazines, 40¢; Panorama of the Hudson, $1; 15 souvenir post cards at 3 for 10¢; morning paper, 5¢. What was the sum of these expenses?



14. Find the whole cost of the trip.

15. The purser told Robert's uncle that there were 1600 passengers on board that night; $\frac{3}{4}$ of them paid full fare ($1.50) and $\frac{1}{5}$ paid half fare. The rest were steerage passengers and paid $1 each. Find the receipts for fares.

16. The staterooms were all occupied. The boat had 110 @ $1, 200 @ $2, 30 @ $3, 2 @ $4, 10 @ $5, and 4 @ $6. What were the receipts for staterooms?

17. Find the receipts from 182 cabin berths at 50¢ each.

18. During the trip 396 meals were served at an average price of $1.25 each. Find the receipts of the dining room.

19. It costs 5¢ to check an article, and the following articles were checked: 138 suit cases, 97 overcoats, and 48 hand satchels. What were the receipts of the check room?

20. The steamer carried freight that night as follows:

Class	Weight	Rate per 100 lb.
First	$29\frac{3}{4}$ tons	20¢
Second	$19\frac{1}{2}$ "	15¢
Third	$51\frac{1}{4}$ "	12¢
Fourth	$127\frac{1}{2}$ "	11¢
Fifth	283 "	10¢
Sixth	472 "	8¢

What were the receipts for freight?

21. Find the total receipts from fares, staterooms, cabin berths, dining room, check room, and freight.

22. If the average salary of each man employed was \$580 for a season of 290 trips, what was the amount paid the entire crew of 135 men for one trip?

23. If the receipts of the dining room amounted to \$495 for the trip and the cost of the supplies was $\frac{3}{5}$ of that amount, how much did the supplies cost?

24. The steamer uses 35 tons of pea coal a trip. Find the cost at \$3.60 per ton.

25. If 9 gallons of lubricating oil are used in the engine room on one trip, find the cost at 67¢ per gallon.

26. Find the cost of the cotton waste used, 8 pounds at $9\frac{1}{2}$¢ a pound.

27. It took 45 men 9 hours to unload and reload the freight. Find the cost, if each man received 30¢ per hour.

28. Find the total cost for wages of crew, dining room supplies, coal, lubricating oil, cotton waste, and handling freight.

29. The distance from Albany to New York is 143 miles. If the steamer makes the trip in 10 hours, find its rate per hour.

30. If the same steamer sails at the rate of 13 miles per hour in returning to Albany from New York, how much longer does it take for the return trip?

NOTATION AND NUMERATION

7. The people of Europe first learned the use of the figures **1**, **2**, **3**, **4**, **5**, **6**, **7**, **8**, **9**, and **0** from the Arabs.

These ten figures are therefore called the **Arabic numerals.**

The figure **0**, representing *no value* and used to fill vacant places, is called **naught**, **zero**, or **cipher**. The others are **significant** figures.

8. The method of representing numbers by means of the ten Arabic numerals is called the **Arabic notation.**

9. Any method of naming or reading numbers is called **numeration**.

10. 1. Count by ones to 9; by tens to 90; by hundreds to 900; by thousands to 9000.

2. Ones are called **units of the first order** or simply **units**; tens are called **units of the second order**; hundreds are called **units of the third order**; and so on.

11. In the Arabic notation:

(1) *The greatest number of units of any order is nine.*

(2) *Therefore ten units of any order are written as one unit of the next higher order.*

The Arabic notation, based on the number ten, is often called the **decimal notation**, from the Latin word *decem*, meaning ten.

12. 1. Read: 8, 28, 46, 104, 208, 500, 987, 1000.

If you have read the numbers correctly, you have illustrated these three facts about reading numbers:

(1) *The figure* 0 *is not read.*

(2) *The word "and" is not used in reading integers.*

(3) *Tens and units and also hundreds, tens, and units are read together, as units.*

Thus 987 is not read 9 hundreds, 8 tens, 7 units. It may be read "nine hundred eighty-seven units," but the word "units" is usually omitted.

2. Hundreds, tens, and units constitute **units' period**.

3. When we reach a thousand, we begin to name the thousands in order as we do the units, thus:

1 thousand, 2 thousand, etc., up to 999 thousand.

Thousands, ten-thousands, and hundred-thousands constitute **thousands' period**.

4. Write with figures 654 thousand 528 (units).

Read 362,475; 502,308; 750,000.

5. Just as 1000 units make 1 thousand, so 1000 thousands make 1 **million** (1,000,000), and 1000 millions make 1 **billion** (1,000,000,000).

BILLIONS' PERIOD				MILLIONS' PERIOD				THOUSANDS' PERIOD				UNITS' PERIOD		
Hundred-billions	Ten-billions	Billions		Hundred-millions	Ten-millions	Millions		Hundred-thousands	Ten-thousands	Thousands		Hundreds	Tens	Units
3	6	5	,	4	8	2	,	7	2	5	,	9	3	8
	7	2	,	0	0	0	,	0	2	4	,	6	0	6
8	0	4	,	2	0	0	,	3	4	1	,	0	0	0

6. The first number in the table is read: "365 *billion*, 482 *million*, 725 *thousand*, 938." Read the other numbers.

13. To read a large integral number:

First, beginning at the right, separate the figures by commas into periods of three figures each.

Next, beginning at the left, read each period as if it stood alone, adding its name.

The highest period may contain less than three figures.

Periods above billions are *trillions*, *quadrillions*, *quintillions*, etc.

14. 1. How many tens are there in 1 hundred? units in 1 ten? tenths in 1 unit? hundredths in 1 tenth? thousandths in 1 hundredth?

2. .5 denotes 5 tenths and is so read; .56 denotes 5 tenths and 6 hundredths, and is read "56 hundredths"; .567 is read "567 thousandths."

3. Read, and tell what each figure denotes:

.8	.62	.37	.80	.094	.140	.005
.7	.60	.07	.84	.095	.014	.301
.9	.78	.48	.92	.105	.404	.070
.6	.51	.34	.09	.125	.382	.800

15. *The Arabic notation is used to express integers and decimals. Ten units of any order, integral or decimal, make one unit of the next higher order.*

A dot called the **decimal point** is placed before tenths' figure.

Tenths occupy the *first* decimal place, *hundredths* the *second*, *thousandths* the *third*, and so on.

16. 1. Read the following mixed numbers:

$$7\tfrac{5}{10},\ 7.5,\ 4\tfrac{7}{100},\ 4.07,\ 5\tfrac{168}{1000},\ 5.168.$$

A mixed number that is expressed as an integer and a decimal is called a **mixed decimal.**

In reading mixed numbers, "and" is used between the name of the integer and that of the fraction, but not elsewhere.

2. Read, and tell what each figure denotes:

5.3	6.32	28.41	1.125	36.404
8.7	4.75	50.67	3.146	80.225
6.5	8.06	32.04	8.073	37.670
3.9	5.88	90.02	6.008	20.006

EXERCISES

17. Read:

1. 425,360.2
2. 632,587.09
3. 1,789.346
4. 426,502.609
5. 2,000.2
6. 55,000.55
7. 1,001,000.001
8. 105,000,005
9. 6,275,200.003
10. 49,482,376.735
11. 928,346,002
12. 3,003,030,300
13. 72,485,364.6
14. 864,375,000

Copy, writing units of the same order in the same column, point off into periods, and read:

15. 4.9, 17.02, 195.005, 3850, 6500000
16. .75, 508.35, 626271, 6424365, 730.402
17. 4.52, 578.96, 24329.05, 1487625.9
18. 3.003, 860.7, 24867.3, 49278564
19. 5781.2, 1.008, 320.75, 48356033.8
20. 73348, 425637, 42235679, 365289073

WRITTEN EXERCISES

18. 1. Write the names of the orders of integers and decimals from hundred-billions down to thousandths.

Express in the Arabic notation:

2. 4 hundreds 6 tens 3 units 5 tenths.
3. 9 tens 9 units 9 tenths 9 thousandths.
4. 3 billions 3 hundred-thousands 3 tens.
5. 7 ten-millions 8 thousands 6 units 8 tenths.

6. Write the numbers that are one greater than:

1000 1199 6229 9089 6011 9999

7. Write the numbers that are 1 less than:

100 500 1621 2000 11,000 100,000

Express in the Arabic notation:

8. 107 million, 875 thousand, 596 (units).

9. 220 billion, 625 million, 340 (units).

10. 4 billion, 362 million, 256 thousand.

11. 400 million, 870, and 2 tenths.

12. 5 billion, 5 million, 5 thousand, 5 hundred.

13. 6 thousand, 325, and 46 hundredths.

14. 1000 thousand, 50, and 4 tenths.

15. 1000 million, 325 thousand, 411.

16. 625 thousand, 75, and 12 hundredths.

17. 48 thousand, and 9 thousandths.

18. 7 million, 7 thousand, 7.

19. 100 million, 1 thousand, 1, and 101 thousandths.

20. Two hundred two thousand, sixty.

21. Sixteen million, forty-five thousand.

22. One hundred million, seven hundred fifty.

23. Forty-three million, seventy-six thousand.

24. Ninety million, ninety thousand, ninety-one.

25. Nine hundred ninety-nine thousand, ninety-nine, and nine hundredths.

26. Sixty-two million, nineteen thousand, seven hundred, and seven hundredths.

27. Seventeen hundred three, and eighteen thousandths.

28. Fourteen hundred million, five hundred thousand, two hundred ninety-seven.

Write in words:

29. 19, 90, 99, 101, 320, 514, 711, 7.11, 6.009

30. 705, 700.5, 700.705, 2000.2, 600.003, .603

31. 33,485,620.033, 650,000,001.010, 428,000.866

United States Money

19. In the money of the United States:

10 mills = 1 cent **10 cents = 1 dime**	**10 dimes = 1 dollar** **10 dollars = 1 eagle**

The **dollar sign** is $; the sign for cents is ¢.

20. Since 10 units of one denomination make one unit of the next higher denomination, United States money is conveniently represented in figures by the decimal notation, dimes and cents being written as hundredths, and mills as thousandths of a *dollar*, which is the unit of the system.

Thus, 4 eagles, 6 dollars, 7 dimes, 8 cents, 5 mills, is written $46.785, and read "46 dollars, 78 cents, 5 mills."

Also, $.01 is read "1 cent"; $.001, "1 mill"; $.256, "25 cents 6 mills"; 25¢, "25 cents"; and 56.7¢, "56 and 7 tenths cents."

21. In business, the seller usually regards any part of a cent as an additional cent.

EXERCISES

1. Read as dollars and cents:

$7.42	$11.25	$256.30	$100.10	$12,645.32
$2.87	$28.06	$809.66	$650.03	$75,050.90

2. Read as dollars, cents, and mills:

$3.845	$1.264	$24.056	$17.283	$100.084
$9.206	$7.605	$30.015	$89.617	$620.908

3. Write in the decimal form, using the sign $:

19 dollars 7 cents	500 dollars 16 cents 3 mills
63 dollars 82 cents	88 dollars 37 cents 5 mills
80 cents; 1 cent 5 mills	740 cents; 23,765 cents

4. Write to the nearest cent: $1.667; $.264; $4.56; $.812.

Roman System

22. The Roman notation uses seven letters, namely:

I	**V**	**X**	**L**	**C**	**D**	**M**
1	**5**	**10**	**50**	**100**	**500**	**1000**

23. These letters are combined to represent other numbers according to the following principles:

1. *When a letter is followed by the same letter or a letter of less value, the values of the letters are to be united.*

Thus, II represents 2; XXX, 30; VI, 6; DC, 600.

2. *When a letter is followed by a letter of greater value, its value is to be taken from that of the greater.*

Thus, IV represents 4; IX, 9; XL, 40; CD, 400.

3. *A letter with a bar placed over it represents a thousand times as much as it does without the bar.*

Thus, $\overline{\text{V}}$ represents 5000; $\overline{\text{X}}$, 10,000; $\overline{\text{IV}}$, 4000.

24. This table further illustrates the method of combination:

III, 3	XIV, 14	XLI, 41	XCIX, 99	MC, 1100
XII, 12	XIX, 19	LXX, 70	DCCC, 800	$\overline{\text{XX}}$, 20,000

EXERCISES

1. Read the following:

$\overline{\text{M}}$	LXXIX	CCV	MDXC
XCIX	CXVIII	DXIX	$\overline{\text{MD}}$CCC
DLXXV	CCXXVI	DCXL	MMDC
CXXXII	CDLXIII	$\overline{\text{D}}$CCL	$\overline{\text{IV}}$CCXL

2. Express the following in the Roman notation:

55	61	101	496	1607
66	58	114	509	5000
79	99	325	1900	10200

ADDITION

EXERCISES

25. Add, giving results quickly:

Add the tens of one number to the whole of the other, and then add the units, thus in exercise 1, "57, 87, 93."

1.	57	28	46	39	22	57	34	62
	36	63	42	54	69	27	46	29
2.	93	75	49	83	26	78	69	34
	38	46	64	45	57	39	63	75
3.	77	84	56	43	64	85	76	99
	47	96	79	58	87	66	98	88
4.	28	65	49	37	78	25	57	18
	39	75	26	84	36	49	64	86

Add rapidly:

5. \$7	**6.** 9¢	**7.** 8 lb.	**8.** 3 bu.	**9.** 5 rd.
8	6¢	5 lb.	9 bu.	8 rd.
5	8¢	6 lb.	8 bu.	7 rd.
10. \$9	**11.** 8¢	**12.** 4 in.	**13.** 7 pt.	**14.** 9 gal.
5	7¢	9 in.	3 pt.	7 gal.
6	5¢	9 in.	6 pt.	8 gal.

26. The process of finding a number that is equal to two or more given numbers is called **addition**.

27. The numbers added are called **addends**, and the result found by adding is called the **sum,** or **amount.**

28. The numbers $2 and $5 are *like numbers;* so also are 6 qt. and 8 qt.; also 3 and 10.

Give other like numbers.

29. 7 sheep and 9 cows are *unlike numbers*, but 7 animals and 9 animals are *like numbers.*

30. Add $2 and $5; 6 qt. and 8 qt. Can you add 7 sheep and 9 cows? 7 animals and 9 animals?

Only like numbers can be added.

EXERCISES IN MAKING CHANGE

31. The picture shows a cash register. When a purchase amounting to, say, $3.57 is made, the clerk presses the keys marked $3, 50, and 7. This registers the purchase on a slip of paper within, pushes up cards that show the amount of the purchase, and opens the cash drawer.

Suppose the purchaser has given the clerk $5. The clerk puts the money into the drawer and may take out the change thus: three one-cent pieces, a dime, a 5-cent piece, a quarter, and a dollar. He then hands the change to the purchaser in the same order, saying, as he does so: "Three fifty-seven, sixty, seventy, seventy-five, four dollars, five dollars."

Following are the amounts of some purchases and the sum of money paid the clerk in each case:

Tell what you would do if you were clerk — the keys of the cash register you would press, the pieces of money you would take out of the drawer, and what you would say to the purchaser as you handed him the change.

	Purchase	Money Paid		Purchase	Money Paid
1.	$1.38	$2.00	13.	$3.69	$5.00
2.	2.13	3.00	14.	1.47	2.00
3.	.42	1.00	15.	2.24	4.00
4.	1.76	5.00	16.	6.45	7.00
5.	2.84	4.00	17.	8.72	10.00
6.	.77	2.00	18.	7.66	9.00
7.	4.09	5.00	19.	5.23	10.00
8.	3.61	5.00	20.	3.17	10.00
9.	1.15	1.50	21.	7.14	8.00
10.	2.03	4.00	22.	8.21	10.00
11.	3.11	3.50	23.	7.25	20.00
12.	2.44	5.00	24.	9.48	20.00

EXERCISES

32. Add rapidly, both columns at once, thus in exercise 1, adding upward, "11, 20, 30, 36, 44, 54, 65":

1.	2.	3.	4.	5.
11	10	11	10	9
10	11	10	8	10
8	5	11	10	8
6	10	9	11	11
10	10	3	4	6
9	7	11	11	10
11	8	6	10	3

Add rapidly, both columns at once:

6.	7.	8.	9.	10.
10	11	9	10	11
9	6	10	8	10
8	10	11	9	9
11	8	7	11	2
10	5	6	3	11
2	10	10	10	5
10	9	4	7	10
5	11	6	10	6

11.	12.	13.	14.	15.
9	10	11	6	11
11	4	7	5	6
6	11	8	11	10
10	9	10	10	11
11	6	2	8	10
7	11	11	10	7
11	10	5	11	6
8	9	10	9	9
10	8	6	10	11

16.

$$\left.\begin{matrix}3\\4\\3\end{matrix}\right\}10 \quad \left.\begin{matrix}5\\6\end{matrix}\right\}11 \quad 9 \quad \left.\begin{matrix}4\\3\end{matrix}\right\}7 \quad \left.\begin{matrix}6\\4\end{matrix}\right\}10 \quad \underline{8} \quad 55$$

Learn to catch the sums of groups of numbers and add them, reading up the column rapidly.

Thus, in the column in the margin read, "8, 18, 25, 34, 45, 55."

Where possible, use groups of 10 or 11, as these sums are more easily added, but combine into any convenient groups. It is best not to pass over numbers in order to group by 10's, for there is danger of neglecting numbers once passed over.

By practice you will be able to use groups that are larger and larger, but for the present they should not be over 11.

Practice until you can add all the following columns in less than $1\frac{1}{2}$ minutes:

17.	**18.**	**19.**	**20.**	**21.**	**22.**	**23.**
5	3	8	4	2	7	5
3	6	3	7	8	6	7
2	7	5	8	3	3	3
6	4	4	6	7	9	4
7	4	6	9	8	4	7
3	6	9	2	3	7	9
2	9	5	5	2	8	2
9	1	2	5	4	3	6
7	3	7	2	4	4	4
5	2	8	3	3	5	9
5	5	2	5	5	5	8
8	7	6	8	6	4	2
2	3	4	2	7	6	3
9	5	5	6	3	9	7
7	6	5	4	5	6	8

WRITTEN EXERCISES

33. In adding numbers of two or more columns, the sum of each column should be retained in some way so that in case of error or interruption while adding any column it will not be necessary to go over all the previous columns.

A convenient method often used by business men is illustrated in the margin, the sums being written on a separate piece of paper. The sums of the columns, so written, give the required sum.

1.	\$4.39
	3.64
	5.98
	7.75
	6.83
	29
	33
	25
	\$28.59

CAUTION. — When this method of keeping the sums of the columns is used, care should be taken not to "carry" from one column to the next.

Add by grouping, and test each result by adding in the opposite direction :

Exercises 2–6 have been added and tested in less than 6 minutes. Can you do better?

2.	3.	4.	5.	6.
$3.46	2648	$46.28	97.38	27.456
8.75	3462	17.32	4.75	46.734
.63	5732	54.59	56.86	8.463
8.96	1897	9.81	37.91	2.057
2.47	314	2.27	24.29	43.591
.64	9285	47.96	8.63	38.762
4.27	8625	63.84	.58	76.357
6.93	3974	15.38	49.63	14.205
5.26	7286	34.02	12.74	7.836
3.72	4738	8.46	56.37	.074
2.48	3473	3.25	82.46	8.698

Practice until you can add and test exercises 7–11 in less than 10 minutes :

7.	8.	9.	10.	11.
28,749	$537.24	$46.32	932.468	684,231
36,451	394.66	24.724	825.510	598,763
54,937	716.84	48.066	253.132	3,821,475
80,783	29.35	79.827	984.749	5,279,635
25,436	82.75	12.283	229.861	2,634,798
93,617	894.33	95.75	671.154	275,824
74,262	323.04	69.31	528.187	743,186
43,231	462.53	24.649	163.728	194,089
54,768	659.21	13.371	429.351	523,716
56,342	51.36	83.18	740.286	8,576,371
79,506	978.03	39.724	967.345	425,843
86,789	792.14	59.869	479.683	176,928
34,321	747.96	54.627	937.427	934,182

WRITTEN EXERCISES

34. 1. A dealer in poultry sold 4800 eggs in January, 2160 in February, and 6708 in March. How many eggs did he sell in the three months?

2. Add: forty-nine thousand eight hundred seventeen, twenty, thirty thousand five hundred forty-five, sixteen thousand eight hundred ninety, and six thousand twelve.

3. What is the sum of 2685 dollars 32 cents, 476 dollars 9 cents, 1020 dollars 10 cents, and 946 dollars 87 cents?

4. Westport is 127.78 mi. north of Albany and 63.13 mi. south of Rouses Point, on a railroad joining these two places. How far is it from Albany to Rouses Point?

5. A farmer sold his potatoes for $1085, wheat for $248.25, rye for $176.45, and hay for $369.50. How much did he receive for all?

6. A builder bought a lot for $584. He built a house upon it costing $4362 and a barn for $974.50. He paid $293.25 for walks, grading, and setting out trees. For how much must he sell the property to gain $775?

7. How many rods of fence are needed to inclose a field whose sides are 27.5 rods, 84.125 rods, 46.08 rods, 62.504 rods, and 18.32 rods long, respectively?

8. In 1904, the popular vote for Roosevelt was 7,624,489; for Parker, 5,082,754; for Debs, 402,286; for Swallow, 258,787; for Watson, 117,935; and for Corrigan, 32,088. What was the total vote for all candidates?

9. One year the number of foreigners entering the United States was as follows: Germans, 46,380; Italians, 193,246; Russians, 145,141; Irish, 36,142; English, 38,626; Japanese, 14,264; all other nationalities, 339,071. What was the total immigration that year?

SUBTRACTION

EXERCISES

35. Subtract, giving results quickly:

Subtract the tens of one number from the whole of the other and then subtract the units, thus in exercise 1, "75, 35, 26."

1.	75 49	52 36	61 29	46 18	88 25	72 37	55 29	99 63
2.	48 19	67 42	81 33	56 27	44 16	91 59	73 25	85 68

3.	\$83 38	78¢ 53¢	56 in. 17 in.	92 oz. 45 oz.	64 mo. 28 mo.	75 doz. 46 doz.
4.	\$1.72 .45	\$1.10 .76	\$2.95 .54	\$1.46 .63	\$2.67 .49	\$1.13 .85

36. The process of taking part of a number away from it, or of finding how much greater one number is than another, is called **subtraction.**

37. The number from which another is subtracted is called the **minuend**; the number subtracted is called the **subtrahend.**

38. The result found by subtracting is called the **difference,** or **remainder.**

39. *The minuend and subtrahend must be like numbers.*

40. Find the difference between 7 and 5; $7+3$ and $5+3$; $7-3$ and $5-3$. How do these differences compare?

Increasing or decreasing the minuend and subtrahend by the same number does not change the difference, or remainder.

EXERCISES

41. 1. Mary's geography cost 75¢ and her reader 39¢. How much more did her geography cost than her reader?

2. Mr. Morton had $85 and spent $46 of it for a rug. How much money had he left?

3. If 74 is the minuend and 35 is the subtrahend, what is the remainder?

4. There were 34 collie dogs at a dog show. All but 18 of them belonged to Mr. Cory. How many did Mr. Cory exhibit?

5. A man earns $112 a month and spends $88 in that time. How much does he save per month?

6. How much change from a dollar bill will a boy receive who buys a ball for 69 cents?

7. Mr. Barnes has 96 sheep in three lots. In one lot there are 25 and in another 34. How many are in the third lot?

8. Find the sum of 48 and 36, and subtract it from 114. What is the remainder?

9. By selling a horse for $125 I lost $32. If I had sold it for $175, should I have gained or lost and how much?

10. Mrs. James bought a pail for 25¢ and a dozen pans for 84¢. How much change did she receive out of $2?

Find, by subtracting the amount of the purchase from the money paid, how much change should be given:

	Purchase	Money Paid		Purchase	Money Paid
11.	$1.45	$2.00	**17.**	$2.33	$5.00
12.	.75	1.00	**18.**	1.12	1.50
13.	2.25	3.00	**19.**	2.61	2.75
14.	3.50	5.00	**20.**	3.35	4.00
15.	.49	2.00	**21.**	2.09	3.00
16.	3.41	4.00	**22.**	2.79	10.00

WRITTEN EXERCISES

42. 1. From 83.9 subtract 26.5.

$$\begin{array}{r} 83.9 \\ \underline{26.5} \\ 57.4 \end{array}$$

Subtract as follows: 5 tenths from 9 tenths leaves 4 tenths or .4; write .4 under the tenths. 6 units from 13 units leaves 7 units; write 7 under the units. 2 tens from 7 tens leaves 5 tens; write 5 under the tens.

The remainder is 57.4.

Another method of subtraction, known as the *Austrian* method, is based on the idea of finding what number added to the subtrahend will produce the minuend.

$$\begin{array}{r} 83.9 \\ \underline{26.5} \\ 57.4 \end{array}$$

Since **.4** added to .5 equals .9, we write .4 in the remainder under tenths.

Since **7** added to 6 equals 13, we write 7 under the units and *carry* 1 ten to the next column. (We carry 1 ten because having added 10 to the minuend to make 13, by § 40 we must add 1 ten to the subtrahend.)

Since **5** added to the sum of 1 and 2 equals 8, we write 5 under the tens.

NOTE. — The teacher is advised not to confuse pupils by teaching both of these methods to the same class. The one that is preferred should be adopted and followed.

Subtract, and test the result by adding it to the subtrahend; the sum should be equal to the minuend.

2. \$7.21 2.98	**3.** 5,836 2,947	**4.** \$46.25 13.81	**5.** 59.63 27.31	**6.** 48,794 16,859
7. \$8.46 3.77	**8.** 7,029 4,634	**9.** \$63.40 28.92	**10.** 95.68 14.09	**11.** 50,623 34,748
12. \$9.03 5.89	**13.** 9,003 2,871	**14.** \$85.00 37.24	**15.** 82.05 54.67	**16.** 70,060 56,789

Subtract and test:

Practice until you can do the work of this page in less than 13 minutes.

17.	$387.43 179.85	**18.**	476.73 298.94	**19.**	876,421 668,376	**20.**	7,894,725 5,698,947
21.	$502.61 346.29	**22.**	624.91 468.36	**23.**	462,385 183,498	**24.**	6,643,489 4,958,397
25.	$750.30 267.41	**26.**	900.00 254.62	**27.**	780,040 562,735	**28.**	8,036,370 3,276,593
29.	$645.00 359.26	**30.**	734.14 469.27	**31.**	948,000 269,123	**32.**	56,914.32 26,347.84
33.	$86.418 27.305	**34.**	53.625 34.579	**35.**	743.981 365.492	**36.**	43,761.26 14,698.77
37.	$54.363 19.875	**38.**	89.684 27.595	**39.**	540.263 274.325	**40.**	74,326.08 35,368.49
41.	$75.285 34.968	**42.**	60.805 13.748	**43.**	673.499 586.944	**44.**	6,234.521 2,578.936
45.	$92.740 58.475	**46.**	70.004 34.052	**47.**	932.068 784.679	**48.**	8,035.040 4,746.806
49.	$40.000 22.743	**50.**	81.206 45.987	**51.**	800.000 245.678	**52.**	9,748.072 5,879.244

WRITTEN EXERCISES

43. 1. From the sum of 5391 and 2645 subtract the difference.

2. George Washington was born in the year 1732 and died in 1799. How old was he when he died?

3. America was discovered by Columbus in the year 1492. How many years have passed since that event?

4. Mr. Burton bought a 1000-mile railroad book to use while on his vacation. When he returned home, he had 479 mile tickets left. How many tickets had he used?

5. A man bought a lot for $978 and built a skating rink on it at a cost of $5345. He then sold the whole property for $8150. How much did he gain?

6. Subtract ten dollars ten cents from ten thousand dollars.

7. A merchant bought some goods for $1637.45 and sold them for $1964.94. How much did he gain?

8. If 13,362 school children are enrolled in a certain city and 6693 of them are boys, how many girls are enrolled?

9. There were 207,977 miles of railway in the United States in a certain year, and 213,904 miles the next year. How much was the gain in one year?

10. The sum of two numbers is 3,042,801, and one of them is 2,400,037. What is the other number?

11. From thirty-four and two hundred forty-five thousandths subtract eighteen and nine hundredths.

12. If it takes a letter 154.6 hours to go from New York to London, and 180.6 hours from New York to Paris, how much more quickly will one go to London than to Paris?

13. Mr. Green had a farm of 342.75 acres. He sold from it at different times 37.5 acres, 126.25 acres, and 88.5 acres. How many acres did he have left?

MULTIPLICATION

44. 1. Count by 6's to 12 times 6. Give the table of 6's.

2. Count by 7's to 12 times 7. Give the table.

3. By counting in a similar way, obtain in order the tables of 8's, 9's, 10's, 11's, and 12's.

4. If you do not already know the multiplication tables perfectly, *commit them to memory now.*

MULTIPLICATION TABLE

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

In this table the 1's are written in the first row, the 2's in the second row, the 3's in the third row, and so on. To find the value of 7 times 8, for example, find 8 in the first column, thus locating the eighth row, in which all the 8's are written. 7 times 8, or 56, will be found in this row in the seventh column, or under the figure 7 in the top row.

DRILL EXERCISES

45. Tell products instantly:

5×6	6×4	8×3	2×5	6×9
3×7	9×3	6×6	8×2	4×4
9×5	3×6	8×5	7×3	8×6
8×7	7×8	9×7	2×9	2×7
3×9	5×7	9×2	9×4	11×2
8×4	7×6	7×4	7×5	11×5

10	11	12	10	11	2	8	2
7	3	2	8	4	12	10	11
11	12	7	11	3	7	11	4
6	5	11	5	12	12	8	12
9	8	12	9	12	11	8	6
7	4	9	5	7	10	12	12
8	9	10	12	9	7	9	11
9	10	12	11	8	6	4	9
6	12	8	12	5	6	9	8
9	8	11	10	12	11	11	5
7	11	10	8	11	10	12	10
7	11	10	8	7	9	6	11
9	7	8	9	12	11	9	12
9	10	6	12	4	12	7	12

46. 1. Find the sum of two 32's; of three 32's; of four 32's; of ten 32's.

2. Tell a short way to find the sum of several 32's.

47. A short process of finding the sum of equal numbers, or the process of taking one number as many times as there are units in another, is called **multiplication.**

48. The number taken or multiplied is called the **multiplicand.**

49. The number that shows how many times the multiplicand is taken is called the **multiplier.**

50. The result obtained by multiplying is called the **product.**

51. 1. How many dollars are 4 times $20? How many trees are 4 times 20 trees? How many miles are 4 times 20 miles?

2. When 20 things of any kind are multiplied by 4, how many things are there in the product? What kind of things compose the product?

52. A number used without reference to any particular thing is called an **abstract number.**

2, 4, 20, etc., are abstract numbers.

53. A number used in connection with some particular thing is called a **concrete number.**

20 is a concrete number when it refers to dollars, or trees, or miles, etc.

54. Point out abstract and concrete numbers:

15 days	15 years	15	$12	12¢	12
6	6	6	5	5	5
90 days	90 years	90	$60	60¢	60

55. *The multiplier must be regarded as abstract. The product is like the multiplicand.*

56. 1. Compare 20 times 2 with 2 times 20; 12 times 4 with 4 times 12.

In finding the product of two abstract numbers, either may be taken for the multiplicand.

2. 5×11 gal. is read "5 **times** 11 gallons"; 11 gal. $\times$ 5 is read "11 gallons **multiplied by** 5"; 6×7 is read either "6 times 7" or "6 multiplied by 7."

The sign of multiplication is read *times* when it *precedes* the multiplicand, and *multiplied by* when it *follows* the multiplicand.

EXERCISES

57. Read and complete the following. Name the multiplicand, the multiplier, and the product. Describe each as concrete or abstract.

1. 3×7 days =
2. 2×16 ounces =
3. 3 feet $\times$ 12 =
4. 15×2 pints =
5. 12 months $\times$ 2 =
6. 4 pecks $\times$ 11 =
7. 10×8 quarts =
8. 4 quarts $\times$ 13 =
9. 12 inches $\times$ 9 =
10. 20×24 hours =
11. How many days are there in 12 weeks?

MODEL SOLUTION

1 week = 7 days;
12 weeks = 12×7 days = 84 days.

Multiplicand and product denote days.

NOTE. — In the following exercises, the tables in the Appendix may be consulted, if necessary.

12. How many ounces are there in 3 pounds?
13. How many feet are there in 13 yards?
14. How many oranges are 8 dozen oranges?

15. How many square feet are there in 9 square yards?

16. A man had 8 rows of cherry trees with 8 trees in each row. How many cherry trees had he?

17. From each of 9 classes, 7 children are chosen to take part in a Christmas play. How many children take part in the play?

18. Edward saw 3 flocks of wild geese, with 15 geese in each flock. How many geese did he see?

19. A boy mailed 12 letters to England with 5-cent stamps. How much did all the stamps cost?

20. How many cents are there in 11 dimes?

21. How far does a train go in 3 hours at the rate of 30 miles an hour?

22. If it takes 17 minutes to go from one place to another by a subway car and twice as long by a surface car, how long will it take by the surface car?

23. Find the cost of a dozen fans at 5 cents each.

24. How many quarts are there in a bushel? in 2 bushels?

25. When laths are tied in bundles of 50 each, how many laths are there in 20 bundles?

26. If a canal boat is towed at the rate of 2 miles per hour, how far will it be towed in 24 hours?

Find the cost of:

27. 30 pairs of rabbits at 15¢ per pair.

28. 16 pairs of canvasback ducks at $2 per pair.

29. 34 boxes of Kiefer pears at $2 per box.

30. A 10-pound box of cherries at 14¢ per pound.

31. 12 pounds of candied cherries at 40¢ per pound.

32. A 25-pound box of apricots at 10¢ per pound.

33. Ten 4-lb. baskets of Catawba grapes at 12¢ per pound.

EXERCISES

58. Multiply:

1.	36 2	48 2	75 2	140 2	250 3	710 3
2.	44 3	65 3	609 3	35 4	216 4	413 4
3.	25 5	64 5	72 5	120 5	312 5	908 5
4.	15 6	43 6	82 6	212 6	13 7	34 7
5.	52 7	612 7	25 8	42 8	212 8	500 8
6.	14 9	22 9	113 9	909 9	56 10	75 10
7.	101 10	44 11	25 11	61 11	104 11	312 11
8.	40 12	32 12	25 12	46 12	203 12	250 12
9.	130 2	140 3	150 4	160 5	117 2	118 2
10.	515 4	225 4	119 2	214 5	320 12	501 11

BUSINESS PROBLEMS

59. 1. Find the cost of sending a 22-word telegram from Kansas City to New York City at 60¢ for the first 10 words and 4¢ for each additional word.

2. The rate mentioned in exercise 1 is the day rate in either direction, and is briefly written 60–4. The night rate is 50–3. How much does it cost to send the same telegram at night?

3. How much less does the telegram cost at night?

4. How much cheaper is a 15-word telegram at a night rate of 50–3 than at a day rate of 60–4?

Find the cost of each telegram between New York City and the place named, at day rates; at night rates:

Place	Number of Words	Day Rate	Night Rate	Place	Number of Words	Day Rate	Night Rate
5. Hoboken	16	25–2	25–1	**17.** St. Paul	18	60–4	50–3
6. Boston *	18	30–2	25–1	**18.** Mobile *	25	60–4	50–3
7. Bangor *	22	40–3	30–2	**19.** Raleigh *	40	50–3	40–3
8. Hartford *	24	25–2	25–1	**20.** Savannah *	60	60–4	50–3
9. Newport *	26	30–2	25–1	**21.** Tampa *	19	60–4	50–3
10. Richmond	17	40–3	30–2	**22.** Topeka	30	60–4	50–3
11. Detroit	19	40–3	30–2	**23.** New Orleans	22	60–4	50–3
12. Toronto	20	40–3	30–2	**24.** Denver *	17	75–5	60–4
13. Chicago	23	50–3	40–3	**25.** Galveston	21	75–5	60–4
14. St. Louis	25	50–3	40–3	**26.** Seattle *	17	100–7	100–7
15. Cleveland	30	40–3	30–2	**27.** Portland, Ore.*	19	100–7	100–7
16. Louisville	32	50–3	40–3	**28.** St. Johns	15	110–9	100–8

*** Other cities in the same state have the same rate.**

29. Find the cost of sending the following telegram, counting only the words of the actual message:

THE WESTERN UNION TELEGRAPH COMPANY

11.30 A.M. *New York, July 10,* 1906.

To *Mrs. William Steel,*

525 West 12th St., Chicago, Ill.

Arrived	*safe*	*steamship*	*Cedric.*	*Reach*
Chicago	*Polk*	*Street*	*Station*	*five*
thirty	*to-morrow.*	*Bring*	*friend*	*Hastings.*
				John Steel.

☞ READ THE NOTICE AND AGREEMENT ON BACK ☜

30. Find the cost of sending this telegram at 9 P.M.

31. How much will it cost to talk over a long-distance telephone for 7 minutes at 25¢ for the first 3 minutes and 5¢ for each additional minute (25–5)?

32. Alfred talked with his father in a distant city from 7:55 P.M. to 8:03 P.M. Find the telephone charges at 50–15.

33. How much would the conversation have cost by day at 100–25?

Find the cost of telephoning for:

34. 10 min. @ 20–5

35. 11 min. @ 40–10

36. 12 min. @ 50–10

37. 6 min. @ 60–15

38. 8 min. @ 100–25

39. 5 min. @ 150–40

60. **1.** Multiply 2 by 10; by 100; by 1000.

2. Multiply 25 by 10; by 100; by 1000.

3. Multiply 156 by 100; by 1000; by 10,000.

4. How may an integer be multiplied by 10? by 100? by 1000? by 10,000? by 100,000?

An integer may be multiplied by 10, 100, 1000, *etc., by annexing to it as many ciphers as there are ciphers in the multiplier.*

WRITTEN EXERCISES

61. Multiply by 10, by 100, by 1000, and by 100,000:

1. 32	**5.** 506	**9.** 247	**13.** 210
2. 56	**6.** 708	**10.** 396	**14.** 400
3. 85	**7.** 124	**11.** 101	**15.** 7386
4. 79	**8.** 365	**12.** 308	**16.** 5490

Multiply:

17.	**18.**	**19.**	**20.**	**21.**
234	234	725	408	760
20	200	120	800	900
4680	***00	****0	****00	****00

Multiply:

22. 38 by 20	**28.** $125 by 400	**34.** 165 by 3000
23. 58 by 70	**29.** $632 by 500	**35.** 287 by 9000
24. 49 by 30	**30.** $496 by 600	**36.** 429 by 8000
25. 75 by 90	**31.** $328 by 800	**37.** 360 by 4000
26. $46 by 110	**32.** $560 by 110	**38.** 798 by 7000
27. $93 by 120	**33.** $784 by 120	**39.** 590 by 1200

40. Find the area of a tennis court 78 ft. long and 30 ft. wide.

41. A barrel of flour weighs 196 pounds. How many pounds of flour are required to fill 40 barrels?

42. If the heart beats 72 times in a minute, how many times does it beat in an hour?

43. A barge ferries 16 cars over the Hudson River in one trip. How many does it ferry over in 20 trips?

44. A certain large city consumes 400 car loads of fruit and vegetables per day. How many tons are consumed in a day, if a car load averages 15 tons?

62. **1.** What part of 10 yards is 5 yards?
What part of 10 times 3 feet is 5 times 3 feet?
Compare 5 times 4 with 10 times 4; 5×8 with 10×8.

2. How is 10×48 found? How may 5×48 be found from 10×48? $5 \times 48 = ?$

$$\begin{array}{r} 2\overline{)480} \\ \hline 240 \end{array}$$

3. Tell an easy way to multiply any integer by 5.

4. How is 100×66 found? How may 50×66 be found from 100×66? $50 \times 66 = ?$
How may any integer be multiplied by 50?

$$\begin{array}{r} 2\overline{)6600} \\ \hline 3300 \end{array}$$

5. What part of 1000 is 500? How is any integer multiplied by 1000? How, then, may any integer be multiplied by 500? $500 \times 24 = ?$ $500 \times 18 = ?$

6. Since any integer may be multiplied by 100 by annexing two ciphers, and since 25 is $\frac{1}{4}$ of 100, how may any integer be multiplied by 25? $25 \times 44 = ?$ $25 \times 32 = ?$

7. Since any integer may be multiplied by 1000 by annexing three ciphers, and since 250 is $\frac{1}{4}$ of 1000, how may any integer be multiplied by 250? $250 \times 44 = ?$ $250 \times 16 = ?$

$$\begin{array}{r} 4\overline{)44{,}000} \\ \hline 11{,}000 \end{array}$$

63. *Any integer may be multiplied*
By 50, *by annexing two ciphers and dividing by* 2.
By 500, *by annexing three ciphers and dividing by* 2.
By 25, *by annexing two ciphers and dividing by* 4.
By 250, *by annexing three ciphers and dividing by* 4.

WRITTEN EXERCISES

64. Multiply by 5; by 25; by 500; by 250; by 50:

1. 64	**7.** 90	**13.** 202	**19.** 990
2. 86	**8.** 220	**14.** 814	**20.** 870
3. 98	**9.** 160	**15.** 735	**21.** 1125
4. 69	**10.** 128	**16.** 916	**22.** 3487
5. 71	**11.** 177	**17.** 497	**23.** 4966
6. 87	**12.** 346	**18.** 318	**24.** 9999

25. How much will it cost to rent a house for 5 years at $1800 a year?

26. A barrel of salt weighs 280 pounds. Find the weight of 25 barrels of salt.

27. A field yielded 50 bales of cotton averaging 496 pounds each. How many pounds of cotton did it yield?

28. A bushel of shelled corn weighs 56 pounds. Find the weight of 500 bushels.

WRITTEN EXERCISES

65.

1.	**2.**	**3.**	**4.**
95	4387	3675	350
350	1204	2008	95
4750	17548	29400	1750
285	52644	7350	3150
33250	5281948	7379400	33250

95 multiplied by 5 tens = 475 *tens.*
4387 multiplied by 12 hundreds = 52644 *hundreds.*
3675 multiplied by 2 thousands = 7350 *thousands.*

Exercise 4 shows a method of testing the answer in exercise 1. Test the other answers.

Multiply and test:

5. 345 × 240	**6.** 427 × 990	**7.** 538 × 705	**8.** 932 × 1004
9. 89 × 4400	**10.** 488 × 1201	**11.** 843 × 480	**12.** 999 × 1212

13. 409 × 45	**21.** 607 × 304	**29.** 7854 × 1210
14. 790 × 68	**22.** 418 × 730	**30.** 4936 × 8020
15. 480 × 99	**23.** 922 × 908	**31.** 3705 × 5550
16. 650 × 85	**24.** 756 × 709	**32.** 4010 × 8750
17. 396 × 430	**25.** 1244 × 202	**33.** 75,073 × 4800
18. 589 × 280	**26.** 1201 × 837	**34.** 49,000 × 3609
19. 278 × 760	**27.** 1108 × 746	**35.** 55,410 × 5040
20. 169 × 970	**28.** 1010 × 890	**36.** 37,962 × 7588

Multiply:

37. \$85.25 by 48	**41.** \$110.75 by 92
38. \$56.62 by 36	**42.** \$434.19 by 206
39. \$83.69 by 77	**43.** \$391.72 by 508
40. \$42.96 by 29	**44.** \$588.93 by 2505

WRITTEN EXERCISES

66. **1.** How far will a train go in 24 hours at the rate of 40 miles an hour?

$$\begin{array}{r} 24 \\ \underline{40} \\ 960 \end{array}$$
960, number of miles.

SOLUTION. — Since the train goes 40 miles an hour, in 24 hours it will go 24 times 40 miles.

Since 24 × 40 = 40 × 24, the number of miles the train will go may be found either by multiplying 40 by 24, or by multiplying 24 by 40.

Since it is easier to multiply by 40 than by 24, we multiply by 40.

2. Find the number of pounds of cheese in 175 cheeses that weigh 40 pounds each.

3. Find the weight of 126 bales of cotton, if the average weight of a bale is 500 pounds.

4. At the bank a boy saw 18 bags, each of which contained $5000 in gold coin. Find the value of the gold in all the bags.

5. A grocer bought 185 cases of eggs. Each case contained 30 dozen. How many dozen eggs did he buy?

6. A coffee plantation of 62 acres contained 500 trees to the acre. Find the whole number of coffee trees.

7. Find the value of 219 Angora goats at $5.75 a head.

8. The clip from a flock of Angora goats was 847 pounds of mohair. Find its value at $.35 per pound.

9. The material for a dress was 14 yards of mohair and cost $1.25 per yard. Making and trimmings cost $18.75. Find the cost of the dress.

10. An upholsterer bought 75 yards of mohair plush at $1.50 per yard. Find the cost.

11. How much must a harness maker pay for 36 mohair carriage robes at $22 each?

12. How much will it cost to drill a gas well to a depth of 1125 feet at $1.05 per foot?

13. A cargo of lemons from Sicily consisted of 25,000 boxes each weighing 92 pounds. Find the weight of the cargo.

14. A jobber bought 225 boxes of lemons at $2.85 a box. Find the cost of his purchase.

How much will it cost to send a message by cable from

15. New York to London, 15 words at $.25 per word?

16. New York to Tokyo, 18 words at $1.33 per word?

17. New York to Manila, 24 words at $1.12 per word?

18. New York to Constantinople, 88 words at $.37 per word?

DIVISION

67. 1. How many times does 150 contain 15? How many times does 152 contain 15, and how many units remain?

2. Separate 150 units into 10 equal parts. How many units are there in each part?

3. If 155 is separated into 10 equal parts, how many units and parts of a unit will there be in each part?

68. The process of finding how many times one number is contained in another is called **division**.

Division is also the process of separating a number into equal parts. This kind of division is sometimes called **partition**.

69. The number divided is called the **dividend**.

70. The number by which we divide is called the **divisor**.

71. The number obtained by dividing is called the **quotient**.

72. The part of the dividend remaining when the division is not exact is called the **remainder**.

DRILL EXERCISES

73. Tell quotients instantly:

32 ÷ 8	40 ÷ 8	56 ÷ 7	72 ÷ 12	120 ÷ 12
42 ÷ 6	45 ÷ 5	54 ÷ 6	60 ÷ 12	110 ÷ 11
30 ÷ 6	42 ÷ 7	56 ÷ 8	81 ÷ 9	108 ÷ 12
35 ÷ 7	40 ÷ 5	60 ÷ 5	99 ÷ 9	132 ÷ 12
24 ÷ 8	45 ÷ 9	64 ÷ 8	84 ÷ 12	100 ÷ 10
28 ÷ 7	48 ÷ 6	63 ÷ 7	84 ÷ 7	121 ÷ 11
36 ÷ 4	49 ÷ 7	72 ÷ 8	96 ÷ 8	120 ÷ 10
35 ÷ 5	54 ÷ 9	72 ÷ 6	96 ÷ 12	132 ÷ 11
36 ÷ 9	36 ÷ 6	72 ÷ 9	99 ÷ 11	144 ÷ 12

74. 1. \$36 may be divided by \$4, but not by 4 years, 4 gallons, 4 pounds, etc.

The dividend and divisor, if concrete, must be like numbers.

2. Divide \$36 by \$4; 60 gal. by 12 gal.; 42 days by 7 days; 56 in. by 8 in.

When the divisor is like the dividend, the quotient is abstract.

3. Divide \$96 by 12; 72 hr. by 8; 81¢ by 9.

When the dividend is concrete and the divisor is abstract, the quotient is like the dividend.

4. $$21 \text{ days} \div 7 \text{ days} = 3$$

means that 21 days contains 7 days 3 times.

$$21 \text{ days} \div 3 = 7 \text{ days}$$

means that $\frac{1}{3}$ of 21 days is 7 days.

The sign ÷ may denote either measuring or dividing into parts, but in either case it is read "**divided by.**"

EXERCISES

75. Divide:

1. 64 qt. ÷ 8	6. 34 pt. ÷ 17
2. 48 lb. ÷ 3	7. 96 ft. ÷ 12
3. 24 hr. ÷ 12 hr.	8. 45 days ÷ 15
4. 32 oz. ÷ 16 oz.	9. 72 in. ÷ 12 in.
5. 700¢ ÷ 100¢	10. 42 bu. ÷ 14 bu.

11. How many weeks are there in 365 days?

SOLUTION

$$7 \text{ days}\overline{)365 \text{ days}}$$
$$52 \text{ times}, + 1 \text{ day}$$
$$52 \text{ wk. } 1 \text{ day}$$

Since 7 days = 1 wk., 365 days are equal to as many weeks as 7 days are contained times in 365 days.

Then, 365 days = 52 wk. 1 day = $52\frac{1}{7}$ wk.

NOTE. — Consult the tables in the Appendix, if necessary.

12. How many pounds are there in 160 ounces?

13. How many yards are there in 34 feet?

14. A box contains 126 oranges. How many dozen oranges does it contain?

15. If 45 cents is divided among 3 boys, how many cents will each boy receive?

16. If an 18-acre field yields 36 tons of clover, how many tons per acre does it yield?

17. How many times does the hour hand of a clock go around the clock face in 72 hours?

18. How many bushels are there in 50 pecks?

19. If a motor car runs 12 miles an hour, how long will it take to run 100 miles?

20. Four boys weigh 283 pounds. Find the average weight.

21. A famous cherry tree in California yielded, in five years, 2000 lb., 1800 lb., 3000 lb., 2200 lb., and 3000 lb. of cherries. Find the average annual yield.

EXERCISES

76. The expressions $\$22 \div \$5$, $\$5\overline{)\$22}$, and $\frac{\$22}{\$5}$ all indicate that \$22 is to be divided by \$5.

The answer should be given in the form:

"4 and \$2 remainder," or "$4\frac{2}{5}$."

Divide:

1. $132 \div 2$	**4.** $\$180 \div 6$	**7.** $\$5\overline{)\$510}$	**10.** $8\overline{)\$404}$
2. $632 \div 7$	**5.** $\$810 \div 9$	**8.** $\$3\overline{)\$132}$	**11.** $7\overline{)\$364}$
3. $722 \div 9$	**6.** $\$132 \div 12$	**9.** $\$5\overline{)\$351}$	**12.** $11\overline{)999}$
13. $\frac{90¢}{6¢}$	**14.** $\frac{341 \text{ wk.}}{7}$	**15.** $\frac{\$282}{4}$	**16.** $\frac{\$639}{9}$

WRITTEN EXERCISES

77. Divide :

1. \$729 ÷ \$8	**2.** \$9656 ÷ 8	**3.** 896 yd. ÷ 8
4. 12)52800	**5.** 11)\$8877	**6.** 8)\$75.94
7. 11)49390	**8.** 10)\$10.10	**9.** 7)\$87.43
10. 12)\$76.80	**11.** 12)\$98.40	**12.** 9)\$89.91
13. \$6)\$5914	**14.** \$11)\$4004	**15.** 6)\$12108
16. $\frac{168 \text{ hr.}}{7 \text{ hr.}}$	**17.** $\frac{110 \text{ sq. in.}}{5 \text{ sq. in.}}$	**18.** $\frac{1320 \text{ cu. in.}}{12 \text{ cu. in.}}$
19. $\frac{1760 \text{ yd.}}{11}$	**20.** $\frac{5280 \text{ ft.}}{8}$	**21.** $\frac{5760 \text{ oz.}}{12 \text{ oz.}}$
22. $\frac{7249 \text{ sq. ft.}}{9 \text{ sq. ft.}}$	**23.** $\frac{3000 \text{ mi.}}{8}$	**24.** $\frac{43360 \text{ in.}}{12 \text{ in.}}$

78. **1.** Divide 20, 60, 210, 350, 600, and 7000 by 10.

How may an integer ending in one or more ciphers be divided by 10?

2. Divide 300, 800, 1100, 2500, and 16,000 by 100.

How may an integer ending in two or more ciphers be divided by 100?

3. Divide 11,000, 225,000, and 3,000,000 by 1000.

Make a rule about dividing certain integers by 1000.

4. Divide 5762 by 10; by 100; by 1000.

What is the remainder in each case?

An integer may be divided by 10, 100, 1000, *etc., by cutting off as many figures from the right as there are ciphers on the right of the divisor. The figures cut off form the remainder if there is any.*

EXERCISES

79. Divide by 10, by 100, by 1000, and by 100,000:

1. 5,200,000	**5.** 9,246,340	**9.** 11,482,000
2. 7,568,000	**6.** 2,756,982	**10.** 58,700,000
3. 4,300,000	**7.** 7,536,289	**11.** 90,000,000
4. 8,000,000	**8.** 4,750,304	**12.** 84,300,750

WRITTEN EXERCISES

80. Divide:

1. 2|0)86|0 43	**2.** 4|0)36|7 $9\frac{7}{40}$	**3.** 3|00)15|23 $5\frac{23}{300}$
4. 30)9300	**5.** 60)4260	**6.** 800)24800
7. 40)1640	**8.** 50)3550	**9.** 700)56700
10. 80)7285	**11.** 600)1271	**12.** 4000)24511
13. 70)8473	**14.** 200)1837	**15.** 5000)45313
16. 90)1087	**17.** 800)9633	**18.** 9000)99919

19. Divide 2457 by 40.

4|0)245|7
$61\frac{17}{40}$

4 tens are contained in 245 tens, 61 times, with 1 ten remainder.

Therefore 40 is contained in 2450, 61 times, with 1 ten remainder; and in 2457, 61 times with 1 ten and 7 units, or 17, remainder.

Divide by 20, 30, 40, 50, 60, 70, 80, and 90:

20. 3433	**22.** 6257	**24.** 5827	**26.** 60,077
21. 1271	**23.** 4739	**25.** 3271	**27.** 97,001

Divide by 200, 300, 400, 500, 600, 700, 800, and 900:

28. 7523	**30.** 42,641	**32.** 73,099	**34.** 356,101
29. 6931	**31.** 38,407	**33.** 65,411	**35.** 701,407.

36. A 40-acre vineyard in California produced grapes worth $4160. Find the value of the crop per acre.

37. How long will it take a fast freight train running 20 miles an hour to go from Charleston to New York, 804 miles?

38. Mr. Hoy bought a car load of grain. It weighed 34,000 pounds. How many tons did it weigh?

39. A car that weighed 22,800 pounds when empty was loaded with wheat. It then weighed 56,460 pounds. How many bushels of wheat did it contain, if 1 bushel weighs 60 pounds?

40. How much are the freight charges on 35,000 pounds of machinery sent from St. Louis to Wichita at 52¢ per 100 pounds?

$$\begin{array}{r} \$.52 \\ \underline{350\cancel{0}\cancel{0}} \\ 2600 \\ \underline{156} \\ \$182.00 \end{array}$$

Dividing 35,000 pounds by 100 pounds by cutting off the last two ciphers of 35,000, it is found that 35,000 pounds = 350 hundredweight. Since the freight on 1 hundredweight is $.52, the freight on 350 hundredweight is 350 times $.52, or $182.

Freight rates are usually quoted per 100 pounds.

Find the freight charges on:

41. 23,200 pounds brick @ 22¢.

42. 45,000 pounds sugar @ 28¢.

43. 36,300 pounds coffee @ 35¢.

44. 28,900 pounds rice @ 38¢.

45. 56,400 pounds molasses @ 75¢.

46. 38,400 pounds cotton piece goods @ 92¢.

47. Find the freight charges on 88,600 lb. of sugar shipped from New Orleans to Minneapolis @ 33¢; from New Orleans to Salt Lake City @ $1.50.

48. Find the freight on 776,400 lb. of corn shipped from Omaha to New York City @ 20¢.

49. Find the freight on 27,000 lb. of raisins shipped from Stockton, Cal., to New York at 95¢.

50. Find the freight on 1400 crates of strawberries shipped from Wilmington, N.C., to Pittsburg at 65¢ per crate.

Find the cost of:

51. 22,000 bricks at $6.50 per 1000.

52. 18,000 strawberry plants at $2.50 per 1000.

53. 44,000 cu. ft. of gas at $.95 per 1000 cu. ft.

WRITTEN EXERCISES

81.

1.	2.	3.
245	$6.44	5063
764)187180	275)$1771.00	427)2161901
1528	1650	2135
3438	1210	2690
3056	1100	2562
3820	1100	1281
3820	1100	1281

When the divisor is so large that each step in the work must be written, the process is called **long division.**

Divide:

4. 8544 by 24

5. 1722 by 42

6. 64,260 by 85

7. 71,916 by 78

8. 62,923 by 89

9. $63,595 by 79

10. $608.16 by 56

11. $899.91 by 99

12. $90,600 by 75

13. 17,088 by 356

14. $102.60 by 285

15. $577.98 by 234

16. $360.36 by 156

17. 158,976 by 414

18. 246,906 by 522

19. 553,491 by 691

20. 348,425 by 385

21. 645,582 by 798

Divide by 658; 782; 496; 879; 1025; 4560; 7854:

22. 594,832	**26.** 358,632	**30.** 6,597,774
23. 316,710	**27.** 930,040	**31.** 3,385,200
24. 314,675	**28.** 792,232	**32.** 4,775,232
25. 450,864	**29.** 848,470	**33.** 8,458,758

34. How many cars of 48 tons capacity are required to receive the cargo of a Lake Erie vessel carrying 8400 tons of iron ore?

35. If a bale of hops weighs 180 lb., how many bales and how many pounds over will make a ton?

36. It cost a man $1275.00 to drill a gas well 1500 ft. deep. How much did the work cost per foot?

37. A rolling mill consumes about 3,500,000 cu. ft. of natural gas per day. How many households would this amount supply, if each uses 875 cu. ft. per day?

38. A spool of barbed wire contains 1500 ft. How much does this length of wire weigh, if 12 ft. weigh 1 lb.?

39. A 75-acre field yielded 3 crops of alfalfa in a year, the first 187 tons, the second 158 tons, and the third 105 tons. Find the average annual yield per acre.

40. A steamship has 2450 tons of coal in her bunkers. If she burns 175 tons per day, how long will the coal last?

41. It required 160 dollars' worth of gypsum to fertilize a hop field 80 rd. by 40 rd. Find the cost per acre.

42. A New England state road cost $6388.80 per mile. How much did it cost per lineal foot?

43. A strawberry farm in New Jersey yielded 140,000 qt. of strawberries. How many bushel crates did it yield?

44. In one season 69,750 crates of strawberries were shipped from one county in Delaware. Find the number of car loads shipped, considering 225 crates as a car load.

COMBINATION OF PROCESSES

82. When $+$ and $-$ are the only signs in an expression, the operations are performed in order, beginning at the left.

$$2+8-6+3=10-6+3=4+3=7.$$

83. When $\times$ occurs in an expression in connection with $+$, $-$, or both, the indicated *multiplications* must be performed first.

$$8+2\times 3-9=8+6-9=5.$$

84. When $\div$ occurs in an expression in connection with $+$, $-$, or both, the indicated *divisions* must be performed first.

$$10-8\div 2+5=10-4+5=11.$$

85. When $\times$ and $\div$ are the only signs in an expression or are succeeding signs in any expression, the indicated multiplications and divisions are usually performed in order from the left.

$$7+10-6\div 3\times 4=7+10-2\times 4=7+10-8=9.$$

86. The parentheses ( ), brackets [ ], braces { }, and the vinculum ——, called **signs of aggregation,** are used to group expressions, each group being regarded as a single number.

All operations within groups should be performed first.

$$10\times\overline{6-2}+[9\div(2+1)]=10\times 4+9\div 3=40+3=43.$$

WRITTEN EXERCISES

87. Find the value of:

1. $5\times 10-7$
2. $5\times(10-7)$
3. $2\times 5+3\times 4$
4. $16-2\times 2\times 12\div 4$
5. $(25-13)\div 4\times 2$
6. $6+2\times 8-4\div 2$
7. $(6+2)\times 8-4\div 2$
8. $(6+2\times 8-4)\div 2$
9. $6+2\times(8-4)\div 2$
10. $6+2\times(8-4\div 2)$
11. $3+[2+\overline{10-8}+\overline{4\times 3-2\times 5}]\div 2$
12. $24-\{16-6\div 3\times(5\times 5-6\times 4)\}$

FACTORS AND DIVISORS

88. 1. How many are 3×4? 6×2? $3 \times 2 \times 2$?

2. Name *two* numbers whose product is 16; 24; *four* numbers whose product is 16; 24.

89. The integers that multiplied together produce a given number are called its **factors**.

90. An integer that will divide a number without leaving a remainder is called an **exact divisor** of the number.

The factors of a number are exact divisors of it.

2, 3, 4, and 6 are exact divisors of 12. *Divisible* means *exactly divisible.*

91. A number that has no factors except itself and 1 is called a **prime number**.

5, 7, 11, are prime numbers.

92. A number that has other factors than itself and 1 is called a **composite number**.

4, 9, 12, are composite numbers.

93. 1. Tell which of the following are divisible by 2:

2, 4, 5, 8, 10, 13, 17, 18, 20, 21, 42, 50.

2. Which of these numbers are not divisible by 2?

94. A number that is divisible by 2 is called an **even number**; a number that is not divisible by 2 is called an **odd number**.

Tests of Divisibility

95. 1. Make a list of numbers from 2 to 50 that contain 2 as a factor. Notice the units' figure of each.

2. Make a list of numbers from 5 to 100 that contain 5 as a factor. What must the units' figure of such numbers be?

3. Make a list of numbers from 3 to 60 that contain 3 as a factor. Find whether 3 is exactly contained in the sum of the figures, or *digits*, of each number.

4. Make a list of numbers from 9 to 90 that contain 9 as a factor. Find whether 9 is exactly contained in the sum of the digits of each number.

96. *A number is divisible by*

2, *if the units' figure is* 2, 4, 6, 8, *or* 0.
5, *if the units' figure is* 5 *or* 0.
3, *if the sum of the digits is divisible by* 3.
9, *if the sum of the digits is divisible by* 9.

EXERCISES

97. By applying the preceding tests tell which of the numbers 2, 5, 3, 9, are exact divisors of:

1. 24	4. 225	7. 567	10. 3705	13. 7964
2. 35	5. 374	8. 654	11. 4839	14. 8730
3. 72	6. 460	9. 864	12. 7080	15. 9828

98. Illustrate these additional tests.

A number is divisible:

1. By 6, if it is *even* and the sum of its digits is divisible by 3.

2. By 4, if its *two* right-hand digits are 0's or if the number expressed by them is divisible by 4.

3. By 25, if its *two* right-hand digits are 0's or if the number expressed by them is divisible by 25.

4. By 8, if its *three* right-hand digits are 0's or if the number expressed by them is divisible by 8.

5. By 11, if the *difference* between the *sums* of its *alternate* digits is zero or is divisible by 11.

EXERCISES

99. Find which of the numbers 2, 3, 4, 5, 6, 8, 9, 11, 25, are exact divisors of:

1. 17,418	3. 23,512	5. 210,705	7. 568,392
2. 97,317	4. 58,914	6. 105,372	8. 890,550

9. If an *even* number is divisible by an *odd* number, it is divisible by *twice* that number. Illustrate.

10. An exact divisor of a number is an exact divisor of any number of times that number. Illustrate.

11. An exact divisor of each of two numbers is an exact divisor of their *sum* and of their *difference*. Illustrate.

Factoring

100. 1. What numbers are exact divisors of 18?

2. If 3 is taken as one of two factors of 18, what is the other factor? How is it found?

In separating a number into two factors, any exact divisor may be taken for one factor and the quotient for the other.

101. The process of separating a number into its factors is called **factoring.**

102. Factors that are prime numbers are called **prime factors.**

2, 2, and 3 are the prime factors of 12.

103. When numbers have no factor, except 1, that is contained in each of them, they are **prime to each other.**

4 and 21 are prime to each other, though neither is a prime number.

104. A small figure written at the right of a number and a little above, to indicate how many times the number occurs as a factor, is called an **exponent.**

In $16 = 2^4$, the 4 is an exponent, indicating that 2 occurs 4 times as a factor of 16; that is, 2^4 means $2 \times 2 \times 2 \times 2$.

WRITTEN EXERCISES

105. 1. Find the prime factors of 2295.

$$\begin{array}{r|r} 5 & 2295 \\ \hline 9 & 459 \\ \hline 3 & 51 \\ \hline & 17 \end{array}$$

$$2295 = 5 \times 3^3 \times 17$$

Since the units' figure of 2295 is 5, by what number is 2295 divisible? Dividing by 5, we find the other factor of 2295 to be 459.

Since the sum of the digits of 459 is divisible by 9, we divide by 9 and find the other factor to be 51. The sum of the digits of 51 is divisible by 3; then 3 is one factor of 51 and 17 is the other.

Hence the factors of 2295 found are 5, 9, 3, and 17, but the *prime factors* are 5, 3, 3, 3, and 17; that is, $2295 = 5 \times 3^3 \times 17$.

2. Find the prime factors of 7000; of 2880; of 8250.

$$\begin{array}{r|r} 10 & 7000 \\ \hline 10 & 700 \\ \hline 10 & 70 \\ \hline & 7 \end{array} \qquad \begin{array}{r|r} 10 & 2880 \\ \hline 9 & 288 \\ \hline 8 & 32 \\ \hline & 4 \end{array} \qquad \begin{array}{r|r} 10 & 8250 \\ \hline 25 & 825 \\ \hline 3 & 33 \\ \hline & 11 \end{array}$$

$7000 = 2^3 \times 5^3 \times 7$ $\quad 2880 = 5 \times 3^2 \times 2^6$ $\quad 8250 = 5^3 \times 2 \times 3 \times 11$

NOTE. — Do not try divisors greater than half the number to be factored.

3. Factor all the composite numbers from 1 to 100 and make a list of the prime numbers.

Find the *prime factors* of:

4. 144	7. 576	10. 1050	13. 64,640
5. 260	8. 891	11. 9702	14. 30,888
6. 315	9. 672	12. 4620	15. 44,000

Cancellation

106. Since 8×5 is contained in 16×5 the same number of times that 8 is contained in 16, and since 8 is contained in 16 the same number of times that 1 is contained in 2, it is evident that the factors 5 and 8 may be omitted or *canceled* from *both* dividend and divisor *without changing the quotient.*

107. The process of shortening computations in division by rejecting equal factors from both dividend and divisor is called **cancellation.**

WRITTEN EXERCISES

108. 1. Divide $8 \times 77 \times 15$ by $4 \times 44 \times 20$.

$$\frac{\overset{\overset{1}{2}}{\cancel{8}} \times \overset{7}{\cancel{77}} \times \overset{3}{\cancel{15}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{4}} \times \underset{4}{\cancel{44}} \times \underset{4}{\cancel{20}}} = \frac{21}{8} = 2\tfrac{5}{8}$$

We indicate the division by writing $8 \times 77 \times 15$ above a line and $4 \times 44 \times 20$ below it (§ 76).

Since dividing both dividend and divisor by 4 does not change the quotient, the factor 4 is canceled from 8 and 4, leaving 2 in the dividend and 1 (not written) in the divisor.

Similarly, 11 is canceled from 77 and 44; 5 from 15 and 20; and 2 from 2, left in the dividend, and from one of the 4's left in the divisor.

Since the factors now left in the dividend are prime to those left in the divisor, we have 7×3 divided by 2×4, or $\frac{21}{8}$. The quotient is $2\frac{5}{8}$.

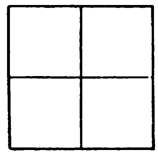
NOTE. — When all the factors of the dividend are canceled, the resulting dividend is 1. The same is true of the divisor. When all the factors of *both* dividend and divisor are canceled, the quotient is $\frac{1}{1}$, or 1.

Divide, using cancellation:

2. $14 \times 27 \times 24 \times 80$ by $32 \times 63 \times 45$
3. $25 \times 42 \times 18 \times 54$ by $12 \times 70 \times 30 \times 9$
4. $16 \times 28 \times 72 \times 50$ by $35 \times 64 \times 24 \times 15$
5. $11 \times 81 \times 26 \times 100$ by $39 \times 15 \times 90 \times 22$
6. $5000 \times 810 \times 1750$ by $625 \times 45 \times 30 \times 50$
7. $63 \times 36 \times 48 \times 27 \times 96 \times 69$ by $81 \times 81 \times 144 \times 46$
8. $48 \times 48 \times 20 \times 65 \times 36 \times 54$ by $2880 \times 1080 \times 936$
9. $792 \times 240 \times 35 \times 756$ by $882 \times 180 \times 800 \times 33$
10. $512 \times 27 \times 720 \times 847$ by $64 \times 144 \times 99 \times 55$
11. $168 \times 144 \times 216 \times 432 \times 484$ by $224 \times 198 \times 576 \times 162$

FRACTIONS

109. 1. Into how many parts is this square divided? How do the parts compare in size?



2. What name is given to one of the four equal parts of anything? to three of the four equal parts?

3. If any one thing is divided into five equal parts, what name is given to one part? to two parts? to three parts? to four parts?

110. Any one thing is called a **unit**. A unit or a collection of units is called a **number**.

111. One or more of the equal parts of a unit is called a **fraction**.

112. 1. Two numbers are used to write a common fraction— one above and the other below a line; thus, three fourths is written $\frac{3}{4}$.

Write four fifths, five sixths, seven eighths.

2. In the fraction $\frac{3}{4}$, which number shows into how many equal parts the thing, or unit, is divided? which shows how many parts form the fraction?

3. In the fraction $\frac{4}{5}$, what does the 5 show? the 4? What does the 8 show in the fraction $\frac{7}{8}$? the 7?

113. The number that shows into how many equal parts the unit is divided is called the **denominator**.

It is written below the line.

114. The number that shows how many parts form a fraction is called the **numerator**.

It is written above the line.

115. The numerator and denominator of a fraction are called its **terms**.

What are the terms of the fraction $\frac{2}{3}$? Which is the numerator? the denominator?

EXERCISES

116. 1. Read $\frac{9}{206}$, $12\frac{3}{20}$, $\$2.66\frac{2}{3}$.

$\frac{9}{206}$ is read, "nine two-hundred-sixths."
$12\frac{3}{20}$ is read, "twelve and three twentieths."
$\$2.66\frac{2}{3}$ is read, "two dollars, sixty-six and two thirds cents."

Read:

2. $\frac{16}{25}$	5. $\frac{37}{160}$	8. $\frac{89}{200}$	11. $34\frac{7}{8}$ in.	14. $\$5.12\frac{1}{2}$
3. $\frac{19}{42}$	6. $\frac{194}{281}$	9. $\frac{111}{1728}$	12. $42\frac{5}{16}$ lb.	15. $\$18.33\frac{1}{3}$
4. $\frac{22}{61}$	7. $\frac{71}{820}$	10. $26\frac{3}{19}$	13. $78\frac{21}{25}$ T.	16. $\$85.46\frac{3}{4}$

WRITTEN EXERCISES

117. Write in figures:

1. Seventy-five eightieths.
2. Nineteen fifty-seconds.
3. Twenty-five hundredths.
4. Eleven four-hundredths.
5. Fourteen thousandths.
6. Sixty-eight ninetieths.
7. Eighty-two thirty-sixths.
8. Five and seven eighths.
9. Five hundred two-thousandths.
10. Seventy-seven one-hundred-fortieths.
11. Nine hundred four six-thousand-two-hundred-tenths.
12. Two hundred one and twenty one-hundred-fourths.
13. Sixty-four dollars forty-two and one fourth cents.

Write in words:

14. $\frac{25}{48}$	17. $\frac{14}{101}$	20. $\frac{175}{2681}$	23. $69\frac{19}{61}$	26. $\$46.62\frac{1}{2}$
15. $\frac{68}{54}$	18. $\frac{37}{144}$	21. $\frac{288}{1728}$	24. $86\frac{21}{37}$	27. $\$125.33\frac{1}{3}$
16. $\frac{73}{90}$	19. $\frac{496}{217}$	22. $\frac{462}{8249}$	25. $50\frac{36}{75}$	28. $\$376.24\frac{5}{8}$

Reduction of Fractions

118. The process of changing the form of a number without changing its value is called **reduction.**

119. Reduction of fractions to higher or lower terms.

1. How many fourths are there in $\frac{1}{2}$? how many eighths?

How may the terms of $\frac{2}{4}$ be obtained from those of $\frac{1}{2}$? the terms of $\frac{4}{8}$ from those of $\frac{1}{2}$? from those of $\frac{2}{4}$?

Which has the larger, or *higher*, terms, $\frac{1}{2}$ or $\frac{2}{4}$? $\frac{2}{4}$ or $\frac{4}{8}$?

2. Change $\frac{1}{2}$ to sixths; to twelfths. Change $\frac{2}{3}$ to ninths.

3. How many eighths are there in $\frac{12}{16}$? how many fourths?

How may the terms of $\frac{6}{8}$ be obtained from those of $\frac{12}{16}$? the terms of $\frac{3}{4}$ from those of $\frac{6}{8}$? from those of $\frac{12}{16}$?

Which has the smallest, or *lowest*, terms, $\frac{12}{16}$, $\frac{6}{8}$, or $\frac{3}{4}$?

4. Reduce $\frac{4}{8}$ to halves; $\frac{2}{6}$ to thirds; $\frac{6}{10}$ to fifths; $\frac{10}{12}$ to sixths.

120. *Multiplying or dividing both terms of a fraction by the same number does not change its value.*

121. A number that will exactly divide two or more numbers is called a **common divisor** or **common factor** of those numbers; and the *greatest* number that will exactly divide them is called their **greatest common divisor (g. c. d.)**. (See Appendix.)

Since (§ 105) $18 = 2 \times 3 \times 3$ and $30 = 2 \times 3 \times 5$, the *common* divisors of 18 and 30 are 2, 3, and 2×3, or 6; hence, their *greatest* common divisor is 6.

122. A fraction is expressed in its **lowest terms** when its terms have no *common divisor* except 1.

123. 1. Change to twelfths: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{6}$; $\frac{5}{6}$.

2. Change $\frac{1}{2}$ to tenths; $\frac{2}{3}$ to eighteenths; $\frac{4}{5}$ to twentieths.

3. Reduce $\frac{4}{8}$ to halves; $\frac{4}{12}$ to thirds; $\frac{9}{12}$ to fourths.

4. Reduce to lower terms : $\frac{3}{9}$; $\frac{2}{10}$; $\frac{8}{12}$; $\frac{7}{14}$; $\frac{10}{51}$; $\frac{8}{16}$.

5. Reduce to lowest terms : $\frac{4}{8}$; $\frac{6}{12}$; $\frac{8}{16}$; $\frac{12}{16}$; $\frac{10}{20}$; $\frac{16}{24}$.

WRITTEN EXERCISES

124. 1. Reduce $\frac{5}{8}$ to thirty-seconds.

$$\frac{5 \times 4}{8 \times 4} = \frac{20}{32}$$

We have learned that it will not change the value of $\frac{5}{8}$ to multiply both terms by any number. $32 \div 8 = 4$; then we multiply both terms by 4.

2. Reduce to forty-eighths : $\frac{1}{2}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{1}{6}$; $\frac{7}{8}$; $\frac{5}{12}$; $\frac{9}{16}$.

3. Reduce to sixtieths : $\frac{1}{3}$; $\frac{1}{4}$; $\frac{2}{5}$; $\frac{5}{6}$; $\frac{3}{10}$; $\frac{7}{12}$; $\frac{8}{15}$; $\frac{11}{20}$.

4. Reduce $\frac{45}{60}$ to its lowest terms.

$$\frac{5)45}{5)60} = \frac{9}{12}$$

$$\frac{3)9}{3)12} = \frac{3}{4}$$

Or, $$\frac{15)45}{15)60} = \frac{3}{4}$$

We have learned that it will not change the value of $\frac{45}{60}$ to divide both terms by any number. By § 96 we see that 5 and also 3 will exactly divide both terms of $\frac{45}{60}$. Dividing both terms by 5, $\frac{45}{60} = \frac{9}{12}$; dividing the terms of $\frac{9}{12}$ by 3, $\frac{9}{12} = \frac{3}{4}$.

The terms of $\frac{3}{4}$ have no common divisor except 1; then $\frac{45}{60}$ reduced to lowest terms is equal to $\frac{3}{4}$.

Or, we may directly divide both terms of the fraction by their greatest common divisor, 15 (§121).

Reduce to lowest terms :

5. $\frac{18}{36}$	7. $\frac{24}{40}$	9. $\frac{25}{125}$	11. $\frac{126}{315}$	13. $\frac{375}{1000}$
6. $\frac{15}{27}$	8. $\frac{32}{60}$	10. $\frac{84}{280}$	12. $\frac{144}{576}$	14. $\frac{288}{1728}$

15. Reduce to fortieths : $\frac{1}{2}$; $\frac{3}{4}$; $\frac{2}{5}$; $\frac{3}{8}$; $\frac{7}{10}$; $\frac{34}{80}$; $\frac{95}{200}$; $\frac{378}{560}$.

16. Reduce to hundredths : $\frac{1}{4}$; $\frac{3}{5}$; $\frac{9}{10}$; $\frac{11}{20}$; $\frac{14}{25}$; $\frac{120}{400}$; $\frac{252}{900}$.

17. Reduce to fractions having the denominator 72 : $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{6}$; $\frac{7}{8}$; $\frac{5}{9}$; $\frac{11}{12}$; $\frac{19}{24}$; $\frac{25}{36}$; $\frac{98}{144}$; $\frac{188}{288}$; $\frac{432}{648}$.

18. Reduce $\frac{2}{3}$ and $\frac{9}{16}$ each to a fraction whose denominator is 48; 96; 144; 240; 384; 528.

125. Reduction of integers or mixed numbers to fractions.

1. How many half dollars are there in one dollar? in two dollars? in \$3? in \$$3\frac{1}{2}$?

2. How many quarters are there in \$1? in \$2? in \$$2\frac{1}{4}$?

3. How many fourths are there in 3? in $3\frac{1}{4}$? in $3\frac{3}{4}$?

4. Express as thirds: 1; $2\frac{1}{3}$; $3\frac{2}{3}$; 8; $6\frac{1}{3}$; 9.

5. Reduce to fifths: 1; 3; $2\frac{1}{5}$; $4\frac{2}{5}$; 6; $5\frac{3}{5}$; $7\frac{4}{5}$.

126. A number expressed by an integer and a fraction is called a **mixed number.**

EXERCISES

127. 1. Reduce to sixths: 4; 5; $2\frac{1}{6}$; $3\frac{5}{6}$; 7; 9; 12.

2. Reduce to eighths: 3; $2\frac{1}{8}$; 7; $4\frac{3}{8}$; 9; $6\frac{5}{8}$; 11.

Reduce to a fraction:

3. $3\frac{1}{2}$	5. $5\frac{1}{6}$	7. $9\frac{4}{5}$	9. $3\frac{3}{8}$	11. $10\frac{1}{3}$
4. $6\frac{2}{3}$	6. $7\frac{2}{5}$	8. $4\frac{5}{6}$	10. $2\frac{5}{9}$	12. $11\frac{1}{2}$

WRITTEN EXERCISES

128. 1. Reduce $24\frac{4}{5}$ to fifths.

$$24 = \frac{120}{5}$$

$$\frac{120}{5} + \frac{4}{5} = \frac{124}{5}$$

How many fifths are there in 1? in 24? How many fifths are 120 fifths and 4 fifths, that is, $\frac{120}{5}$ and $\frac{4}{5}$?

Then how many fifths are $24\frac{4}{5}$?

Reduce to a fraction:

2. $23\frac{1}{2}$	6. $28\frac{1}{8}$	10. $52\frac{3}{5}$	14. $48\frac{5}{12}$	18. $142\frac{3}{4}$
3. $14\frac{1}{4}$	7. $44\frac{2}{5}$	11. $81\frac{5}{6}$	15. $65\frac{7}{15}$	19. $375\frac{4}{5}$
4. $37\frac{1}{3}$	8. $75\frac{3}{4}$	12. $93\frac{7}{8}$	16. $72\frac{9}{16}$	20. $561\frac{5}{6}$
5. $16\frac{1}{6}$	9. $63\frac{3}{8}$	13. $78\frac{3}{10}$	17. $86\frac{17}{24}$	21. $826\frac{7}{8}$

129. Reduction of improper fractions to integers or mixed numbers.

1. How many quarter dollars are equal to \$1? to \$2?

To how many dollars are 8 quarters equal? 12 quarters? \$$\frac{8}{4}$? \$$\frac{12}{4}$? \$$\frac{16}{4}$?

2. How many ones and how many thirds over are $\frac{7}{3}$? $\frac{10}{3}$? $\frac{14}{3}$? $\frac{16}{3}$? $\frac{20}{3}$?

3. Reduce to a mixed number: $\frac{9}{2}$; $\frac{13}{3}$; $\frac{17}{4}$; $\frac{19}{4}$; $\frac{21}{5}$.

4. Find the value of: \$$\frac{6}{2}$; \$$\frac{7}{2}$; \$$\frac{9}{4}$; \$$\frac{15}{4}$; $\frac{20}{5}$; $\frac{24}{5}$.

130. A fraction *indicates division*, and its **value** is the quotient of the numerator divided by the denominator.

131. A fraction whose numerator is less than its denominator is called a **proper fraction**.

The value of a proper fraction is less than 1.

132. A fraction whose numerator equals or exceeds its denominator is called an **improper fraction**.

The value of an improper fraction is 1 or more than 1.

EXERCISES

133. 1. Change to an integer: $\frac{9}{3}$; $\frac{10}{2}$; $\frac{10}{5}$; $\frac{12}{4}$; $\frac{12}{6}$; $\frac{18}{6}$; $\frac{20}{4}$.

2. Express as a mixed number: $\frac{5}{2}$; $\frac{11}{5}$; $\frac{13}{4}$; $\frac{11}{3}$; $\frac{13}{6}$; $\frac{15}{2}$; $\frac{16}{3}$; $\frac{17}{6}$; $\frac{21}{4}$; $\frac{23}{5}$; $\frac{25}{4}$.

3. Reduce to an integer or a mixed number: $\frac{14}{2}$; $\frac{17}{3}$; $\frac{24}{4}$; $\frac{26}{5}$; $\frac{35}{5}$; $\frac{39}{4}$; $\frac{24}{6}$; $\frac{40}{8}$; $\frac{27}{8}$; $\frac{29}{6}$.

WRITTEN EXERCISES

134. 1. Reduce $\frac{94}{4}$ to a mixed number.

$\frac{94}{4} = 94 \div 4 = 23\frac{2}{4} = 23\frac{1}{2}$

How many fourths are equal to 1? How many times does 94 contain 4? How many ones, then, are there in 94 fourths? how many fourths over? Express $\frac{2}{4}$ in its lowest terms. Then $\frac{94}{4} = 23\frac{1}{2}$.

Reduce to an integer or a mixed number:

2. $\frac{77}{3}$	**5.** $\frac{125}{6}$	**8.** $\frac{226}{12}$	**11.** $\frac{608}{32}$	**14.** $\frac{2000}{125}$
3. $\frac{98}{5}$	**6.** $\frac{294}{8}$	**9.** $\frac{462}{16}$	**12.** $\frac{925}{75}$	**15.** $\frac{2844}{144}$
4. $\frac{84}{4}$	**7.** $\frac{848}{7}$	**10.** $\frac{500}{24}$	**13.** $\frac{876}{96}$	**16.** $\frac{6000}{256}$

135. Reduction to least common denominator.

1. Name some numbers that are divisible by 2; by 3.

2. What is the smallest number that is divisible by both 3 and 2?

136. A number that is divisible by another number is a **multiple** of that number; a number that is divisible by each of several numbers is a **common multiple** of those numbers; and the *least* number that is divisible by each of several numbers is their **least common multiple**. (See Appendix.)

137. 1. Change $\frac{1}{2}$ and $\frac{2}{3}$ each to sixths; to twelfths; to some other *common* denominator.

Which common denominator is the smallest, or *least?*

2. What is the least number that will exactly contain the denominator of each of the fractions $\frac{1}{3}$ and $\frac{3}{4}$?

Reduce $\frac{1}{3}$ and $\frac{3}{4}$ each to twelfths, that is, to fractions having the least common denominator.

138. Fractions that have the same denominator are said to have a **common denominator**, and are called **similar fractions.**

139. Similar fractions that have the smallest common denominator possible are said to have the **least common denominator.**

EXERCISES

140. Reduce to similar fractions:

1. $\frac{1}{2}$ and $\frac{1}{6}$	**4.** $\frac{5}{12}$ and $\frac{1}{2}$	**7.** $\frac{2}{3}$ and $\frac{5}{6}$	**10.** $\frac{3}{4}$ and $\frac{5}{12}$
2. $\frac{3}{8}$ and $\frac{1}{2}$	**5.** $\frac{1}{4}$ and $\frac{5}{8}$	**8.** $\frac{1}{3}$ and $\frac{4}{9}$	**11.** $\frac{4}{5}$ and $\frac{9}{10}$
3. $\frac{1}{2}$ and $\frac{3}{10}$	**6.** $\frac{1}{12}$ and $\frac{1}{4}$	**9.** $\frac{7}{12}$ and $\frac{2}{3}$	**12.** $\frac{11}{12}$ and $\frac{5}{6}$

Reduce to fractions having the least common denominator:

13. $\frac{1}{2}$ and $\frac{3}{5}$	**15.** $\frac{1}{2}, \frac{5}{6}, \frac{1}{12}$	**17.** $\frac{2}{5}, \frac{1}{2}, \frac{9}{10}$
14. $\frac{1}{4}$ and $\frac{1}{6}$	**16.** $\frac{1}{3}, \frac{3}{4}, \frac{5}{12}$	**18.** $\frac{1}{2}, \frac{2}{3}, \frac{7}{12}$

WRITTEN EXERCISES

141. **1.** Reduce $\frac{5}{8}$ and $\frac{7}{12}$ to similar fractions.

$$\frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$

$$\frac{7 \times 2}{12 \times 2} = \frac{14}{24}$$

To change these fractions to similar fractions, a number must be selected for a common denominator that will exactly contain each of the given denominators. It is always desirable to select the smallest number that will contain them.

It is seen that 24 will contain 8 and 12, and that no smaller number will contain both without a remainder.

Since $24 \div 8 = 3$, the terms of $\frac{5}{8}$ must be multiplied by 3, and since $24 \div 12 = 2$, the terms of $\frac{7}{12}$ must be multiplied by 2, giving for results the similar fractions $\frac{15}{24}$ and $\frac{14}{24}$.

Reduce to similar fractions:

2. $\frac{3}{4}$ and $\frac{1}{5}$	**6.** $\frac{5}{8}, \frac{1}{4}, \frac{7}{16}$	**10.** $\frac{1}{2}, \frac{3}{5}, \frac{1}{6}$
3. $\frac{1}{6}$ and $\frac{3}{8}$	**7.** $\frac{1}{3}, \frac{7}{8}, \frac{1}{4}$	**11.** $\frac{3}{4}, \frac{2}{9}, \frac{5}{12}$
4. $\frac{5}{9}$ and $\frac{5}{6}$	**8.** $\frac{4}{5}, \frac{2}{3}, \frac{9}{10}$	**12.** $\frac{2}{5}, \frac{1}{8}, \frac{11}{20}$
5. $\frac{7}{8}$ and $\frac{3}{7}$	**9.** $\frac{3}{4}, \frac{5}{8}, \frac{4}{5}$	**13.** $\frac{7}{8}, \frac{7}{32}, \frac{9}{16}$

142. It is not always easy to discover by inspection the *least common multiple* of the given denominators, that is, the *least common denominator* (l. c. d.).

The l. c. d. may be found, however, by factoring the denominators, for it is the product of all their *different prime* factors, each factor used the greatest number of times that it occurs in any denominator.

Thus, if the given denominators are 6, 4, and 16, factoring we find: $6 = 2 \times 3$; $4 = 2 \times 2$; and $16 = 2 \times 2 \times 2 \times 2$.

Then, the factors of the l. c. d. are 2, 2, 2, 2 (the greatest number of 2's found in any denominator), and 3 (the only factor of any of them not already taken).

Hence, the l. c. d. $= 2 \times 2 \times 2 \times 2 \times 3 = 48$.

WRITTEN EXERCISES

143. 1. Reduce $\frac{7}{12}$, $\frac{5}{8}$, and $\frac{9}{10}$ to fractions having the l. c. d.

Factoring the denominators, $12 = 2 \times 2 \times 3$; $8 = 2 \times 2 \times 2$; and $10 = 2 \times 5$. The factors of the l. c. d., then, are 2, 2, 2, 3, and 5.

Hence the l. c. d. $= 2 \times 2 \times 2 \times 3 \times 5 = 120$, and the fractions become $\frac{70}{120}$, $\frac{75}{120}$, and $\frac{108}{120}$.

NOTE. — Fractions should first be reduced to their lowest terms.

Reduce to fractions having the l. c. d.:

2. $\frac{3}{4}, \frac{7}{12}, \frac{9}{20}$	5. $\frac{4}{15}, \frac{23}{30}, \frac{16}{20}$	8. $\frac{3}{8}, \frac{5}{16}, \frac{3}{4}, \frac{7}{32}$
3. $\frac{2}{5}, \frac{15}{25}, \frac{8}{15}$	6. $\frac{7}{12}, \frac{11}{24}, \frac{13}{18}$	9. $\frac{1}{4}, \frac{2}{3}, \frac{4}{5}, \frac{1}{2}, \frac{5}{6}$
4. $\frac{7}{8}, \frac{9}{14}, \frac{43}{56}$	7. $\frac{5}{36}, \frac{15}{16}, \frac{19}{24}$	10. $\frac{5}{12}, \frac{1}{6}, \frac{1}{3}, \frac{5}{36}, \frac{3}{4}$

Addition and Subtraction of Fractions

144. 1. How many fifths are $\frac{3}{5} + \frac{2}{5}$? how many ones?

2. How many fourths are $\frac{3}{4} - \frac{1}{4}$? how many halves?

3. How many tenths are $\frac{9}{10} + \frac{7}{10}$? how many ones and how many tenths over? how many fifths over?

4. How many eighths are there in $\frac{1}{2} + \frac{3}{8}$? in $\frac{1}{2} - \frac{3}{8}$? in $\frac{5}{8} + \frac{1}{4}$? in $\frac{5}{8} - \frac{1}{4}$? in $\frac{3}{4} + \frac{3}{8}$? in $\frac{3}{4} - \frac{3}{8}$?

5. How much is $\frac{5}{6} + \frac{2}{3}$? $\frac{5}{6} - \frac{2}{3}$? $\frac{1}{2} + \frac{1}{3}$? $\frac{1}{2} - \frac{1}{3}$? $\frac{3}{4} + \frac{2}{3}$? $\frac{3}{4} - \frac{2}{3}$? $\frac{1}{4} + \frac{1}{6}$? $\frac{1}{4} - \frac{1}{6}$?

145. *Fractions must be made similar before they can be added or subtracted.*

EXERCISES

146. Add or subtract as the signs indicate:

1. $\frac{1}{4} + \frac{1}{2}$	5. $\frac{2}{9} + \frac{2}{3}$	9. $\frac{1}{12} + \frac{2}{3}$	13. $\frac{1}{2} + \frac{2}{3}$
2. $\frac{1}{2} - \frac{1}{6}$	6. $\frac{3}{4} - \frac{5}{8}$	10. $\frac{3}{4} - \frac{7}{12}$	14. $\frac{4}{5} - \frac{1}{2}$
3. $\frac{1}{8} + \frac{1}{2}$	7. $\frac{7}{10} + \frac{1}{2}$	11. $\frac{2}{5} + \frac{3}{10}$	15. $\frac{1}{4} + \frac{1}{3}$
4. $\frac{1}{3} - \frac{1}{6}$	8. $\frac{1}{2} - \frac{5}{12}$	12. $\frac{11}{12} - \frac{5}{6}$	16. $\frac{5}{6} - \frac{3}{4}$

First add ; then subtract :

17. $4\frac{3}{4}$ $\underline{2\frac{1}{2}}$	**18.** $7\frac{5}{8}$ $\underline{3\frac{1}{2}}$	**19.** $6\frac{1}{2}$ $\underline{4\frac{1}{10}}$	**20.** $5\frac{7}{12}$ $\underline{1\frac{1}{3}}$	**21.** $8\frac{1}{4}$ $\underline{4\frac{1}{12}}$
22. $9\frac{5}{6}$ $\underline{3\frac{2}{3}}$	**23.** $3\frac{3}{4}$ $\underline{1\frac{3}{8}}$	**24.** $5\frac{11}{12}$ $\underline{2\frac{1}{2}}$	**25.** $7\frac{4}{5}$ $\underline{4\frac{3}{10}}$	**26.** $6\frac{7}{12}$ $\underline{3\frac{1}{6}}$
27. $7\frac{1}{2}$ $\underline{5\frac{1}{3}}$	**28.** $8\frac{3}{5}$ $\underline{3\frac{1}{2}}$	**29.** $10\frac{3}{4}$ $\underline{6\frac{1}{3}}$	**30.** $12\frac{5}{6}$ $\underline{7\frac{1}{4}}$	**31.** $9\frac{9}{10}$ $\underline{4\frac{2}{5}}$

32. A newsboy earned $\$\frac{3}{4}$ and gave his mother $\$\frac{1}{2}$. What part of a dollar did he have left?

33. How wide must a piece of silk be cut to make a band $\frac{1}{2}$ in. wide with $\frac{1}{8}$ in. turned in on each side?

34. Mary spent $\frac{1}{8}$ and $\frac{1}{4}$ of her money. What part of it had she left?

35. Find the perimeter of an envelope 6 in. by $3\frac{1}{4}$ in.

36. A $3\frac{1}{2}$-inch nail is driven through a $1\frac{1}{4}$-inch board into a post. How far does it extend into the post?

WRITTTEN EXERCISES

147. 1. Find the sum of $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$.

$$\frac{5}{6} + \frac{7}{8} + \frac{11}{12} =$$

$$\frac{20}{24} + \frac{21}{24} + \frac{22}{24} = \frac{63}{24} = 2\frac{5}{8}$$

You have learned that the fractions must be changed to similar fractions before they can be added. The l. c. d. of the given fractions is 24; then, $\frac{5}{6} = \frac{20}{24}$, $\frac{7}{8} = \frac{21}{24}$, and $\frac{11}{12} = \frac{22}{24}$. Adding these similar fractions, the sum is found to be $\frac{63}{24} = 2\frac{15}{24}$ or $2\frac{5}{8}$.

Add:

2. $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{12}$	**6.** $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{10}$	**10.** $\frac{2}{3}$, $\frac{4}{5}$, $\frac{1}{2}$, $\frac{5}{6}$
3. $\frac{1}{4}$, $\frac{5}{6}$, $\frac{7}{12}$	**7.** $\frac{5}{6}$, $\frac{2}{3}$, $\frac{9}{16}$	**11.** $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{6}$, $\frac{1}{3}$
4. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{11}{15}$	**8.** $\frac{8}{9}$, $\frac{3}{4}$, $\frac{17}{18}$	**12.** $\frac{2}{3}$, $\frac{3}{8}$, $\frac{5}{6}$, $\frac{3}{4}$
5. $\frac{3}{4}$, $\frac{7}{8}$, $\frac{13}{16}$	**9.** $\frac{7}{8}$, $\frac{5}{16}$, $\frac{11}{12}$	**13.** $\frac{5}{9}$, $\frac{7}{8}$, $\frac{11}{12}$, $\frac{71}{72}$

14. Add $23\frac{1}{2}$, $49\frac{4}{5}$, and $36\frac{3}{4}$.

$$\begin{aligned} 23\tfrac{1}{2} &= 23\tfrac{10}{20} \\ 49\tfrac{4}{5} &= 49\tfrac{16}{20} \\ 36\tfrac{3}{4} &= \underline{36\tfrac{15}{20}} \\ &\ 110\tfrac{1}{20} \end{aligned}$$

Reducing the fractions to fractions having the l. c. d., they become $\frac{10}{20}$, $\frac{16}{20}$, and $\frac{15}{20}$, whose sum is $\frac{41}{20}$ or $2\frac{1}{20}$. Writing $\frac{1}{20}$ under the fractions and adding the 2 to the integers, the whole sum is found to be $110\frac{1}{20}$.

Add:

15.	**16.**	**17.**	**18.**	**19.**
$19\frac{1}{3}$	$25\frac{1}{5}$	$44\frac{1}{2}$	$37\frac{1}{6}$	$56\frac{2}{3}$
$28\frac{1}{2}$	$38\frac{5}{8}$	$68\frac{2}{3}$	$91\frac{1}{2}$	$75\frac{3}{4}$
$32\frac{1}{6}$	$74\frac{1}{2}$	$47\frac{3}{4}$	$84\frac{1}{4}$	$94\frac{5}{6}$

20.	**21.**	**22.**	**23.**	**24.**
$24\frac{1}{2}$	$41\frac{1}{3}$	$96\frac{1}{4}$	$77\frac{2}{3}$	$48\frac{7}{10}$
$85\frac{3}{4}$	$9\frac{1}{2}$	$74\frac{5}{6}$	$13\frac{3}{4}$	$64\frac{7}{12}$
$62\frac{1}{3}$	$58\frac{1}{6}$	$26\frac{2}{3}$	$5\frac{7}{8}$	$92\frac{7}{15}$
$70\frac{1}{6}$	$37\frac{3}{5}$	$42\frac{3}{8}$	$84\frac{1}{10}$	$58\frac{7}{20}$

25. Find the difference between $\frac{7}{8}$ and $\frac{5}{12}$.

$$\tfrac{7}{8} - \tfrac{5}{12} =$$
$$\tfrac{21}{24} - \tfrac{10}{24} = \tfrac{11}{24}$$

We have learned that the fractions must be changed to similar fractions before one can be subtracted from the other. Reducing them to their least common denominator, $\frac{7}{8}$ becomes $\frac{21}{24}$ and $\frac{5}{12}$ becomes $\frac{10}{24}$; then, the difference between $\frac{21}{24}$ and $\frac{10}{24}$ is seen to be $\frac{11}{24}$.

Find the difference between:

26. $\frac{3}{4}$ and $\frac{5}{16}$ **29.** $\frac{4}{5}$ and $\frac{1}{4}$ **32.** $\frac{14}{15}$ and $\frac{1}{2}$ **35.** $\frac{15}{16}$ and $\frac{5}{12}$

27. $\frac{19}{20}$ and $\frac{2}{5}$ **30.** $\frac{3}{8}$ and $\frac{1}{6}$ **33.** $\frac{11}{12}$ and $\frac{1}{8}$ **36.** $\frac{17}{20}$ and $\frac{4}{15}$

28. $\frac{7}{8}$ and $\frac{11}{24}$ **31.** $\frac{5}{6}$ and $\frac{2}{5}$ **34.** $\frac{9}{20}$ and $\frac{1}{4}$ **37.** $\frac{7}{12}$ and $\frac{7}{30}$

38. From $84\frac{3}{8}$ subtract $27\frac{3}{4}$.

$$\begin{aligned} 84\tfrac{3}{8} &= 84\tfrac{3}{8} = 83\tfrac{11}{8} \\ 27\tfrac{3}{4} &= 27\tfrac{6}{8} = \underline{27\tfrac{6}{8}} \\ &\phantom{= 27\tfrac{6}{8} =}\ 56\tfrac{5}{8} \end{aligned}$$

Reducing the fractions to similar fractions, it is seen that $\frac{6}{8}$ cannot be subtracted from $\frac{3}{8}$; hence, 1 is taken from 84, changed to eighths, and combined with $\frac{3}{8}$; then $\frac{6}{8}$ is subtracted from $\frac{11}{8}$ and 27 from 83, giving the remainder $56\frac{5}{8}$.

Subtract:

39. $35\frac{1}{2}$ $\underline{19\frac{3}{4}}$	**43.** $42\frac{3}{10}$ $\underline{26\frac{4}{5}}$	**47.** $77\frac{1}{3}$ $\underline{49\frac{1}{2}}$	**51.** $54\frac{7}{8}$ $\underline{25\frac{1}{6}}$	**55.** $275\frac{3}{8}$ $\underline{184\frac{13}{24}}$
40. $86\frac{1}{6}$ $\underline{22\frac{1}{2}}$	**44.** $68\frac{5}{6}$ $\underline{35\frac{5}{12}}$	**48.** $95\frac{1}{2}$ $\underline{54\frac{4}{5}}$	**52.** $36\frac{1}{4}$ $\underline{17\frac{3}{5}}$	**56.** $347\frac{7}{9}$ $\underline{159\frac{19}{36}}$
41. $57\frac{1}{4}$ $\underline{28\frac{5}{8}}$	**45.** $97\frac{1}{3}$ $\underline{42\frac{7}{12}}$	**49.** $38\frac{2}{3}$ $\underline{15\frac{3}{4}}$	**53.** $71\frac{1}{3}$ $\underline{43\frac{3}{8}}$	**57.** $742\frac{14}{25}$ $\underline{475\frac{33}{50}}$
42. $74\frac{2}{9}$ $\underline{39\frac{2}{3}}$	**46.** $59\frac{3}{4}$ $\underline{21\frac{11}{12}}$	**50.** $82\frac{1}{4}$ $\underline{64\frac{5}{6}}$	**54.** $96\frac{3}{7}$ $\underline{78\frac{3}{4}}$	**58.** $503\frac{7}{20}$ $\underline{324\frac{5}{12}}$

WRITTEN EXERCISES

148. 1. John could run a hundred yards in $14\frac{1}{5}$ seconds and George in $13\frac{4}{5}$ seconds. How much more quickly could George run the distance than John?

2. A wagon with its load weighs $2\frac{1}{4}$ tons. The wagon alone weighs $\frac{3}{5}$ of a ton. Find the weight of the load.

3. A man left $\frac{1}{3}$ of his property to his wife, $\frac{3}{8}$ of it to his daughters, and the rest to his son. What part of the property did the son receive?

4. Add seventy-five and seven eighths, forty-three and five sixths, and ninety-one and five twelfths.

5. Mr. Brown burned $12\frac{3}{4}$ tons of coal in his furnace last winter, $11\frac{13}{20}$ tons the winter before, and $11\frac{4}{5}$ tons the winter before that. How much coal did he use in three winters?

6. From the sum of thirty-nine and four fifths and sixty-two and three eighths subtract their difference.

7. A miller shoveled $38\frac{1}{2}$ bushels of corn from a bin containing $75\frac{3}{8}$ bushels into a bin containing $27\frac{3}{4}$ bushels. How many bushels of corn were there then in each bin?

8. A farmer drew his potatoes to market in four loads, containing, respectively, $60\frac{5}{8}$ bu., $57\frac{3}{4}$ bu., $62\frac{1}{2}$ bu., and $54\frac{7}{8}$ bu. How many bushels did he take to market?

9. If it costs on the average $4\frac{19}{20}$¢ per pound to raise cotton and get it ready for market, what is the profit per pound when it sells at $8\frac{7}{8}$¢? at $9\frac{3}{4}$¢? at $10\frac{3}{16}$¢? at $12\frac{1}{2}$¢? at $13\frac{1}{4}$¢?

Multiplication of Fractions

149. Multiplication of integers by fractions.

1. How many are $\frac{1}{3}$ of 9? $\frac{2}{3}$ of 9? $\frac{1}{4}$ of 12? $\frac{3}{4}$ of 12?

2. Find $\frac{2}{5}$ of 10; $\frac{1}{5}$ of 20. How does $\frac{2}{5}$ of 10 compare with $\frac{1}{5}$ of 20? 2 times $\frac{1}{5}$ of 10 with $\frac{1}{5}$ of 2 times 10?

Tell two ways of finding $\frac{2}{5}$ of 10.

3. Find $\frac{2}{5}$ of 11 by finding $\frac{1}{5}$ of 2 times 11; $\frac{3}{4}$ of 9 by finding $\frac{1}{4}$ of 3 times 9.

4. Find $\frac{2}{3}$ of 8; $\frac{2}{3}$ of 9; $\frac{3}{4}$ of 8; $\frac{3}{4}$ of 9; $\frac{4}{5}$ of 10; $\frac{5}{6}$ of 10.

150. Finding a fractional part of a number is called **multiplying by a fraction.**

WRITTEN EXERCISES

151. 1. Find $\frac{7}{8}$ of 16; $\frac{7}{8}$ of 18.

$\frac{1}{8}$ of $16 = 16 \div 8 = 2$;
$\frac{7}{8}$ of $16 = 7$ times $2 = 14$.

Since 8 is exactly contained in 16, in finding $\frac{7}{8}$ of 16 it is easier and shorter first to *divide* the integer 16 by the *denominator* 8 and then to *multiply* the quotient by the *numerator* 7, obtaining the result 14.

$\frac{7}{8}$ of $18 = \frac{1}{8}$ of $7 \times 18 = \frac{1}{8}$ of 126;
$\frac{1}{8}$ of $126 = 126 \div 8 = 15\frac{6}{8} = 15\frac{3}{4}$.

Since 8 is *not* exactly contained in 18, in finding $\frac{7}{8}$ of 18 it is easier and shorter first to *multiply* the integer 18 by the *numerator* 7 and then to *divide* the product by the *denominator* 8.

It is still shorter to indicate the work and cancel.

$$\frac{7 \times \overset{9}{\cancel{18}}}{\underset{4}{\cancel{8}}} = \frac{63}{4} = 15\frac{3}{4}.$$

Find, in the shortest way:

2. $\frac{3}{4}$ of 36	**6.** $\frac{3}{5}$ of 75	**10.** $\frac{3}{8}$ of 46	**14.** $\frac{2}{9}$ of 81
3. $\frac{3}{4}$ of 38	**7.** $\frac{4}{5}$ of 89	**11.** $\frac{4}{7}$ of 63	**15.** $\frac{7}{8}$ of 98
4. $\frac{2}{3}$ of 96	**8.** $\frac{5}{6}$ of 93	**12.** $\frac{5}{8}$ of 77	**16.** $\frac{8}{10}$ of 85
5. $\frac{2}{3}$ of 97	**9.** $\frac{3}{4}$ of 81	**13.** $\frac{2}{5}$ of 59	**17.** $\frac{5}{12}$ of 92

Multiply, using cancellation where possible:

18. 48 by $\frac{5}{6}$	**23.** 146 by $\frac{2}{3}$	**28.** 295 by $\frac{7}{10}$
19. 50 by $\frac{5}{6}$	**24.** 421 by $\frac{3}{4}$	**29.** 507 by $\frac{5}{12}$
20. 84 by $\frac{3}{8}$	**25.** 937 by $\frac{2}{5}$	**30.** 348 by $\frac{15}{16}$
21. 68 by $\frac{4}{5}$	**26.** 742 by $\frac{5}{8}$	**31.** 864 by $\frac{13}{20}$
22. 76 by $\frac{3}{4}$	**27.** 894 by $\frac{5}{6}$	**32.** 928 by $\frac{17}{24}$

33. Multiply 285 by $27\frac{5}{6}$.

$$
\begin{array}{rl}
 & 285 \\
 & \underline{27\frac{5}{6}} \\
6) & \underline{1425} \\
 & 237\frac{1}{2}, \text{ product by } \frac{5}{6} \\
 & 1995\ \ , \text{ product by } 7 \\
 & \underline{570\ \ \ }, \text{ product by } \underline{20} \\
 & 7932\frac{1}{2}, \text{ product by } 27\frac{5}{6}
\end{array}
$$

Multiplying 285 by the numerator 5 and dividing the product by 6, $237\frac{1}{2}$ is found to be the product by $\frac{5}{6}$.

Multiplying by 7 and by 20 (2 tens), writing these partial products in their proper places as in the process, and then adding, the product of 285 multiplied by $27\frac{5}{6}$ is found to be $7932\frac{1}{2}$.

Multiply:

34. 37 by $17\frac{3}{4}$	**41.** 134 by $36\frac{1}{2}$	**48.** 346 by $125\frac{2}{3}$
35. 46 by $25\frac{2}{3}$	**42.** 341 by $91\frac{1}{4}$	**49.** 517 by $206\frac{3}{4}$
36. 54 by $42\frac{3}{5}$	**43.** 604 by $43\frac{2}{3}$	**50.** 628 by $409\frac{4}{5}$
37. 79 by $38\frac{1}{6}$	**44.** 467 by $56\frac{3}{4}$	**51.** 497 by $342\frac{1}{6}$
38. 294 by $18\frac{1}{5}$	**45.** 528 by $87\frac{2}{5}$	**52.** 836 by $570\frac{5}{8}$
39. 723 by $26\frac{3}{8}$	**46.** 279 by $68\frac{5}{6}$	**53.** 288 by $735\frac{7}{10}$
40. 618 by $17\frac{5}{6}$	**47.** 842 by $74\frac{7}{8}$	**54.** 969 by $678\frac{11}{12}$

EXERCISES

152. Find the cost of:

1. $\frac{3}{4}$ pound of ginger at 28 cents a pound.
2. $2\frac{1}{2}$ pounds of coffee at 30 cents a pound.
3. $1\frac{1}{4}$ gallons of vinegar at 24 cents a gallon.
4. $3\frac{2}{3}$ dozen bananas at 12 cents a dozen.
5. $3\frac{1}{2}$ pounds of figs at 16 cents a pound.
6. $1\frac{3}{4}$ gallons of molasses at 48 cents a gallon.
7. $4\frac{1}{2}$ pounds of cheese at 12 cents a pound.
8. If I am on a train that goes at the rate of 40 miles an hour, how far do I travel in $1\frac{1}{2}$ hours?
9. How far is it from Hurst to Troy, if John uses up $\frac{2}{5}$ of a thousand-mile railroad book in traveling that distance?
10. If it takes Mary 45 minutes to walk to school and Clara $\frac{2}{3}$ as long, how many minutes does it take Clara?
11. A lesson in arithmetic contained 24 exercises. Elmer solved $\frac{5}{6}$ of them. How many did he solve?
12. How many cents are there in $\frac{3}{5}$ of a dollar?
13. A child should sleep at least $\frac{3}{8}$ of the time. How many of the 24 hours in a day should every child sleep?
14. A farmer who had 21 sheep sold $\frac{1}{3}$ of them at \$4 each. How much did he receive for the sheep he sold?
15. Eleanor bought $4\frac{1}{4}$ yards of ribbon at 20 cents a yard. How much did it cost her?
16. Mr. Williams paid 32 cents a bushel for a bag of oats containing $2\frac{1}{8}$ bushels. How much did the oats cost?
17. I bought $1\frac{3}{4}$ pounds of 40-cent tea and handed the grocer a dollar. How much change did he give me?
18. Find the cost of $5\frac{1}{2}$ yards of percale at 12¢ a yard.

WRITTEN EXERCISES

153. Find the cost of:

1. 25 shovels at $\$\frac{3}{4}$ each.

SUGGESTION. — At $\$\frac{3}{4}$ each the cost is $\frac{3}{4}$ of what it would be at \$1 each.

2. 278 barrels of cement at $\$1\frac{1}{4}$ a barrel.

3. $6\frac{2}{5}$ tons of coal at \$6.25 a ton.

4. $37\frac{5}{8}$ acres of land at \$244 an acre.

5. 465 bushels of apples at $\$\frac{7}{10}$ a bushel.

6. $4\frac{5}{6}$ miles of wire fencing at \$174 per mile.

7. 2548 pounds of iron pipe at $8\frac{3}{4}$ cents a pound.

8. 32,864 pounds of cotton at $9\frac{7}{16}$ cents a pound.

9. If it requires $1\frac{3}{4}$ bushels of seed wheat to sow one acre, how many bushels are needed to sow 26 acres?

10. An apple tree yielded $12\frac{3}{8}$ bushels of apples that sold for 72 cents a bushel. Find the income from this tree.

11. A meadow of 96 acres produced on the average $1\frac{4}{5}$ tons of hay per acre. What was the total production?

12. A man rented a house at the rate of \$480 a year. He lived in it $4\frac{5}{12}$ years. How much rent did he pay?

13. A chimney measuring 150 feet from the bottom of its foundation is $\frac{7}{8}$ above ground. How high is it above ground?

14. If Ruth has read $\frac{9}{14}$ of the 336 pages in her book, how many pages has she read?

15. At $\$\frac{9}{10}$ a foot how much will it cost to drill an oil well 2256 feet deep?

16. Each sheep of a flock of 440 yielded on the average $3\frac{15}{16}$ pounds of wool. Find the total yield.

17. Find the total cost of a crop of coffee from a plantation of 24,000 trees that yield an average of $1\frac{3}{5}$ pounds each, if the cost of raising is $4\frac{3}{8}$ cents per pound.

154. Multiplication of fractions by integers.

1. How many fifths are 3 times 1 fifth? 3 times $\frac{1}{5} = ?$

2. How many eighths are 4 times $\frac{1}{8}$? Express the result in its lowest terms.

You have multiplied $\frac{1}{8}$ by 4, obtaining the product $\frac{1}{2}$. Tell how you did it.

3. Divide the denominator of $\frac{1}{8}$ by 4. How does the result compare with the product found by multiplying $\frac{1}{8}$ by 4?

In what other way, then, may we sometimes multiply a fraction by an integer?

4. Multiply $\frac{1}{4}$ by 2 in two ways; $\frac{5}{6}$ by 3; $\frac{3}{8}$ by 2; $\frac{2}{9}$ by 3; $\frac{5}{12}$ by 6.

5. Multiply $\frac{1}{2}$ by 3; $\frac{2}{3}$ by 4; $\frac{3}{4}$ by 5; $\frac{4}{5}$ by 3; $\frac{7}{8}$ by 5.

6. Find 2 times 4; 2 times $\frac{1}{3}$; 2 times $4\frac{1}{3}$; 3 times $2\frac{1}{2}$; $5 \times 3\frac{1}{3}$; $6 \times 2\frac{1}{4}$; $3 \times 5\frac{2}{3}$.

155. *Multiplying the numerator or dividing the denominator of a fraction by an integer multiplies the fraction by that integer.*

EXERCISES

156. Multiply:

1. $\frac{2}{3}$ by 8	4. $\frac{5}{6}$ by 2	7. $\frac{3}{10}$ by 5	10. $5\frac{1}{2}$ by 4
2. $\frac{2}{5}$ by 4	5. $\frac{7}{8}$ by 4	8. $\frac{7}{12}$ by 4	11. $2\frac{3}{4}$ by 2
3. $\frac{3}{4}$ by 2	6. $\frac{8}{9}$ by 3	9. $\frac{9}{16}$ by 8	12. $7\frac{1}{6}$ by 6

13. At $\$\frac{2}{5}$ a yard, how much will 4 yards of cloth cost?

14. A family uses $\frac{3}{8}$ of a pound of butter at each meal. How much butter is used at the table each day?

15. Dr. Jones pays $\$3\frac{1}{2}$ a month for his telephone. How much does it cost him a year?

16. If a man earns $\$2\frac{3}{4}$ a day, how much does he earn in a week of 6 working days?

17. How far can James walk in 4 hours, walking $2\frac{7}{8}$ miles per hour?

18. Mr. Seymour bought a set of 6 books for his library. How much did they cost, if the price was $\$1\frac{3}{4}$ each?

19. George earned $\$2\frac{7}{10}$ a week delivering papers. How much did he earn in 4 weeks?

20. Find the cost of 5 tons of coal at $\$6\frac{1}{4}$ a ton; 3 barrels of flour at $\$5\frac{4}{5}$ a barrel.

21. If the cost of 4 yards of ribbon is $\$1\frac{2}{5}$, what is the cost of 12 yards of the same kind?

SOLUTION. 12 is three times 4; then, the cost of 12 yards is 3 times the cost of 4 yards; that is, the cost of 12 yards is 3 times $\$1\frac{2}{5}$ or $\$4\frac{1}{5}$.

22. If a girl in a factory receives $\$1\frac{3}{4}$ for making 4 shirtwaists, how much will she receive for making 12 shirtwaists?

23. If she makes 3 shirtwaists in $10\frac{1}{2}$ hours, how long will it take her to make 12 shirtwaists?

24. A man paid $\$3\frac{1}{2}$ for the use of a boat for 7 days. At that rate how much would it cost to hire it for 21 days?

25. A farmer sold 6 bushels of oats for $\$2\frac{1}{10}$. At that rate how much should he receive for 48 bushels?

26. A fruit dealer sold 4 dozen oranges for $\$1\frac{1}{5}$. How much should he receive for 16 dozen at the same rate?

WRITTEN EXERCISES

157. 1. Find 8 times $\frac{13}{24}$.

$$8 \times \frac{13}{24} = \frac{\cancel{8} \times 13}{\underset{3}{\cancel{24}}} = \frac{13}{3} = 4\frac{1}{3}$$

We have learned that to find 8 times $\frac{13}{24}$ we may multiply the numerator 13 by 8. Indicating this as in the process, canceling, and reducing, the result is found to be $4\frac{1}{3}$.

Find:

2. $5 \times \frac{11}{25}$
3. $6 \times \frac{15}{16}$
4. $8 \times \frac{13}{18}$
5. $7 \times \frac{17}{28}$
6. $4 \times \frac{21}{32}$
7. $9 \times \frac{38}{60}$
8. $7 \times \frac{33}{42}$
9. $8 \times \frac{65}{72}$

Multiply:

10. $\frac{9}{10}$ by 85
11. $\frac{8}{21}$ by 49
12. $\frac{24}{35}$ by 28
13. $\frac{25}{96}$ by 72
14. $\frac{7}{88}$ by 33
15. $\frac{19}{100}$ by 75
16. $\frac{66}{132}$ by 96
17. $\frac{87}{200}$ by 80

18. Multiply $45\frac{3}{8}$ by 26.

$$\begin{array}{r} 45\frac{3}{8} \\ 26 \\ \hline 9\frac{3}{4} \\ 270\phantom{\frac{3}{4}} \\ 90\phantom{0\frac{3}{4}} \\ \hline 1179\frac{3}{4} \end{array}$$

Multiplying the fraction $\frac{3}{8}$ by 26, the product is found to be $9\frac{3}{4}$.

Multiplying the integer 45 by 6 and by 20 (2 tens), writing these partial products in their proper places and adding, the product of $45\frac{3}{8}$ multiplied by 26 is found to be $1179\frac{3}{4}$.

Multiply:

19. $68\frac{2}{3}$ by 14
20. $94\frac{3}{5}$ by 57
21. $247\frac{3}{4}$ by 6
22. $468\frac{5}{6}$ by 9
23. $306\frac{1}{2}$ by 27
24. $184\frac{7}{8}$ by 78
25. $536\frac{1}{6}$ by 49
26. $845\frac{4}{5}$ by 96
27. $628\frac{5}{8}$ by 124
28. $459\frac{3}{4}$ by 205
29. $738\frac{7}{10}$ by 462
30. $906\frac{5}{12}$ by 369

WRITTEN EXERCISES

158. Find the cost of:

NOTE. — For any part of a cent the seller collects a whole cent.

1. 5 pounds of butter at $23\frac{3}{4}$ cents a pound.
2. 8 cans of tomatoes at $16\frac{2}{3}$ cents a can.
3. 2 turkeys weighing $14\frac{1}{2}$ lb. and $16\frac{3}{4}$ lb. at 24¢ a pound.
4. $18\frac{3}{4}$ yards of velvet at $3 a yard.
5. $24\frac{5}{6}$ dozen buttons at 36 cents a dozen.
6. $85\frac{7}{8}$ yards of matting at 42 cents a yard.
7. A cubic foot of water weighs $62\frac{1}{2}$ pounds. Find the weight of 25 cubic feet of water.
8. If a steamer burns $457\frac{3}{5}$ tons of coal a day, how many tons will it burn on a voyage of 7 days?

9. How far will a train run in 18 hours at the rate of $53\frac{3}{8}$ miles an hour?

10. When potatoes are selling so that a man gets \$$.37\frac{1}{2}$ for a load of 50 bushels, how much is his crop of 450 bushels worth?

11. Mr. Joy has 12 bins in his granary, and they hold $192\frac{7}{8}$ bushels each. How many bushels of grain do they all hold?

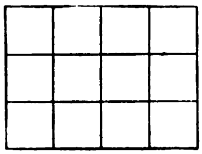
12. If the multiplicand is forty-two and five twelfths, and the multiplier thirty-six, what is the product?

13. A farmer had 6 stacks of hay that contained on the average $16\frac{5}{8}$ tons each. How much was the hay worth at \$14 a ton?

14. Mr. Holmes had a farm of $125\frac{3}{4}$ acres, and he bought an adjoining lot of $86\frac{4}{5}$ acres. How much was the whole farm then worth at \$75 an acre?

Division of Fractions

159. Finding what part one number is of another.



1. How many horizontal rows of squares are there in this oblong? What part of the oblong is one row?

How many squares are there in the oblong? in one row? Then what part of 12 is 4?

How does this result compare with $4 \div 12$, or with $\frac{4}{12}$ reduced to its lowest terms?

Then how may we find what fractional part 4 is of 12 without looking at the diagram?

2. What part of 12 is 8? 3? 6? 9? 10?

3. What part of 16 is 8? 4? 2? What part of 20 is 4? 8? 12? 5? 10? 15?

160. To find what part the second of two numbers is of the first, *the second is divided by the first.*

EXERCISES

161. What part of

1. 18 is 6?	4. 15 is 10?	7. 24 is 12?
2. 24 is 8?	5. 16 is 12?	8. 30 is 20?
3. 30 is 5?	6. 18 is 15?	9. 36 is 18?

10. A boy who had 20 problems to work solved 10 of them. What part of his problems did he solve?

11. A man had 10 miles to drive. When he had driven 8 miles, what part of his journey had he completed?

12. What part of 12 is 4? If 12 spools of thread cost 60 cents, what part of 60 cents will 4 spools cost? how much will 4 spools cost?

13. What part of 15 is 6? If 15 apples cost 25 cents, how much will 6 apples cost?

14. Find the cost of 20 marbles, if 30 marbles cost 12 cents.

15. What part of 75 is 25? If 6 dozen bananas can be bought for 75 cents, what part of 6 dozen can be bought for 25 cents? how many dozen bananas?

16. What part of 50 is 30? If 10 bottles of ink cost 50 cents, how many can be bought for 30 cents?

17. Find how many sheep can be bought for $12, if 9 sheep cost $36.

WRITTEN EXERCISES

162. What part of

1. 48 is 36?	4. 120 is 90?	7. 250 is 100?
2. 64 is 16?	5. 144 is 84?	8. 432 is 288?
3. 96 is 72?	6. 225 is 75?	9. 768 is 336?

10. A farmer raised 180 bushels of oats and sold 120 bushels. What part of his crop did he sell?

11. A grocer bought 400 dozen eggs. When he had sold 144 dozen, what part of them had he sold?

12. Of the 375 bushels of potatoes that Mr. Avery raised 125 bushels decayed. What part of his crop decayed?

13. What part of 64 is 40? I sold 64 head of cattle for $1472. At this rate how much were 40 head worth?

14. If a train runs 860 miles in 20 hours, how far at the same rate will it run in 12 hours?

15. When 28 acres of land sell for $2688, how much are 21 acres worth at the same rate?

16. If a man earns $3570 in 42 months, how much does he earn in 12 months?

17. Mr. Doty sold 64 tons of hay for $1004.80. At that rate, find the value of the rest of his crop, 24 tons.

163. Division of fractions by integers.

1. If $\frac{4}{5}$ of an orange is divided equally between 2 girls, what part of the orange will each girl have? $\frac{4}{5} \div 2 = ?$

In dividing $\frac{4}{5}$ by 2, which term of the fraction is divided?

2. Multiply the denominator of $\frac{4}{5}$ by 2. Express the resulting fraction in its lowest terms. Compare this result with the quotient of $\frac{4}{5}$ divided by 2.

3. In what two ways, then, can $\frac{4}{5}$ be divided by 2?

4. Divide $\frac{3}{4}$ by 3 in two ways; $\frac{4}{6}$ by 2; $\frac{6}{8}$ by 3; $\frac{8}{9}$ by 4; $\frac{10}{12}$ by 5.

5. Find $\frac{2}{3} \div 4$; $\frac{4}{5} \div 6$; $\frac{9}{10} \div 3$; $\frac{7}{8} \div 2$.

6. How many fourths are $2\frac{3}{4}$? Divide $\frac{11}{4}$ by 3. Then how may we divide $2\frac{3}{4}$ by 3?

7. Divide $1\frac{2}{3}$ by 4; $2\frac{1}{2}$ by 3; $4\frac{1}{4}$ by 2; $1\frac{7}{8}$ by 3; $3\frac{3}{4}$ by 5; $3\frac{3}{5}$ by 6.

164. *Dividing the numerator or multiplying the denominator of a fraction by an integer divides the fraction by that integer.*

Always divide the numerator when it is divisible by the integer.

EXERCISES

165. Divide:

1. $\frac{2}{3}$ by 6	**4.** $\frac{5}{6}$ by 5	**7.** $1\frac{4}{5}$ by 3	**10.** $5\frac{1}{4}$ by 7
2. $\frac{1}{4}$ by 2	**5.** $\frac{3}{8}$ by 6	**8.** $3\frac{1}{3}$ by 2	**11.** $4\frac{1}{6}$ by 5
3. $\frac{3}{5}$ by 3	**6.** $\frac{3}{4}$ by 9	**9.** $7\frac{1}{2}$ by 5	**12.** $3\frac{3}{8}$ by 9

13. If $\frac{7}{8}$ of a bushel of oats will feed a horse 7 times, what part of a bushel will feed him once?

14. If $\$\frac{1}{2}$ is divided equally among 5 boys, what part of a dollar will each boy receive?

15. When 6 dozen eggs cost $\$1\frac{1}{5}$, what is the price per dozen?

16. Eleanor paid $\$2\frac{1}{4}$ for 9 yards of challis. How much did it cost her per yard?

17. Frederick earned $\$5\frac{2}{5}$ in 6 days. How much did he earn per day?

WRITTEN EXERCISES

166. 1. Divide $\frac{24}{25}$ by 8; $\frac{7}{8}$ by 6; $\frac{15}{16}$ by 10.

$$\frac{24}{25} \div 8 = \frac{24 \div 8}{25} = \frac{3}{25}$$

You have learned that dividing the numerator of a fraction divides the fraction; then, since the numerator 24 is exactly divisible by the integer 8, you should divide the numerator by 8. The result is $\frac{3}{25}$.

$$\frac{7}{8} \div 6 = \frac{7}{6 \times 8} = \frac{7}{48}$$

You have learned that multiplying the denominator of a fraction divides the fraction; then, since the numerator 7 is *not* divisible by the integer 6, you should multiply the denominator by 6. The result is $\frac{7}{48}$.

$$\frac{15}{16} \div 10 = \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{10}} \times 16} = \frac{3}{32}$$

Usually the *shortest* way to divide a fraction by an integer is to indicate the multiplication of the denominator by the integer and then cancel whenever possible. Indicating and canceling, the result is $\frac{3}{32}$.

Divide:

2. $\frac{11}{12}$ by 4	**6.** $\frac{3}{4}$ by 24	**10.** $\frac{13}{16}$ by 26
3. $\frac{20}{21}$ by 5	**7.** $\frac{4}{5}$ by 32	**11.** $\frac{5}{18}$ by 25
4. $\frac{24}{35}$ by 8	**8.** $\frac{5}{6}$ by 40	**12.** $\frac{19}{20}$ by 38
5. $\frac{36}{42}$ by 9	**9.** $\frac{7}{8}$ by 42	**13.** $\frac{22}{48}$ by 33

14. Divide $13\frac{3}{4}$ by 5;

$$13\frac{3}{4} = \frac{55}{4}$$
$$\frac{55}{4} \div 5 = \frac{11}{4} = 2\frac{3}{4}$$

The integral part of the dividend is *small;* in such a case the mixed number may be reduced to an improper fraction and the division performed as before.

The quotient is $2\frac{3}{4}$.

$213\frac{3}{4}$ by 5.

$$\begin{array}{r} 5\underline{)213\frac{3}{4}} \\ 42\frac{3}{4} \end{array}$$

The integral part of the dividend is *large* and the divisor is small; in this case short division may be used: thus, 5 is contained in $213\frac{3}{4}$, 42 times with $3\frac{3}{4}$, or $\frac{15}{4}$, undivided; $\frac{15}{4} \div 5 = \frac{3}{4}$; then the entire quotient is $42\frac{3}{4}$.

Divide:

15. $18\frac{2}{3}$ by 4
16. $12\frac{5}{6}$ by 7
17. $21\frac{3}{8}$ by 6
18. $53\frac{3}{4}$ by 5
19. $241\frac{1}{2}$ by 3
20. $474\frac{2}{5}$ by 8
21. $738\frac{4}{7}$ by 5
22. $815\frac{5}{8}$ by 9
23. $1347\frac{1}{6}$ by 2
24. $3476\frac{1}{4}$ by 6
25. $5286\frac{7}{8}$ by 9
26. $6304\frac{2}{7}$ by 5

27. Divide $768\frac{3}{4}$ by 15.

$$\begin{array}{l} 15)\ 768\frac{3}{4} \\ \underline{\ 4 \quad\ \ 4\ } \\ 60)3075(51\frac{1}{4} \\ \quad\ \underline{300} \\ \qquad 75 \\ \qquad \underline{60} \\ \qquad \frac{15}{60} = \frac{1}{4} \end{array}$$

Here it is necessary to use long division. Before dividing, change both divisor and dividend to fourths, obtaining 60 (fourths) and 3075 (fourths), respectively.

Dividing as in integers, the quotient is found to be $51\frac{1}{4}$.

Divide:

28. $246\frac{2}{3}$ by 16
29. $597\frac{3}{5}$ by 24
30. $789\frac{3}{4}$ by 18
31. $408\frac{4}{5}$ by 28
32. $376\frac{5}{6}$ by 21
33. $613\frac{3}{8}$ by 35
34. $54\frac{7}{12}$ by 25
35. $942\frac{6}{7}$ by 75
36. $2619\frac{3}{4}$ by 56
37. $4294\frac{2}{5}$ by 48
38. $9877\frac{7}{8}$ by 63
39. $5054\frac{2}{9}$ by 32

WRITTEN EXERCISES

167. Find in the shortest way:

1. $\frac{7}{12} \div 5$	**6.** $372\frac{1}{3} \div 6$	**11.** $2796\frac{2}{3} \div 4$
2. $\frac{24}{25} \div 8$	**7.** $654\frac{1}{4} \div 9$	**12.** $4271\frac{3}{4} \div 7$
3. $9\frac{3}{8} \div 25$	**8.** $536\frac{5}{8} \div 27$	**13.** $8429\frac{2}{5} \div 9$
4. $\frac{14}{15} \div 21$	**9.** $743\frac{1}{6} \div 35$	**14.** $5686\frac{7}{8} \div 45$
5. $8\frac{2}{5} \div 12$	**10.** $942\frac{6}{7} \div 44$	**15.** $829\frac{7}{12} \div 55$

Find the cost of each when

16. 18 chafing dishes cost $\$94\frac{1}{2}$.

17. 36 alarm clocks cost $\$82\frac{4}{5}$.

18. 25 mantel clocks cost $\$78\frac{3}{4}$.

19. 24 cabinet clocks cost $\$87\frac{3}{5}$.

20. 21 opera glasses cost $\$186\frac{9}{10}$.

21. 15 open-face watches cost $\$521\frac{1}{4}$.

22. 33 velour couches cost $\$874\frac{1}{2}$.

23. 52 sewing machines cost $\$998\frac{2}{5}$.

24. 45 cooking stoves cost $\$1091\frac{1}{4}$.

25. 71 nickeled parlor stoves cost $\$1370\frac{3}{10}$.

26. If a train runs $19\frac{1}{5}$ miles in 24 minutes, how far does it run in 1 minute?

27. A bicycle rider went around a track 51 times, thus riding a distance of $12\frac{3}{4}$ miles. How far is it around the track?

28. Mr. Hobbs received $\$1698\frac{3}{4}$ for 45 cows. What was the average price per cow?

29. The receipts of an excursion were $\$3307\frac{1}{2}$. If 945 tickets were sold, what was the fare?

30. If $123\frac{1}{4}$ quarts of milk make 17 pounds of butter, how many quarts of milk are required to make 1 pound of butter?

168. Division of integers by fractions.

1. How many times is $\$\frac{1}{4}$ contained in $1? in $2? in $3? Then $1 \div \frac{1}{4} =$ how many? $2 \div \frac{1}{4} = ?$ $3 \div \frac{1}{4} = ?$

2. What is the value of $1 \div \frac{1}{5}$? $2 \div \frac{1}{5}$? $1 \div \frac{1}{6}$? $3 \div \frac{1}{6}$? $1 \div \frac{1}{3}$? $5 \div \frac{1}{3}$? $4 \div \frac{1}{2}$?

3. Compare the result of $2 \div \frac{1}{3}$ with 2 multiplied by 3; $5 \div \frac{1}{4}$ with 5 multiplied by the denominator of $\frac{1}{4}$. Then how can you divide an integer by a fraction whose numerator is 1?

4. Divide 6 by 1; by 2; by 3. Notice that *as the divisor becomes greater the quotient becomes smaller.*

In $6 \div \frac{1}{5}$ and $6 \div \frac{2}{5}$, which divisor is greater? Which quotient, then, is greater? How can you find the second quotient from the first?

5. What part of $3 \div \frac{1}{4}$ is $3 \div \frac{3}{4}$? From $3 \div \frac{1}{4}$ find $3 \div \frac{3}{4}$.

6. By what number must you divide the result of $2 \div \frac{1}{6}$ to find $2 \div \frac{5}{6}$? Find $2 \div \frac{5}{6}$.

7. Divide 4 by $\frac{2}{3}$; 6 by $\frac{3}{4}$; 3 by $\frac{4}{5}$; 4 by $\frac{5}{6}$.

169. *An integer is divided by a fraction by multiplying the integer by the denominator of the fraction and dividing the product by the numerator.*

EXERCISES

170. Divide:

1. 2 by $\frac{1}{4}$
2. 4 by $\frac{1}{3}$
3. 5 by $\frac{1}{2}$
4. 6 by $\frac{2}{3}$
5. 9 by $\frac{3}{4}$
6. 8 by $\frac{4}{5}$

Find:

7. $5 \div \frac{1}{7}$
8. $4 \div \frac{1}{8}$
9. $7 \div \frac{1}{6}$
10. $2 \div \frac{3}{8}$
11. $6 \div \frac{4}{7}$
12. $8 \div \frac{5}{6}$

13. When tea is worth $\$\frac{1}{2}$ a pound, how many pounds can be bought for $3?

14. How many dozen eggs can be bought for $2 at $\$\frac{1}{4}$ a dozen?

15. A man pays $\$\frac{3}{4}$ a day for his board. How many days can he board for \$6?

16. If a cobbler can heel a pair of shoes in $\frac{4}{5}$ of an hour, how many pairs can he heel in 8 hours?

17. Mr. Hay divided 5 pounds of candy among his children, giving each $\frac{5}{8}$ of a pound. How many children had he?

18. How many steps will a man take in walking 10 yards, if his step is $\frac{5}{6}$ of a yard long?

171. 1. Divide 1 by $\frac{3}{4}$ and express the quotient as a fraction.

$1 \div \frac{3}{4} = ?$ $\qquad 1 \div \frac{4}{5} = ?$ $\qquad 1 \div \frac{2}{3} = ?$ $\qquad 1 \div \frac{7}{6} = ?$

2. The fraction $\frac{4}{3}$ is the *reciprocal* of $\frac{3}{4}$; $\frac{5}{4}$ is the reciprocal of $\frac{4}{5}$. What is the reciprocal of $\frac{2}{3}$? of $\frac{7}{6}$?

172. The **reciprocal** of a fraction is 1 divided by the fraction, or it is the fraction inverted.

WRITTEN EXERCISES

173. 1. Divide 18 by $\frac{4}{5}$.

$$18 \div \frac{4}{5} = \frac{\overset{9}{\cancel{18}} \times 5}{\underset{2}{\cancel{4}}} = \frac{45}{2} = 22\frac{1}{2}$$

Since $1 \div \frac{1}{5} = 5$, $18 \div \frac{1}{5} = 18 \times 5$; then, $18 \div \frac{4}{5} = \frac{1}{4}$ of 18×5 (§ 168, 4), or $\frac{18 \times 5}{4}$. Canceling and reducing, the result is found to be $22\frac{1}{2}$.

Observe that dividing 18 by $\frac{4}{5}$ is the same as multiplying 18 by $\frac{5}{4}$. That is, dividing an integer by a fraction is the same as *multiplying the integer by the reciprocal of the fraction.*

Divide: | | Find: |
|---|---|---|---|
| **2.** 15 by $\frac{2}{3}$ | **6.** 45 by $\frac{6}{7}$ | **10.** $78 \div \frac{9}{10}$ | **14.** $245 \div \frac{14}{15}$ |
| **3.** 24 by $\frac{3}{4}$ | **7.** 62 by $\frac{3}{5}$ | **11.** $81 \div \frac{5}{12}$ | **15.** $456 \div \frac{10}{21}$ |
| **4.** 28 by $\frac{2}{5}$ | **8.** 57 by $\frac{7}{8}$ | **12.** $95 \div \frac{15}{16}$ | **16.** $372 \div \frac{16}{5}$ |
| **5.** 35 by $\frac{5}{6}$ | **9.** 68 by $\frac{8}{9}$ | **13.** $63 \div \frac{18}{25}$ | **17.** $657 \div \frac{27}{4}$ |

18. Divide 28 by $4\frac{3}{8}$.

SUGGESTION. — Reduce the mixed number to an improper fraction and divide as before.

Divide:

19. 21 by $4\frac{1}{2}$	**22.** 48 by $3\frac{3}{5}$	**25.** 126 by $6\frac{3}{4}$
20. 39 by $2\frac{1}{4}$	**23.** 65 by $4\frac{1}{6}$	**26.** 395 by $5\frac{5}{6}$
21. 44 by $6\frac{2}{3}$	**24.** 90 by $1\frac{7}{8}$	**27.** 625 by $9\frac{3}{8}$

28. Divide 225 by $18\frac{3}{4}$.

$$\begin{array}{l} 18\frac{3}{4})225 \\ \underline{\ \ 4 \quad\ \ 4} \\ 75\ )900(12 \end{array}$$

By changing both dividend and divisor to *fourths*, the numerator 900 may be divided by the numerator 75 as in integers.

In some cases, especially when the divisor is large, it is convenient to employ this method.

Divide:

29. $345 \div 26\frac{2}{3}$	**31.** $725 \div 45\frac{5}{6}$	**33.** $609 \div 14\frac{7}{10}$
30. $528 \div 38\frac{2}{5}$	**32.** $910 \div 56\frac{7}{8}$	**34.** $825 \div 27\frac{1}{12}$

WRITTEN EXERCISES

174. Find by the method best adapted:

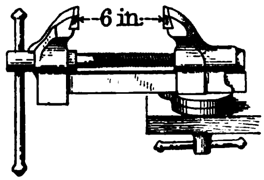
1. $17 \div \frac{1}{4}$	**5.** $164 \div \frac{6}{7}$	**9.** $775 \div 20\frac{5}{6}$
2. $31 \div \frac{2}{3}$	**6.** $327 \div \frac{7}{8}$	**10.** $925 \div 34\frac{3}{8}$
3. $64 \div \frac{5}{12}$	**7.** $462 \div \frac{15}{16}$	**11.** $2396 \div 6\frac{2}{5}$
4. $91 \div 8\frac{1}{6}$	**8.** $695 \div 3\frac{3}{4}$	**12.** $5285 \div 46\frac{2}{3}$

13. There are $16\frac{1}{2}$ feet in a rod. How many rods are there in 792 feet?

14. If a glacier in the Alps moves at an average rate of $1\frac{3}{8}$ feet a day, how many days will it take to move 660 feet, or an eighth of a mile?

15. How many pounds of cotton must a man pick to earn \$1, if he receives \$$28\frac{1}{8}$ for picking 7200 pounds?

16. If each turn of the screw brings the jaws of this vise $\frac{3}{16}$ in. nearer together, how many turns will it take to close the vise?



17. If it requires $4\frac{3}{4}$ yards of silk to make a waist, how many waists can be made from a piece containing 38 yards?

18. A barrel of flour weighs 196 pounds. If it takes $\frac{3}{4}$ of a pound for a loaf of bread, how many loaves can a baker make out of 3 barrels of flour?

19. A man and his wife took a trip in a gasoline launch, sailing 6 hours a day. The launch went at the rate of $8\frac{5}{8}$ miles an hour, and covered a distance of 1311 miles. How many days were they on the trip?

175. Finding the whole when a fractional part of it is given.

EXERCISES

1. If $\frac{3}{5}$ of the cost of a ball is 30 cents, find the whole cost.

SOLUTION.
$\frac{3}{5}$ of the cost = 30¢.
$\frac{1}{5}$ of the cost = $\frac{1}{3}$ of 30¢, or 10¢.
$\frac{5}{5}$ of the cost = 5×10¢, or 50¢.

2. Helen paid 12 cents for $\frac{3}{4}$ of a pound of mixed nuts. At that rate how much would a pound cost?

3. Our baseball team won $\frac{2}{3}$ of the games that it played. If it won 14 games, how many did it play?

4. How deep is a well that goes through 18 feet of rock, if $\frac{3}{8}$ of its depth is through rock?

5. If $\frac{5}{6}$ of the passengers on a car were men, and there were 35 men, how many passengers were there?

6. A football player ran $\frac{3}{5}$ of the length of the field. If he ran 66 yards, how long was the field?

WRITTEN EXERCISES

176. **1.** If 679 is $\frac{7}{8}$ of a number, what is the number?

SOLUTION. $\frac{7}{8}$ of the number $= 679$.

$\frac{1}{8}$ of the number $= \frac{1}{7}$ of 679, or 97.

$\frac{8}{8}$ of the number $= 8 \times 97$, or 776.

Or the work may be done more quickly by indicating it and then canceling; thus:

$$679 \div \frac{7}{8} = \frac{\overset{97}{\cancel{679}} \times 8}{\cancel{7}} = 776.$$

Find the number of which

2. 68 is $\frac{2}{3}$	**5.** 224 is $\frac{7}{10}$	**8.** 1347 is $\frac{3}{8}$
3. 81 is $\frac{3}{5}$	**6.** 605 is $\frac{11}{12}$	**9.** 4684 is $\frac{4}{5}$
4. 95 is $\frac{5}{6}$	**7.** 747 is $\frac{9}{16}$	**10.** 8298 is $\frac{6}{7}$

11. How far is it from Chicago to Detroit, if $\frac{3}{5}$ of the distance is 171 miles?

12. A man's expenses are $\frac{4}{5}$ of his earnings. If his expenses are $816 a year, how much does he earn?

13. If a steer when killed and dressed weighs $\frac{7}{10}$ as much as when alive, what was the live weight of one that dressed 742 pounds?

14. If a duck, flying $\frac{3}{5}$ as fast as a hawk, flies 90 miles in an hour, how fast does the hawk fly?

15. A printing press can print $\frac{3}{8}$ as many copies of a 20-page paper as of an 8-page paper. If it prints 36,000 20-page papers per hour, how many 8-page papers can it print in that time?

16. The bell of the Houses of Parliament in London weighs 30,000 pounds, and is $\frac{5}{26}$ as heavy as the heaviest bell in the world, which is at Moscow. How much does the bell at Moscow weigh?

PROBLEMS IN REVIEW

177. 1. When purchasing Ada's boarding-school outfit, her mother examines two trunks, one 36 in. by $20\frac{1}{2}$ in. by 23 in., the other 34 in. by 21 in. by 24 in. How many more cubic inches does the larger trunk contain than the smaller one?

2. The first trunk weighs 63 lb., the second 56 lb. Compare their weights in two ways.

3. At one store she selects 2 suits at $10.75 each, a walking skirt for $6.50, 3 shirtwaists at $2.25 each, and a Sunday suit for $35. Find her change from four 20-dollar bills.

4. At another store she buys a dozen towels and a dozen wash cloths for $3.30. If the price of the wash cloths is 2 for 5¢, what is the price of each towel?

5. She orders 2 pairs of shoes at $3.50 a pair and a pair of Oxford ties for $2.50. Find the cost of shoes and ties.

6. At Hill's she buys a pair of kid gloves for $1.25, a dozen pairs of stockings at 3 for $1, a dozen handkerchiefs at $12\frac{1}{2}$¢ each, 2 stocks at 49¢ each, 2 belts at 35¢ each, and $6\frac{3}{4}$ yards of ribbon at 16¢ a yard. Find the cost.

7. Find the amount of these additional purchases:

Sunday hat,	$6.00	Raincoat,	$12.00	Party dress,	$18.00
Week-day hat,	2.50	Umbrella,	3.00	Silk waist,	6.00
Toilet articles,	5.19	Overshoes,	.65	Miscellaneous,	30.67

8. Find the total cost of fitting out Ada for boarding school, including $6.75 for a trunk and $2.49 for a suit case.

9. The Girls' Sunshine Club held a fair for a Fresh Air Fund. Elsa made for it 10 lb. stuffed dates, paying 15¢ a pound for the dates, 72¢ a peck for the peanuts, and 5¢ for the sugar. If 2 lb. dates and 2 qt. peanuts make $2\frac{1}{2}$ lb. stuffed dates, what quantities did Elsa buy and how much did her contribution cost her?

10. Miriam made 5 lb. cream walnuts. She used the whites of 4 eggs, as much water as egg, 2 lb. sugar, 3 lb. English walnuts, and 5 cents' worth of vanilla flavoring. Find the cost if eggs were 24¢ a dozen, confectioners' sugar 8¢ a pound, and walnuts 18¢ a pound.

11. Helen made 12 fern baskets, paying 36¢ for the reed and 10¢ apiece for the ferns to put into the baskets. What was the cost of each filled basket?

12. Alice made 5 laundry bags, using $1\frac{1}{4}$ yd. of toweling and 3 yd. of ribbon for each bag. At 40¢ a yard for the toweling, and 10¢ a yard for the ribbon, find the cost of all the bags.

13. The stuffed dates were sold at 40¢ a pound, the cream walnuts at 45¢ a pound, the fern baskets at 20¢ apiece, and the laundry bags at $1.25 apiece. The receipts from the refreshment booth were $7.65, from the flower booth $9.35, and from other booths $37.15. Find the total receipts from the fair.

DECIMAL FRACTIONS

178. Review definitions.

1. Fractions that express tenths or hundredths or thousandths, etc., are called either **decimal fractions**, or simply **decimals.**

The term "decimals" is more commonly applied to decimal fractions that are expressed with a *decimal point.*

.3 and .25 are decimal fractions, or decimals.

2. Fractions that do not express tenths, or hundredths, or thousandths, etc., are called **common fractions.**

$\frac{3}{8}$, $\frac{25}{32}$, $\frac{9}{16}$ are common fractions.

Reduction of Decimals

179. Reduction of decimals to common fractions.

WRITTEN EXERCISES

1. Reduce 7.025 to a mixed number having its decimal part expressed as a common fraction in its lowest terms.

$$7.025 = 7\tfrac{25}{1000} = 7\tfrac{1}{40}.$$

Reduce each of the following to a common fraction in its lowest terms, or to a mixed number with the fractional part in its lowest terms:

2. .5	**7.** .15	**12.** 7.05	**17.** 7.625
3. .4	**8.** .25	**13.** 3.75	**18.** 4.875
4. 1.6	**9.** .32	**14.** .125	**19.** 9.324
5. 7.8	**10.** .65	**15.** .375	**20.** 6.075
6. 8.4	**11.** .95	**16.** .875	**21.** 5.012

180. Reduction of common fractions to decimals.

1. How many tenths are there in 1? in 3? Then how many tenths are there in $\frac{1}{5}$ of 3, or in $\frac{3}{5}$?

$$5\overline{)30} \text{ tenths} \qquad\qquad 5\overline{)3.0}$$
$$6 \text{ tenths} \qquad\qquad .6$$

2. How many hundredths are there in 3? Then how many hundredths are there in $\frac{1}{4}$ of 3, or in $\frac{3}{4}$?

$$4\overline{)300} \text{ hundredths} \qquad\qquad 4\overline{)3.00}$$
$$** \text{ hundredths} \qquad\qquad .**$$

3. Reduce $\frac{5}{8}$ to thousandths in this way:

$$8\overline{)5000} \text{ thousandths} \qquad\qquad 8\overline{)5.000}$$
$$*** \text{ thousandths} \qquad\qquad .***$$

WRITTEN EXERCISES

181. 1. Reduce $\frac{2}{3}$ to a decimal.

$$3\overline{)2.000}$$
$$.666\tfrac{2}{3}$$

Since $\frac{2}{3} = 2 \div 3$, divide 2 by 3.
$\frac{1}{3}$ of 20 tenths = 6 tenths and 2 tenths remaining.
2 tenths = 20 hundredths.
$\frac{1}{3}$ of 20 hundredths = 6 hundredths and 2 hundredths remaining.
2 hundredths = 20 thousandths.
$\frac{1}{3}$ of 20 thousandths = $6\frac{2}{3}$ thousandths.
Therefore, $\frac{2}{3} = .666\frac{2}{3}$.

One decimal cipher is annexed for each decimal place required. At present results need not be carried beyond three decimal places.

Reduce to decimals:

2. $\frac{2}{5}$	7. $\frac{3}{8}$	12. $\frac{5}{6}$	17. $\frac{5}{12}$
3. $\frac{3}{5}$	8. $\frac{5}{8}$	13. $\frac{2}{7}$	18. $\frac{17}{20}$
4. $\frac{4}{5}$	9. $\frac{7}{8}$	14. $\frac{4}{9}$	19. $\frac{13}{500}$
5. $\frac{1}{4}$	10. $\frac{1}{3}$	15. $\frac{7}{9}$	20. $\frac{77}{200}$
6. $\frac{1}{2}$	11. $\frac{1}{6}$	16. $\frac{3}{11}$	21. $\frac{6}{700}$

Addition and Subtraction of Decimals

WRITTEN EXERCISES

182. The following have been added and tested in 16 minutes. Practice until you can do as well.

1.	2.	3.	4.
14.386	27.909	642.8	6.775
7.545	6.7	37.95	29.22
12.564	85.387	42.44	63.72
90.387	146.962	68.308	698.285

5.	6.	7.	8.
49.837	45.555	72.499	71.473
37.922	77.6	36.309	66.206
256.39	60.72	108.758	48.68
70.683	84.966	728.025	92.375
96.007	67.386	88.989	190.806

9.	10.	11.	12.
$868.369	$ 89.73	$ 77.49	$4708.95
397.487	448.29	345.18	16.884
549.20	569.375	806.725	472.492
305.868	92.447	49.875	5848.13
844.796	999.96	72.125	429.418
84.34	47.775	450.50	484.38
256.634	38.248	81.475	91.656

13.	14.	15.	16.
913.216	$ 75.45	$194.19	$9999.99
300.999	888.88	63.72	8888.88
20.075	8.09	.05	7777.77
.009	459.68	80.09	.66
154.073	43.35	908.05	5.55
13.95	799.99	5.17	44.44
621.79	81.04	64.31	3333.33

Write, with units of the same order in the same column, add, and test results. Preserve your answers.

17. 3.656 + 4.381 + 49.244 + 58.9 + 76.75 + 97.975

18. 6.258 + 3.849 + 66.495 + 37.4 + 49.25 + 9.346

19. 7.437 + 9.000 + 42.835 + 56.7 + 38.28 + 45.305

20. 4.888 + 7.495 + 13.625 + 98.5 + 49.56 + 99.999

21. Add the sums obtained in exercises 17–20.

22. Add each of the six columns seen in exercises 17–20; then add the six sums thus obtained.

The answer should be the same as for exercise 21.

23. Add: seventy and six tenths; forty-five and nine hundredths; six hundred and sixteen hundredths; six hundred sixteen, six hundred sixteen thousandths; five thousandths.

24. Add: four hundred fifty-six; four hundred and fifty-six thousandths; nine thousand and ninety-five hundredths; eighty-six and eighty-six thousandths.

Subtract:

25. 728.249 575.655	26. 400.289 38.491	27. 540.00 235.68	28. 408.30 182.94
29. 200.00 96.48	30. 700 47.75	31. 412 62.628	32. 5247 399.99
33. 428 73.86	34. 525.4 96.275	35. 600.02 48.003	36. 500 28.265
37. 1000 39.25	38. 1000 56.625	39. 1000 107.112	40. 1000 11.089

WRITTEN EXERCISES

183. 1. How much larger is the English bushel of 2218.192 cu. in. than the American bushel of 2150.42 cu. in.?

2. A gallon of water weighs 8.355 pounds, and a gallon of milk 8.622 pounds. Which is the heavier, and how much?

3. A chimney stands 220.25 ft. above ground and extends 18.50 ft. below, where it rests on a foundation 7.75 ft. deep. Find the total height of chimney and foundation.

4. A baseball was thrown from the home plate to second base, 127.28 ft.; then from second base to first base, 90 ft.; then from first base to third base, 127.28 ft. How far did the ball travel?

5. The Times building extends 143 ft. along Broadway, $58.33\frac{1}{3}$ ft. along 42d Street, $137.83\frac{1}{3}$ ft. along 7th Avenue, and 20 ft. along 43d Street. Find the distance around it.

6. The increase in price of cotton during one season lasted eight months. The average price per pound each month was:

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.
8.95¢	9.65¢	10.08¢	10.44¢	11.46¢	12.40¢	12.74¢	12.75¢

Find the advance in price from each month to the next.

Test your results by adding them and comparing the sum with the total advance of the August price over the January price.

7. One year the rainfall in a certain section of Oregon was 35.69 in. during the first four months, 9.47 in. the next four, and 31.82 in. the last four. Find the rainfall for the year.

8. A certain railroad had 1631.03 miles of track in Illinois, 1472.99 in Iowa, 1388.58 in Missouri, 233.10 in Wisconsin, 38.45 in Minnesota, 2632.25 in Nebraska, 199.90 in Montana, 260.44 in Kansas, 429.35 in Colorado, 286.66 in South Dakota, and 351.53 in Wyoming. Find the total mileage.

Multiplication of Decimals

WRITTEN EXERCISES

184. Multiply:

1. \$256.75	**2.** 256.75	**3.** 437.45	**4.** 95.364
4	12	6	5
$\$1027.\cancel{0}\cancel{0}$	$3081.\cancel{0}\cancel{0}$	$2624.7\cancel{0}$	$476.82\cancel{0}$

Ciphers at the right of a decimal may be *omitted.*

5. \$614.38	**6.** 427.65	**7.** \$396.25	**8.** 66.666
5	8	12	5
9. 306.46	**10.** 38.695	**11.** 40.625	**12.** 79.694
11	9	8	7

13. Multiply 42.46 by $6\frac{2}{3}$.

$$\begin{array}{r} 42.46 \\ 6\frac{2}{3} \\ \hline 3\,)\,84.92 \\ \hline 28.30\frac{2}{3} \\ 254.76 \\ \hline 283.06\frac{2}{3} \end{array}$$

42.46 multiplied by $6\frac{2}{3}$ is equal to the sum of $\frac{2}{3}$ of 42.46 and 6 times 42.46.

$$\begin{aligned} \tfrac{2}{3} \text{ of } 42.46 &= \tfrac{1}{3} \text{ of } 2 \text{ times } 42.46 \\ &= \tfrac{1}{3} \text{ of } 84.92 \\ &= 28.30\tfrac{2}{3} \\ 6 \times 42.46 &= 254.76 \end{aligned}$$

Therefore the product is $28.30\frac{2}{3} + 254.76$, or $283.06\frac{2}{3}$.

Multiply:

14. 37.72 by $8\frac{2}{5}$

15. 4.875 by $6\frac{3}{4}$

16. 9.624 by $9\frac{5}{8}$

17. 83.55 by $7\frac{1}{5}$

18. 7.233 by $6\frac{2}{3}$

19. 6.364 by $5\frac{2}{7}$

20. 385.61 by $7\frac{5}{6}$

21. 94.395 by $11\frac{7}{8}$

22. 87.366 by $12\frac{4}{5}$

23. 342.99 by $10\frac{1}{9}$

24. 81.128 by $12\frac{3}{8}$

25. 71.006 by $11\frac{3}{5}$

185. 1. How many thousandths make 1 hundredth? How many hundredths make 1 tenth? How many tenths make 1 unit?

2. Since 10 times $.1 = 1$, what is the value of

$10 \times .2$? $10 \times .3$? $10 \times .4$? $10 \times .5$?

Multiplying tenths by 10 *gives units.*

3. Since 10 times $.01 = .1$, what is the value of

$10 \times .03$? $10 \times .05$? $10 \times .07$? $10 \times .08$?

Multiplying hundredths by 10 *gives tenths.*

4. Since 10 times $.001 = .01$, what is the value of

$10 \times .005$? $10 \times .006$? $10 \times .012$? $10 \times .015$?

Multiplying thousandths by 10 *gives hundredths.*

5. Since $10 \times .005 = .05$, $10 \times .03 = .3$, and $10 \times .2 = 2$, when .235 is multiplied by 10 each figure of .235 is advanced into the next higher place. A short way to do this is to *move the decimal point one place toward the right.*

$$\begin{array}{r} .235 \\ \underline{10} \\ 2.35\not{0} \end{array}$$

6. Multiply .235 by 10 and the product by 10, and so obtain $100 \times .235$. Multiply by 10 again and obtain $1000 \times .235$.

How many places and in which direction is the decimal point moved for each multiplication by 10? How may a decimal be multiplied by 100 by moving the decimal point? by 1000?

186. *A decimal may be multiplied by* 10, 100, 1000, *etc., by moving the decimal point toward the right as many places as there are ciphers in the multiplier.*

EXERCISES

Multiply by 10:		Multiply by 100:	
1. .4	4. $4.65	7. .48	10. 5.315
2. .35	5. $3.96	8. 3.90	11. 7.56
3. .09	6. $0.075	9. 0.009	12. 3.004

Multiply by 1000:

13. .016	**15.** 9.060	**17.** $5.75	**19.** $2.375
14. 3.050	**16.** 4.06	**18.** $2.16	**20.** 230.6

Multiply by 10, 100, 1000:

21. 3.025	**23.** 7.011	**25.** 15.603	**27.** 100.11
22. 4.365	**24.** 8.009	**26.** 243.62	**28.** 4256.3

29. Multiply .785 by 1000; by 10,000; by 100,000.

WRITTEN EXERCISES

187. 1. Multiply 7.628 by 200.

$$\begin{array}{r} 762.8 \\ 2 \\ \hline 1525.6 \end{array}$$

100 times 7.628 = 762.8.
200 times 7.628 = 2 times 762.8.
= 1525.6.

Multiply by 20, 30, 40, 50, 60:

2. 3.42	**4.** 4.48	**6.** $2.365	**8.** 87.1
3. .567	**5.** .007	**7.** $24.82	**9.** 48.6

Multiply by 700, 800, 900, 1200:

10. .382	**12.** $4.29	**14.** 25.05	**16.** 79.6
11. .796	**13.** $16.41	**15.** 122.8	**17.** 40.9

Multiply by 6000, 70,000, 11,000,000:

18. 2.254	**19.** .079	**20.** $4.18	**21.** 9.9

22. Multiply $48.57 by 84; 3.845 by 96.

$$\begin{array}{r} \$48.57 \\ 84 \\ \hline 194\ 28 \\ 3885\ 6 \\ \hline \$4079.88 \end{array} \qquad \begin{array}{r} 3.845 \\ 96 \\ \hline 23\ 070 \\ 346\ 05 \\ \hline 369.12\cancel{0} \end{array}$$

Dollar signs and decimal points are omitted in the partial products.

Multiply:

23. $36.62 by 89

24. $94.05 by 66

25. 37.088 by 75

26. 42.094 by 69

27. 78.889 by 98

28. 39.675 by 125

29. 84.719 by 280

30. 78.616 by 675

31. 90.008 by 845

32. 468.77 by 1208

33. Find the weight of a slate roof 40 feet long and 20 feet wide, if every square foot weighs 9.25 pounds.

34. If it takes 7.5 cu. ft. of natural gas to produce as much heat as 1 pound of coal, how many cubic feet of gas are required to give as much heat as 1 ton of coal?

35. A cubic foot of water weighs 62.5 lb. Find the weight of water that a tank 8 ft. by 4 ft. by 3 ft. will hold.

Division of Decimals

WRITTEN EXERCISES

188.

1.	2.	3.
$7\overline{)378.240}$	$9\overline{)456.392}$	$12\overline{)62.574}$
$54.034\frac{2}{7}$	$50.710\frac{2}{9}$	$5.214\frac{1}{2}$

Divide, finding quotients to three decimal places:

4. 34.234 by 6

5. 56.712 by 8

6. 95.203 by 7

7. 84.425 by 9

8. 50.806 by 7

9. 61.753 by 9

10. 5.8 (or 5.800) by 8

11. 7.6 by 11

12. 345.5 by 9

13. 46.37 by 7

14. 1.044 by 12

15. 3.058 by 11

16. Find $\frac{1}{8}$ of 4.512; of 6.2; of \$14.64.

17. Find $\frac{1}{12}$ of \$17.28; of 72.096; of 11.20.

189. 1. How many tenths are there in $\frac{1}{10}$ of 10 tenths, or in $\frac{1}{10}$ of 1? in $\frac{1}{10}$ of 2? in $\frac{1}{10}$ of 3?

$1 \div 10 = .1 \qquad 2 \div 10 = ? \qquad 3 \div 10 = ?$

Dividing units by 10 *gives tenths.*

2. How many hundredths are there in $\frac{1}{10}$ of .10, or in $\frac{1}{10}$ of .1? in $\frac{1}{10}$ of .4? in $\frac{1}{10}$ of .5?

$.1 \div 10 = .01 \qquad .4 \div 10 = ? \qquad .5 \div 10 = ?$

Dividing tenths by 10 *gives hundredths.*

3. How many thousandths are there in $\frac{1}{10}$ of .010, or in $\frac{1}{10}$ of .01? in $\frac{1}{10}$ of .06? in $\frac{1}{10}$ of .07?

$.01 \div 10 = .001 \qquad .08 \div 10 = ? \qquad .09 \div 10 = ?$

Dividing hundredths by 10 *gives thousandths.*

4. Since $2 \div 10$, or $\frac{1}{10}$ of 2, is equal to .2, $\frac{1}{10}$ of $.5 = .05$, and $\frac{1}{10}$ of $.06 = .006$, dividing 2.56 by 10 has the effect of moving each figure into the next lower place. A short way to do this is to *move the decimal point one place toward the left.*

$$\begin{array}{r} 10\overline{)2.56} \\ \hline .256 \end{array}$$

5. If 2.5 is divided by 10 and the quotient by 10, how many places toward the left will the decimal point be moved?

How may a number be divided by 10×10, or by 100?

6. Divide 3, or 3.0, by 10; divide the quotient by 10; divide a third time by 10. How many places and in which direction has the decimal point been moved?

How may a number be divided by 1000?

190. *A number may be divided by* 10, 100, 1000, *etc., by moving the decimal point toward the left as many places as there are ciphers in the divisor.*

EXERCISES

191. Divide:

By 10	By 100	By 1000
1. 72	8. 225	15. 3244
2. 3.5	9. 47.6	16. 8640
3. 4.2	10. 39.6	17. 966
4. .8	11. 1.07	18. 170
5. .09	12. 58.2	19. 49
6. 75	13. 396	20. 5
7. 4.25	14. 44.05	21. 6220

WRITTEN EXERCISES

192. 1. Divide 568 by 30.

$$\begin{array}{r} 3\not{0})\underline{56|8.00} \\ 18.933\frac{1}{3} \end{array}$$

30 is contained in 568 as many times as 3 is contained in 56.8.

Dividing 56.8 by 3, the division is found to be inexact in tenths' place. Therefore, one cipher is annexed to the dividend, and the division is carried to hundredths. The division being inexact in hundredths' place, another cipher is annexed and the division is carried to thousandths, giving 3 thousandths and 1 thousandth remainder.

It is not necessary to carry this division beyond thousandths. The result of the last division is written in the quotient as $3\frac{1}{3}$ thousandths, or the quotient to thousandths may be written 18.933.

In the following exercises, find quotients to thousandths.

Divide by 20, 40, 60, 80, 120:

2. 4	4. .5	6. 35	8. .96
3. 71	5. 4.2	7. 5.60	9. 3.63

Divide by 700, 900, 1100:

10. 25	12. 2763	14. 37.5	16. .6
11. 160	13. 4840	15. 51.2	17. 14.4

Divide by 8000, 11,000, 12,000:

18. 42	19. 132	20. 1680	21. 1056

22. Divide 9.588 by 47; $10,010 by 125.

$$\begin{array}{r} .204 \\ 47\overline{)9.588} \\ \underline{9\,4} \\ 188 \\ \underline{188} \end{array} \qquad \begin{array}{r} 80.08 \\ 125\overline{)\$10010.00} \\ \underline{1000} \\ 1000 \\ \underline{1000} \end{array}$$

Divide:

23. 292.4 by 85

24. 811.22 by 98

25. $400 by 625

26. $328.80 by 48

27. $860.30 by 176

28. 300.933 by 87

29. 141.401 by 327

30. 351.14 by 388

31. 16637.5 by 275

32. 1449.02 by 265

Reduce to decimals, not beyond thousandths:

33. $\frac{5}{16}$ **34.** $\frac{11}{24}$ **35.** $\frac{3}{32}$ **36.** $\frac{153}{48}$ **37.** $\frac{100}{64}$

38. A man sheared 88 Angora goats and obtained 330 pounds of mohair. Find the average clip from a goat.

39. A 35-acre vineyard produced 259 tons of grapes, which were sold for $4014.50. Find the average yield per acre, and the value of the crop per ton and per acre.

40. A watch factory employed 2400 persons 8 hours a day. The weekly pay-roll for a week of 6 days was $20,160. Find the average wage per hour in cents.

41. During April Mr. Hanna's 30 cows produced 18,800 lb. of milk. Find the average number of pounds of milk produced by 1 cow in 1 day.

42. The milk contained 656 pounds of butter fat, which was sold at 27¢ a pound. Find the average value of the butter fat produced by 1 cow in 1 day.

USEFUL PARTS OF A DOLLAR

193. Halves, fourths, and eighths of a dollar.

1. What part of a dollar is \$.50? \$.25? \$.75?

2. Since \$.25 is $\frac{1}{4}$ of a dollar, what part of a dollar is $\frac{1}{2}$ of \$.25, or \$.12$\frac{1}{2}$?

3.

\$.12$\frac{1}{2}$	\$.25	\$.37$\frac{1}{2}$	\$.50	\$.62$\frac{1}{2}$	\$.75	\$.87$\frac{1}{2}$
+.12$\frac{1}{2}$	+.12$\frac{1}{2}$	+.12$\frac{1}{2}$	+.12$\frac{1}{2}$	+.12$\frac{1}{2}$	+.12$\frac{1}{2}$	+.12$\frac{1}{2}$
\$.25	\$.37$\frac{1}{2}$	\$.50	\$.62$\frac{1}{2}$	\$.75	\$.87$\frac{1}{2}$	\$1.00

How many eighths of a dollar is \$.12$\frac{1}{2}$ + \$.12$\frac{1}{2}$, or \$.25? \$.25 + \$.12$\frac{1}{2}$, or \$.37$\frac{1}{2}$? \$.50? \$.62$\frac{1}{2}$? \$.75? \$.87$\frac{1}{2}$?

4. Count by eighths of a dollar to \$1.25 in this way:

\$.12$\frac{1}{2}$ = \$$\frac{1}{8}$; \$.25 = \$$\frac{2}{8}$, or \$$\frac{1}{4}$; \$.37$\frac{1}{2}$ = \$$\frac{3}{8}$; etc.

EXERCISES

194. 1. Find the cost of 32 hats @ \$.37$\frac{1}{2}$.

SOLUTION. — At \$1 each, 32 hats would cost \$32.
Therefore at \$.37$\frac{1}{2}$, or \$$\frac{3}{8}$, each, 32 hats will cost $\frac{3}{8}$ of \$32, or \$12.

Find the cost of:

2. 28 sleds @ \$.50
3. 54 knives @ \$.50
4. 8 baseball bats @ \$.25
5. 14 shin guards @ \$.25
6. 24 neckties @ \$.75
7. 40 mufflers @ \$.75
8. 48 yd. lawn @ \$.12$\frac{1}{2}$
9. 84 yd. gingham @ \$.12$\frac{1}{2}$
10. 48 tennis balls at \$.37$\frac{1}{2}$
11. 32 pr. tennis shoes @ \$.37$\frac{1}{2}$
12. 72 pocketbooks @ \$.62$\frac{1}{2}$
13. 400 bass lines @ \$.62$\frac{1}{2}$
14. 80 hammocks @ \$.87$\frac{1}{2}$
15. 64 boys' sweaters @ \$.87$\frac{1}{2}$

\$1	\$.12½	\$.25
\$.87½		\$.37½
\$.75	\$.62½	\$.50

16. Write 24 in the empty square. Then tell the cost of 24 articles at the prices named.

17. Do the same with the following numbers written successively in the empty square: 32, 48, 72, 80, 96.

195. Thirds and sixths of a dollar.

$$\begin{array}{r|l} 3 & \$1.00 \\ \hline 2 & \$\ .33\tfrac{1}{3} \\ \hline & \$\ .16\tfrac{2}{3} \end{array}$$

1. What part of a dollar is $\$.33\frac{1}{3}$? $\frac{1}{2}$ of $\$.33\frac{1}{3}$, or $\$.16\frac{2}{3}$?

2. Since $\$.33\frac{1}{3}$ is $\frac{1}{3}$ of a dollar, what part of a dollar is $\$.33\frac{1}{3} + \$.33\frac{1}{3}$, or $\$.66\frac{2}{3}$?

3. $\begin{array}{r} \$.16\frac{2}{3} \\ +.16\frac{2}{3} \\ \hline \$.33\frac{1}{3} \end{array}$ $\begin{array}{r} \$.33\frac{1}{3} \\ +.16\frac{2}{3} \\ \hline \$.50 \end{array}$ $\begin{array}{r} \$.50 \\ +.16\frac{2}{3} \\ \hline \$.66\frac{2}{3} \end{array}$ $\begin{array}{r} \$.66\frac{2}{3} \\ +.16\frac{2}{3} \\ \hline \$.83\frac{1}{3} \end{array}$ $\begin{array}{r} \$.83\frac{1}{3} \\ +.16\frac{2}{3} \\ \hline \$1.00 \end{array}$

How many sixths of a dollar is $\$.33\frac{1}{3}$? $\$.50$? $\$.66\frac{2}{3}$? $\$.83\frac{1}{3}$?

4. Count by sixths of a dollar to $\$1.33\frac{1}{3}$ in this way:

$$\$.16\tfrac{2}{3} = \$\tfrac{1}{6};\ \$.33\tfrac{1}{3} = \$\tfrac{1}{3};\ \$.50 = \$\tfrac{1}{2};\ \text{etc.}$$

EXERCISES

196. Find the value of each of the following lots of cloth:

1. 60 yd. @ $\$.33\frac{1}{3}$
2. 42 yd. @ $\$.16\frac{2}{3}$
3. 30 yd. @ $\$.66\frac{2}{3}$
4. 36 yd. @ $\$.16\frac{2}{3}$
5. 120 yd. @ $\$.33\frac{1}{3}$
6. 600 yd. @ $\$.83\frac{1}{3}$
7. 360 yd. @ $\$.66\frac{2}{3}$
8. 144 yd. @ $\$.83\frac{1}{3}$
9. Find the cost of 1200 baskets @ $\$.16\frac{2}{3}$
10. Find the cost of 27 doz. crocks @ $\$.66\frac{2}{3}$
11. Find the cost of 24 gross of clothespins @ $\$.33\frac{1}{3}$

197. Commit to memory the following table showing the decimal parts of $1, or of any other unit, that are advantageously changed to the common fractional form in computations.

TABLE

PARTS OF $1	PARTS OF $1	PARTS OF $1	PARTS OF $1
.10 = $\frac{1}{10}$	.80 = $\frac{4}{5}$	.50 = $\frac{1}{2}$	.$16\frac{2}{3}$ = $\frac{1}{6}$
.20 = $\frac{1}{5}$	.$12\frac{1}{2}$ = $\frac{1}{8}$	.$62\frac{1}{2}$ = $\frac{5}{8}$	.$33\frac{1}{3}$ = $\frac{1}{3}$
.40 = $\frac{2}{5}$	.25 = $\frac{1}{4}$	.75 = $\frac{3}{4}$	.$66\frac{2}{3}$ = $\frac{2}{3}$
.60 = $\frac{3}{5}$	.$37\frac{1}{2}$ = $\frac{3}{8}$	.$87\frac{1}{2}$ = $\frac{7}{8}$	.$83\frac{1}{3}$ = $\frac{5}{6}$

WRITTEN EXERCISES

198. Find the cost of:

1. 60 plates @ $.10
2. 34 yd. braid @ $.20
3. 36 collars @ $.$12\frac{1}{2}$
4. 64 lb. pecans @ $.$12\frac{1}{2}$
5. 140 yd. percale @ $.$12\frac{1}{2}$
6. 135 baseball bats @ $.20
7. 110 baseball mitts @ $.50
8. 144 bottles olives @ 25¢
9. 66 thimbles @ $33\frac{1}{3}$¢
10. 18 handkerchiefs @ 25¢
11. 22 hairbrushes @ $37\frac{1}{2}$¢
12. 42 toy engines @ $62\frac{1}{2}$¢
13. 75 yards of sateen @ $16\frac{2}{3}$¢
14. 54 yards of damask @ $66\frac{2}{3}$¢
15. 140 gallons of sirup @ $37\frac{1}{2}$¢
16. 125 phonograph records @ $.50
17. Find the cost of 15 whips at $1.25 each.

$15
3.75
$18.75

At $1 each, 15 whips would cost $15.

At $1.25, or $$\frac{5}{4}$, each, they will cost $\frac{5}{4}$ of $15, or $18.75.

The business man, however, obtains the answer in a more simple way. He first writes $15, the cost at $1 each; then below $15 he writes $3.75, the cost at $.25, or $$\frac{1}{4}$, found by dividing $15 by 4. Adding the cost at $.25 to the cost at $1, he obtains the cost at $1.25.

18. Find the cost of 44 yards of silk at \$1.12$\frac{1}{2}$ per yard.

$$\begin{array}{r} \$44 \\ 5.50 \\ \hline \$49.50 \end{array}$$

What is the cost at \$1 per yard? at \$.12$\frac{1}{2}$, or \$$\frac{1}{8}$, per yard? at \$1.12$\frac{1}{2}$ per yard?

19. Find the cost of 22 yards of carpet at \$.87$\frac{1}{2}$ per yard.

$$\begin{array}{r} \$22 \\ 2.75 \\ \hline \$19.25 \end{array}$$

What is the cost at \$1 per yard? at \$.12$\frac{1}{2}$, or \$$\frac{1}{8}$, per yard? at \$1 − \$.12$\frac{1}{2}$, or \$.87$\frac{1}{2}$, per yard?

20. How much must be paid for 3548 bushels of wheat at 90 ¢ per bushel?

$$\begin{array}{r} \$3548 \\ 354.80 \\ \hline \$3193.20 \end{array}$$

How much would the wheat cost at \$1 per bushel? at 10 ¢, or \$$\frac{1}{10}$, per bushel? What, then, is the cost at 90 ¢, or \$1 − \$.10, per bushel?

Find the cost of:

21. 56 croquet sets @ \$1.50

22. 36 tennis nets @ \$1.25

23. 45 tennis rackets @ \$2.50

24. 15 baseballs @ \$1.25

25. 24 bathing suits @ \$1.75

26. 33 fishing rods @ \$2.25

27. 420 caps @ 37$\frac{1}{2}$ ¢

28. 58 yd. silk @ \$1.12$\frac{1}{2}$

29. 63 yd. serge @ 83$\frac{1}{3}$ ¢

30. 48 yd. velvet @ \$1.16$\frac{2}{3}$

31. 28 lawn settees @ 87$\frac{1}{2}$ ¢

32. 120 toy watches @ \$.40

33. Find the cost of 45 articles @ 20 ¢; @ 60 ¢; @ 80 ¢.

34. Find the cost of 288 articles @ 50 ¢; @ 51 ¢; @ 49 ¢.

35. Find the cost of 216 articles @ 25 ¢; @ 26 ¢; @ 24 ¢; @ 25$\frac{1}{2}$ ¢; @ 24$\frac{1}{2}$ ¢.

36. Find the cost of 240 articles @ 33$\frac{1}{3}$ ¢; @ 33 ¢; @ 33$\frac{1}{2}$ ¢.

37. A grocer bought 650 pounds of coffee at 22$\frac{1}{2}$ cents per pound, and sold it at 35 cents per pound. How much did he gain?

EXERCISES

199. 1. How many articles costing 50¢ each can be bought for each of the following sums:

$1? $2? $5? $12? $1.50? $3.50? $7.50?

2. How many 25-cent articles can be bought for:

$1?	$4?	$.50?	$2 + $.25?	$3.50?
$2?	$11?	$.75?	$2 + $.75?	$6.25?

3. How many $12\frac{1}{2}$-cent articles can be bought for:

$1?	$5?	$.25?	$1 + $.50?	$3.25?
$3?	$12?	$2 + $.25?	$2 + $.75?	$5.75?

4. How many $33\frac{1}{3}$-cent articles can be bought for:

$1?	$4?	$10?	$13?	$33?
$2?	$6?	$12?	$15?	$50?

How many $16\frac{2}{3}$-cent articles can be bought for the same amounts?

WRITTEN EXERCISES

200. 1. How many yards of silk costing $$1.12\frac{1}{2}$ per yard can be bought for $315?

$$\overset{35}{\cancel{315}} \times \frac{8}{\cancel{9}} = 280$$

Since $$1.12\frac{1}{2}$ = $$1\frac{1}{8}$, or $$\frac{9}{8}$, the number of yards is equal to the number of times $315 contains $$\frac{9}{8}$, which may be found by dividing 315 by $\frac{9}{8}$, or by multiplying 315 by $\frac{8}{9}$.

Hence, 280 yards may be bought for $315.

Find the number of articles that can be bought for:

2. $65 at $1.25 each
3. $85 at $2.50 each
4. $28 at $3.50 each
5. $27 at $$.37\frac{1}{2}$ each
6. $98 at $$.66\frac{2}{3}$ each
7. $455 at $$1.16\frac{2}{3}$ each
8. $540 at $2.25 each
9. $160 at $$1.33\frac{1}{3}$ each

BILLS AND ACCOUNTS

201. The following shows a common form of **bill**.

CHICAGO, ILL., *Sept. 12*, 190*6.*

Mr. *John L. McLean,*

39 Roanoke Ave.

Bought of STONE BROS., 275 WATER ST.

DEALERS IN FRUITS OF ALL KINDS

TERMS: *Cash.* PHONE 147-D

		4 bu. Peaches	*1.65*	*6*	*60*		
		3 crates Muskmelons	*1.50*	*4*	*50*		
		12 baskets Grapes	*.18*	*2*	*16*	*13*	*26*

Received payment,

Stone Bros.,

Per Ralph Stone.

A bill should contain the following facts:

1. The name and address of the purchaser, or **debtor.**

2. The name and address of the person to whom the money is due, or the **creditor.**

3. The date when the debt was incurred.

4. A bill for goods sold should mention the articles sold, their number and price, the amount of each sale and the total amount, or **footing.** A bill for services rendered should show the nature of the services and the footing.

5. When a bill is paid, the words "Received payment," or "Paid," and the creditor's name, are written at the foot, either by the creditor or by some one authorized by him.

202. The following shows an **account**.

BUFFALO, N.Y., *Oct. 1,* 190*6*.

M*r. C. L. Thomas, 180 Western Ave.*

To EMPIRE FURNITURE CO., DR.

FURNITURE, CARPETS, OILCLOTH, REFRIGERATORS, ETC.

TELEPHONE 946-A. 96 & 98 SOUTH PEARL STREET

			Dr.		Cr.	
Sept.	*1*	*Bed, Spring, & Mattress*	*20*			
		1 Washstand	*3*	*50*		
		1 Table	*3*	*50*		
		Cash			*15*	
Sept.	*15*	*1 Refrigerator*	*14*	*25*		
		Cash			*15*	
Oct.	*1*	*15 Stair Pads @ 12½¢*	*1*	*88*		
		1 Parlor Stove	*16*	*50*		
		Cash to balance			*29*	*63*
			59	*63*	*59*	*63*

203. The difference between the sum of the amounts owed and the sum of the amounts paid is called the **balance**.

WRITTEN EXERCISES

204. Make out, foot or balance, and receipt the following, supplying dates, names, and addresses, when lacking:

1. R. G. Palmer bought of Albert P. Hunt, 40 lb. sugar @ $6\frac{1}{2}$¢; $1\frac{1}{4}$ lb. mixed spices @ 20¢; $5\frac{1}{4}$ lb. butter @ 32¢.

2. Chas. Sumner bought of Oscar Strauss, 11 pieces of piano music @ 25¢; 79 pieces for orchestral instruments @ 10¢.

3. George A. Harris sold to Peter Harper, 3 doz. fruit jars @ 65¢; 25 lb. sugar @ 6¢; $1\frac{1}{2}$ bu. tomatoes @ 60¢; $\frac{1}{4}$ lb. tea @ 65¢; 5 lb. coffee @ 35¢; 6 packages oatmeal @ $12\frac{1}{2}$¢.

4. Jas. Osgood is debtor to Dr. Ellis Cook for 8 prescriptions @ \$.50; 25 day visits @ \$2; and 7 night visits @ \$3.

5. Mar. 15, Pearl Roberts bought 1 encyclopedia @ \$48 and 3 books @ \$1.50, of the Haller Publishing Co. She paid \$20 Mar. 15, \$10 Apr. 15, \$10 June 15, and the balance July 1.

6. Reuben Case is debtor to the Municipal Gas Co. for 6300 cu. ft. of gas for lighting @ 95¢ per 1000 cu. ft., and 11,200 cu. ft. of gas for fuel @ 80¢ per 1000 cu. ft.

7. 14 yd. silk @ \$1.75; 2 pairs gloves @ \$1.50; 9 yd. flannel @ $37\frac{1}{2}$¢; 4 doz. handkerchiefs @ \$2.75; 3 tablecloths @ \$3.75; 1 rug at \$32.50; 4 hassocks @ 75¢.

8. Porterhouse steak, $2\frac{1}{4}$ lb. @ 25¢; leg of veal, 14 lb. @ 15¢; pot roast, 5 lb. @ $12\frac{1}{2}$¢; bacon, $6\frac{1}{2}$ lb. @ 18¢; rib roast, $10\frac{3}{4}$ lb. @ 20¢; chickens, $8\frac{1}{2}$ lb. @ 25¢.

9. Turkey, $11\frac{1}{2}$ lb. @ 26¢; 2 qt. cranberries @ $12\frac{1}{2}$¢; $\frac{1}{2}$ pk. sweet potatoes @ 35¢; $1\frac{1}{2}$ lb. sausage @ 16¢; $2\frac{1}{2}$ qt. oysters @ 40¢; $1\frac{3}{4}$ gal. apple butter @ \$1.25; $1\frac{1}{2}$ doz. oranges @ 35¢.

10. 4 sets knives and forks @ \$22.50; 4 sets fruit knives @ \$4.85; 6 carving sets @ \$7.50; 24 butcher knives @ \$.50; 16 shears @ \$.85; 60 pocketknives @ \$.$37\frac{1}{2}$. Paid on account \$75 at the time of purchase, and the balance one month later.

11. 24 baseballs @ \$.$87\frac{1}{2}$; $1\frac{1}{2}$ doz. baseball bats @ \$.55 each; 4 catchers' mitts @ \$4.75; 5 fielders' gloves @ \$.95; 3 catchers' masks @ \$2.25; 3 doz. tennis balls @ \$4.50.

12. 3 shotguns @ \$14.75; 12 shell bags @ 60¢; 3 shotgun covers @ \$1.50; 12 hunting jackets @ \$3.75; 16 pairs Indian clubs @ \$.13; 6 basket balls @ \$4.

PROBLEMS IN REVIEW

205. 1. Eva Crane, who lives in a suburb, decides to do her Christmas shopping in a city department store. She lives 15 minutes from the station. A train leaving at 8:37 A.M. reaches the city at 9:26 A.M. Thence it is 35 minutes to the store. How long does the entire journey take?

2. At the neckwear counter Eva buys 2 stocks at $37\frac{1}{2}$¢ each, 6 collars at $12\frac{1}{2}$¢ each, and 6 pieces of ruching, $\frac{3}{8}$ yd. each, at 24¢ a yard. How much change should she receive from $10?

3. She buys a $4\frac{7}{8}$-yd. remnant of silk worth 72¢ a yard for $3.15. How much does she save on the remnant?

The following articles Eva has charged to her mother's account. Find the cost of the purchases in each department.

4. In the toy department: a doll for 85¢, 2 boxes of blocks @ 35¢, a doll carriage for $1.25, and a toy motor car for 25¢.

5. In the book department: "Alice in Wonderland" for 42¢, "Little Women" and "Little Men," each at 95¢, one Bible for $1.50, and Shakespeare's Works, 16 volumes, at 35¢ each.

6. At the ribbon counter: $4\frac{3}{4}$ yd. ribbon at 36¢ a yard, and 10 yd. baby ribbon at $1\frac{1}{2}$¢ a yard.

7. Grocery and meat department: $1\frac{1}{2}$ lb. nuts @ 20¢, 2 lb. raisins @ 25¢, 3 qt. cranberries @ 9¢, an 8-lb. turkey @ $21\frac{1}{2}$¢.

8. Make out the bill that Mrs. Crane will receive on the first of the month for the articles that have been charged.

9. Mr. and Mrs. Clinton paid \$93.68 for papering and painting the rooms of their new house, and \$19.57 for staining the floors. How much did it cost them?

Find the cost of furnishings for:

10. Parlor		11. Hall	
Rug	\$30.00	Hatrack	\$9.25
Stand	9.00	Settle	16.50
Desk and chair	18.50	Bookshelves	14.00
Rocking-chairs	10.00	Stand	8.00
Morris chair	12.25	Turkish rug	35.40
Flemish chair	9.75	Rope portière	6.65
Curtains	25.00	Oak bookcase	18.00
Portières	21.50	Lace curtains	11.50
Pictures	72.00	Tabouret	2.35

12. Dining room: a rug, \$18.60; a table, \$16.00; 6 dining chairs @ \$2.75; a table cover, \$6.85; a sideboard, \$12.25; and 3 pairs of window curtains @ \$1.75.

13. Best bedroom: bed and mattress, \$18; bolster roll, \$2.75; dressing table, \$10.45; cretonne bed set, \$4; matting, 20 yd. @ 25¢; 2 pairs window curtains @ \$.95; 2 chairs @ \$1.65.

14. Nursery: matting, $17\frac{1}{2}$ yd. @ 20¢; bed and mattress, \$12; spread, \$1.85; bookshelves, \$3; couch and cover, \$10.00; rocking-chair, \$3.45; window curtains, \$1.75; 8 pictures @ 75¢; 3 nursery posters @ 60¢.

15. Veranda: 3 porch chairs @ \$1.50; hammock, \$2.70; porch shades, \$9; rug, \$10.75.

16. Furnishing the kitchen, laundry, and cellar cost \$49.85; 2 bedrooms, \$56.75; bath room, \$15; upper hall and stairs, \$19.35. Pictures not already mentioned cost \$25. How much did it cost Mr. and Mrs. Clinton to decorate and furnish their whole house?

MEASUREMENTS

EXERCISES

206. 1. Give the table* of *measures of length.*

2. How many inches are there in a yard? in $1\frac{1}{4}$ yd.?

3. What part of a yard is 18 in.? 27 in.?

4. How many feet are there in $1\frac{1}{2}$ yd.? What part of a foot is $\frac{1}{4}$ yd.? What part of a yard is $\frac{1}{2}$ ft.?

5. How many feet are there in 1 rd.? in 2 rd.? If a road is 66 ft. wide, how many rods wide is it?

6. How many rods are there in $\frac{1}{2}$ mi.? in $\frac{1}{4}$ mi.? in $1\frac{1}{2}$ mi.? What part of a mile is 40 rd.?

7. Give the table of *measures of surface.*

8. How many square inches are there in $\frac{1}{2}$ sq. ft.? What part of a square foot is 24 sq. in.? 60 sq. in.?

9. How many square feet are there in $2\frac{1}{2}$ sq. yd.? How many square yards are there in 189 sq. ft.?

10. How many acres are there in a plot of land 160 rd. long and 1 rd. wide? 320 rd., or 1 mile, long and 1 rd. wide? 1 mile long and 10 rd. wide? 1 mile long and 1 mile wide, or 1 mile square?

11. The area of a square, each side of which is a mile, is called a **square mile.**

1 square mile (sq. mi.) = 640 acres (A.).

A square mile of land is sometimes called a **section.**

12. How many acres are there in a quarter section?

13. Give the table of *measures of volume.*

14. How is the number of square inches in a square foot obtained? the number of cubic inches in a cubic foot?

*Tables will be found in the Appendix.

15. Give the table of *liquid measures.*

16. How many pints are there in a gallon? in 1 gal. 2 qt.? in 3 gal. 1 qt.?

17. What part of a gallon is 1 qt.? 1 qt. 1 pt.? 2 qt. 1 pt.?

18. Give the table of *dry measures.*

19. How many quarts are there in $1\frac{1}{2}$ pk.? in $2\frac{1}{2}$ pk.? in 3 pk. 2 qt.? in 1 bu. 1 pk.? in 2 bu.?

20. Blackberries are shipped from North Carolina in bushel boxes, and in 2-bushel boxes. How many quarts does a bushel box hold? a 2-bushel box?

21. Strawberries are shipped from the South in crates of the following capacities: 24 qt., 32 qt., 36 qt., 42 qt., 45 qt., 48 qt., 60 qt. Express the different capacities in pecks.

How much less than 2 bushels does a 60-quart crate hold?

Express the capacity of a 48-quart crate in bushels.

22. The capacity of a refrigerator box for shipping Florida strawberries varies from 64 qt. to 128 qt. How many bushels does a 64-quart box hold? a 96-quart box? a 128-quart box?

23. Beans and other vegetables are often sold in $1\frac{1}{2}$-bushel boxes. How many pecks does such a box hold?

24. Tomatoes are often shipped in $\frac{1}{2}$-bushel crates. How much is such a crate of tomatoes worth at 10¢ per quart? How many quarts are there in a $\frac{5}{8}$-bushel crate?

25. Give the table of *time measures.*

26. Name the months and tell the number of days in each.

27. What part of a year is 2 months? 3 mo.? 9 mo.?

28. To how many years are 24 months equal? 30 mo.? 40 mo.? 51 mo.? 56 mo.?

29. Alfred had a high fever for 54 hours. How many days and hours did the fever last?

30. If my watch gains 3 seconds a day, how many minutes will it gain during April?

31. Two watchmen relieved each other every 6 hours. How many times did each one watch during 6 days? How many hours did each watch during that time?

32. Jan. 1, 1906, came on Monday. Since one year = 52 weeks and 1 day, on what day of the week did Jan. 1, 1905, come?

Find the day of the week for Jan. 1, 1907.

33. What units of *weight* are used in weighing coal, iron, potatoes, meat, hay, etc.? Give the table.

34. How many ounces are there in $1\frac{1}{4}$ lb.? in $2\frac{3}{4}$ lb.?

35. Express in pounds 12 oz.; 20 oz.; 36 oz.; 50 oz.

36. How many pounds are there in $1\frac{1}{2}$ T.? in $2\frac{1}{4}$ T.?

37. A large block of marble weighed 14 tons. How many pounds did it weigh?

38. It takes 200 lb. of ice to keep a box of strawberries cool during shipment. What part of a ton does it take?

39. Find the cost of 3000 lb. of hay at $9 per ton.

40. A man bought a ton of bran for $14 and sold it at 90¢ per hundredweight. How much did he gain?

41. How many pounds are there in 2 T. 4 cwt. 75 lb.?

Counting

207. 1. How many gloves make a *pair* of gloves?

2. How many persons make a *score* of persons?

3. How many oysters make a *dozen* oysters?

4. How many buttons are there in 12 dozen buttons, or in 1 *gross* of buttons?

5. How many pens are there in 12 boxes each of which contains 1 gross of pens? This number is a *great gross*.

6. Memorize this **counting table:**

2 = 1 pair
20 = 1 score
12 = 1 dozen
12 dozen = 144 = 1 gross
12 gross = 1728 = 1 great gross

WRITTEN EXERCISES

208. Find the gain from each purchase and sale:

	Articles	Quantity and Cost	Selling Price
1.	Pens	1 great gross @ \$9.60	90¢ per gross
2.	Cuffs	1 doz. pairs @ \$2.70	30¢ per pair
3.	Collars	20 doz. @ \$1.60	15¢ each
4.	Hinges	1 doz. pairs @ \$1.75	25¢ per pair
5.	Screws	100 gross @ 25¢	5¢ per doz.
6.	Pencils	4 gross @ \$3	2 for 5¢
7.	Brackets	2 doz. pairs @ \$7.50	80¢ per pair
8.	Door knobs	5 doz. pairs @ 96¢	10¢ per pair
9.	Coat hooks	18 gross @ \$1.75	25¢ per doz.
10.	Indian clubs	30 pairs @ 28¢	35¢ per pair
11.	Rubber bands	25 gross @ 11¢	15¢ per gross
12.	Eggs	1 case (30 doz.), \$5.10	20¢ per doz.
13.	Soap	1 box (240 cakes), \$7.50	6 for 25¢
14.	Corn	1 case (24 cans), \$1.70	3 for 25¢
15.	Beans	1 case (24 cans), \$2.50	2 for 25¢
16.	Peaches	1 case (24 cans), \$3.20	3 for 50¢
17.	Oranges	1 crate (96 oranges), \$3	50¢ per doz.
18.	Oranges	1 crate (126 oranges), \$2.80	40¢ per doz.

Stationers' Measures

209. Paper is sold in tablets by the 100, 200, 250, 500, and 1000 sheets, and by the pound or 100 pounds.

Folded sheets of paper are often sold by the following measures:

	24 sheets = 1 quire
	20 quires = 480 sheets = 1 ream

EXERCISES

210. 1. If a girl uses 1 doz. sheets of paper and 1 doz. envelopes per month, how many boxes of stationery, each containing 1 quire of paper and 24 envelopes, will she use in a year?

2. A boy buys a $2\frac{1}{2}$-quire box of writing paper. How many letters of 1 sheet each can he write on the paper?

3. Mr. Stahl bought a 5-quire box of writing paper and a "$\frac{1}{8}$-thousand" box of envelopes. How many more envelopes than sheets of paper did he buy?

4. Find the cost of 5 pounds of paper at 80¢ per pound and 500 envelopes at $3.40 per thousand.

5. How many quires of foolscap paper are required to furnish 48 school children 2 sheets apiece during an examination?

Find the cost of the paper at $1.60 per ream.

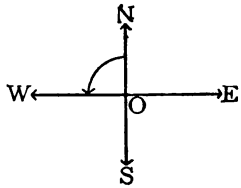
6. If legal cap that weighs 14 pounds to the ream costs 45¢ per $\frac{1}{4}$ ream, find the weight and cost of a quire.

7. The writing paper used by a boys' club cost 18¢ per quire. How much did it cost per sheet? per 100 sheets?

The envelopes cost $3 per thousand. Find the cost of sending 100 invitations, each requiring a sheet of paper, an envelope, and a 2-cent stamp.

Measurement of Angles and Arcs

211. 1. Face toward the north, then turn until you face the west. Continue to turn until you face the north again.

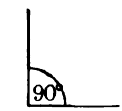


2. What part of a complete turn do you make in turning from north to west? from west to south? from south to east? from north to south?

3. Which is the greater amount of turning or change of direction — from north to northwest or from north to west? from north to west or from north to southwest?

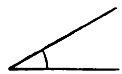
212. The amount of turning, represented by the difference of direction of two lines, is called an **angle**.

213. The size of an angle is usually expressed in **degrees**, a degree being $\frac{1}{360}$ of a complete turn. A complete turn, then, measures 360 degrees (360°).



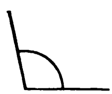
RIGHT ANGLE

214. One fourth of a complete turn, or an angle of 90°, is called a **right angle**.



ACUTE ANGLE

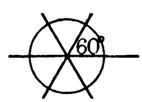
215. An angle that is less than a right angle is called an **acute angle**.



OBTUSE ANGLE

216. An angle that is greater than a right angle but less than two right angles is called an **obtuse angle**.

217. The size of an angle does not depend on the length of the lines that form its *sides*, but it depends only on the difference of direction of the sides, or on the amount of turning required to change the direction of one side to that of the other.

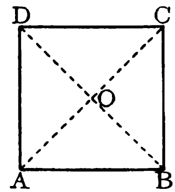


For example, these six angles are all equal, each being 60°, although the sides are of different lengths.

EXERCISES

218. 1. Make a paper square. What kind of angles do the sides form? How many degrees are there in each?

2. Draw two lines connecting the opposite corners, as shown in the diagram.



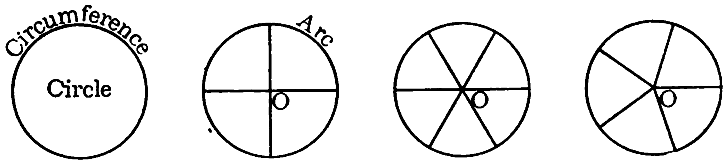
These lines are called **diagonals.**

3. Fold the paper square so that *A* lies on *C*. Crease it, then unfold it and notice whether the diagonal *BD* coincides with the crease.

How, then, are the right angles at *B* and *D* divided by the diagonal *BD*? How many degrees are there in each smaller angle at *B* and at *D*?

4. Fold the square along *BD*, and then so that *D* lies on *B*. Compare the angles at *O*. How many degrees are there in each?

219. The boundary line of a circle is called its **circumference.** Any part of the circumference is called an **arc.**



220. 1. In the second figure, what part of 360° is each angle? What part of the circumference is each arc?

2. What part of 360° is each angle and what part of the circumference is each arc in the third figure? in the fourth?

3. In these figures *O*, the *center* of each circle, is also the *vertex* of each angle. Such angles are called **central angles.**

4. Since an arc is the same part of a circumference that its central angle is of 360°, we measure *arcs* as well as *angles* in *degrees*, calling a circumference 360°.

5. How many degrees are there in an arc that is $\frac{1}{4}$ of a circumference? $\frac{1}{6}$ of a circumference? $\frac{1}{10}$ of a circumference?

6. What part of 360° is an angle or an arc of 1 degree?

7. Angle degrees and arc degrees are divided into 60 parts called **minutes**, and minutes into 60 parts called **seconds.**

8. Memorize this table of **circular measures**:

60 seconds (60″)	**= 1 minute (1′)**
60 minutes	**= 1 degree (1°)**
360 angle degrees	**= 4 right angles**
360 arc degrees	**= 1 circumference**

EXERCISES

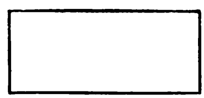
221. 1. How many degrees are there in $\frac{1}{8}$ of a circumference? in $\frac{1}{12}$ of a circumference? in 1200′? in 150′? in 2° 15′?

2. How many minutes are there in a right angle? in 5°? in 3° 40′? in 90″?

3. How many seconds are there in 40′? in 1°? in 10′ 20″? in 5′ 30″? in 2°? in $\frac{3}{4}$ of a minute?

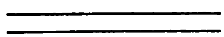
Measurement of Surfaces

222. A figure that has four straight sides and four right angles is called a **rectangle.**



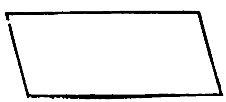
If the sides of a rectangle are all equal, it is called a **square.**

223. Lines that cannot meet, however far they are extended, are called **parallel lines.**



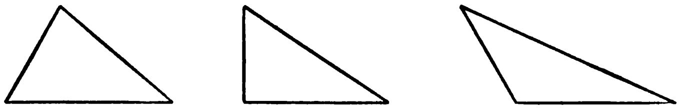
224. A four-sided figure whose opposite sides are parallel is called a **parallelogram.**

A rectangle is a right-angled parallelogram.



225. A three-sided figure is called a **triangle.**

A triangle is *acute-angled* if all its angles are acute; *right-angled* if it has one right angle; *obtuse-angled* if it has one obtuse angle.

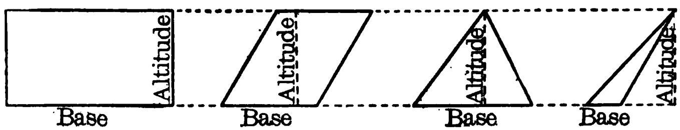


ACUTE-ANGLED TRIANGLE RIGHT-ANGLED TRIANGLE OBTUSE-ANGLED TRIANGLE

226. The two lines that form a right angle are said to be **perpendicular** to each other.

227. The side of a figure on which it is assumed to stand is called the **base**, and the height of the figure measured on a line perpendicular to the base is called the **altitude.**

The following figures have different bases but the same altitude.



228. Rectangles.

1. What is the area of a rectangle whose base is 24 in. and altitude 2 in.?

Find the area if the base is 2 ft. and the altitude is 2 in.

2. How is the area of a rectangle found?

What must be done if the base and altitude are expressed in different units?

3. Find the area of a rectangle whose base is 12 ft. and altitude 1 ft. How many times as great must the altitude be to make the area 48 sq. ft.?

4. The area of a certain rectangle is 48 sq. ft. If the base is 6 ft., what is the altitude?

If the base is 12 ft., what is the altitude?

If the altitude is 3 ft., what is the base?

229. If the base and altitude of a rectangle are expressed in *inches*, the area is found by multiplying the *number* of inches in the base by the *number* of inches in the altitude and calling the result *square inches*. If both dimensions are in *feet*, the area is found by multiplying one dimension by the other and calling the result *square feet*.

This is what is meant by the following statement:

The area of a rectangle is equal to the product of its base and altitude, expressed in like units.

It is seen that *either dimension is the quotient of the area and the other dimension, expressed in corresponding units.*

WRITTEN EXERCISES

230. 1. Find the area of a rectangle 12 ft. by 2 ft. 6 in.

SOLUTION

Base = 12 ft.

Alt. $= 2\frac{1}{2}$ ft.

Area $= (2\frac{1}{2} \times 12)$ sq. ft. $= 30$ sq. ft.

Since the base and altitude must be expressed in *like units*, 2 ft. 6 in. is first changed to *feet*. Then the number of *square feet* in the area is found by multiplying 12 by $2\frac{1}{2}$.

2. If the area of a rectangular field is 24 acres and the length is 60 rods, what is the width?

SOLUTION

Area $= 24$ A. $= 24 \times 160$ sq. rd.

Length $= 60$ rd.

Width $= \dfrac{\overset{4}{\cancel{24}} \times 16\cancel{0}}{\cancel{6}\cancel{0}}$ rd. $= 64$ rd.

Since the area and the length must be expressed in *corresponding units*, 24 A. is first changed to *square rods*. Then, the number of *rods* in the width is found by dividing 24×160 by 60.

The following incomplete table refers to various rectangles; complete the missing areas and dimensions:

	Base	Altitude	Area
3.	21 ft. 6 in.	13 ft.	—— sq. ft.
4.	16.4 ft.	15 ft.	—— sq. ft.
5.	35 ft. 2 in.	12 ft.	—— sq. ft.
6.	——	7 in.	59.5 sq. in.
7.	18 yd.	——	252 sq. yd.
8.	$44\frac{3}{4}$ ft.	12 yd.	—— sq. ft.
9.	24 yd.	32 yd. 2 ft.	—— sq. yd.
10.	$12\frac{1}{2}$ yd.	60 ft.	—— sq. ft.
11.	85 rd.	——	42.5 acres
12.	11 rd.	117 ft.	—— sq. ft.
13.	——	10 ft. 6 in.	6048 sq. in.

14. Draw a square whose area is 1 sq. ft. Draw a rectangle 16 in. long, having the same area as the square.

15. A rectangular farm containing 80 acres was 160 rods long. How wide was the farm?

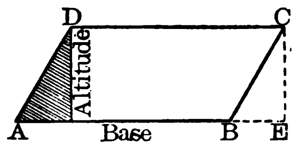
16. A football field is 160 ft. wide, and its area is 52,800 sq. ft. Find the length of such a field.

17. An acre is equal to 43,560 sq. ft. and a mile is equal to 5280 ft. If a gang plow throws 4 furrows, each 9 in. wide, how many miles must the plow travel to overturn an acre of land?

18. How many miles does a man walk in plowing 4 acres with a plow that throws a furrow 12 in. wide?

19. A dealer bought 5000 sheets of Imperial wove bond paper, 10 in. by 16 in., and made them into commercial note tablets, each containing 100 sheets 5 in. by 8 in. Find the number of tablets that he made.

231. Parallelograms.



1. If the shaded part of the parallelogram $ABCD$ is cut off and placed in the position BCE, what kind of a figure will be obtained?

2. Compare the base of the parallelogram with the base of the rectangle. Compare the altitudes. Compare the areas.

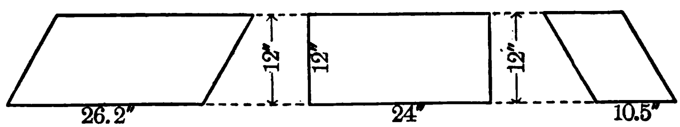
3. Draw any parallelogram. How does its area compare with that of a rectangle having the same base and altitude?

How, then, may the area of a parallelogram be found?

The area of a parallelogram is equal to the product of its base and altitude, expressed in like units.

WRITTEN EXERCISES

232. 1. Find the area of each of these parallelograms.



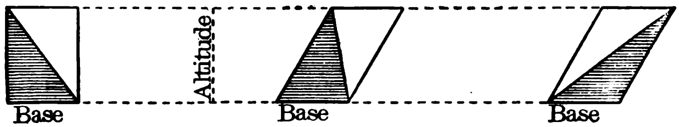
2. If the altitude of a parallelogram is 14 in. and the area is 308 sq. in., what is the base? Draw such a parallelogram.

Compute the missing areas and dimensions:

	Base	Altitude	Area
3.	22 ft.	3 ft. 6 in.	—— sq. ft.
4.	42 in.	16 ft.	—— sq. ft.
5.	125.1 ft.	40 yd.	—— sq. ft.
6.	——	87 rd.	4263 sq. rd.
7.	4 rd.	88 ft.	—— sq. ft.
8.	160 rd.	—— rd.	95 acres

233. Triangles.

1. What part of each of these three parallelograms is the shaded triangle it contains?



2. How is the area of a parallelogram found? Then how is the area of a triangle found?

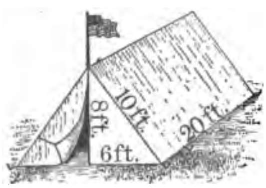
The area of a triangle is equal to half the product of its base and altitude, expressed in like units.

WRITTEN EXERCISES

234. Find the areas of triangles having these dimensions:

1. Base 25 in., altitude 18 in.
2. Base 42 ft., altitude 16 ft.
3. Base 33.4 ft., altitude 48 ft.
4. Base 106 ft. 3 in., altitude 60 ft.
5. Base 84 rd., altitude 80 rd. (Area in acres.)
6. Base 120 rd., altitude 72 rd.
7. Base 111.8 miles, altitude 96 miles.

8. The roof of this tower is composed of five triangular parts, each having a base of 7 ft. and an altitude of 9 ft. Find the area of the roof.



9. A man wishes to make a tent like this one, 20 ft. long, 12 ft. wide, 8 ft. high, and 10 ft. along the slanting edge. How many square yards of canvas does he need for each end? for each side? for the whole tent?

10. Find the area of the floor of the tent in square feet.

Measurement of Rectangular Solids

235. 1. If the part of the wall just begun is 12 ft. long, 2 ft. wide, and 1 ft. high, how many cubic feet of stone masonry has the mason laid?

2. How is this number of cubic feet found?

3. How many cubic feet of masonry will the wall contain when it is $4\frac{1}{2}$ times as high as it is now?

4. How is the volume of a *rectangular solid* 12 ft. long, 2 ft. wide, and $4\frac{1}{2}$ ft. high found?

The volume of a rectangular solid is equal to the product of its length, breadth, and thickness, all expressed in like units.

WRITTEN EXERCISES

236. Find the volume of rectangular solids having the following dimensions:

1. Length $19\frac{1}{4}$ ft., breadth 4 ft., thickness 2 ft.
2. Length 28 in., breadth 16 in., thickness $7\frac{1}{2}$ in.
3. Length 49 ft., breadth 14 ft., thickness 4 ft. 6 in.
4. Length 36 ft., breadth $2\frac{1}{2}$ yd., thickness 5 yd.
5. Length 22.75 ft., breadth 6 ft., thickness 4 ft.
6. Length $112\frac{1}{2}$ ft., breadth 5 ft., thickness 3 ft.
7. Find the volume of a stone sill 15 ft. long, 2 ft. wide, and 8 in. thick.
8. Find the volume of a cake of ice 3 ft. 2 in. long, 1 ft. 8 in. wide, and 14 in. thick.

PROBLEMS IN REVIEW

237. 1. To measure distances on this map make a paper scale 3 in. long. Divide the first inch into 20 equal parts, as shown on the map. What distance does $\frac{1}{20}$ in. represent? $\frac{1}{5}$ in.?

Find the approximate air-line distance between:

2. Omaha and Chicago.
3. Chicago and Boston.
4. St. Paul and Buffalo.
5. Portland and Baltimore.
6. Kansas City and Detroit.
7. Norfolk and New York.

8. Cleveland and Washington; Chicago and Philadelphia; Minneapolis and St. Louis; Indianapolis and Montreal.

9. Find the approximate area of Kansas; of Pennsylvania.

10. How much wider is Ohio than Indiana?

11. The distance by rail from New York to Cleveland is 568 miles. How much less than this is the air-line distance?

12. The distance from Buffalo to Troy is 303 miles by rail, and 345 miles by the Erie Canal. Compare each of these distances with the air-line distance.

13. Ralph was examining the fire engine *Steamer 16* one day, when the fire alarm sounded. In 25 seconds the steamer and chemical wagon were out on the street; in 54 seconds more they had reached the scene of the fire. It took 28 seconds to connect with the hydrant; and 17 seconds later the steamer was throwing water upon the fire. How many minutes and seconds

was it from the sounding of the alarm to the time when the water reached the fire?

14. The chemical wagon unreeled a 250-foot length of chemical hose and five 150-foot lengths of steamer hose. *Steamer 11* and *Chemical* then arrived with 500 feet of steamer hose and 250 feet of chemical hose. How much hose of each kind was used?

15. The first steamer pumped 900 gallons of water per minute for 1 hr. 40 min.; the second 500 gallons per minute for 1 hr. 19 min.; and a third steamer 750 gallons per minute for $1\frac{1}{2}$ hr. How much water was used to put out the fire?

16. Find the value of this water at \$.10 per 1000 gallons.

17. During the fire the three steamers used 1200 pounds of cannel coal costing \$18 per ton. Find the expense for fuel.

18. Between fires the water in the boiler of a fire engine is kept hot continually at an average cost of about 8¢ per day for fuel. Find the cost per year (365 days).

19. A hook-and-ladder truck weighs about 9600 pounds. Express the weight in tons.

FRACTIONS

Review

238. 1. Write a fraction with figures.

Define a fraction; the terms of a fraction. What does the denominator show? the numerator?

2. Write seventeen eighteenths; twenty-five fortieths; sixty-five seventy-fifths; eleven one-hundred-twentieths.

3. Reduce $\frac{8}{12}$ to its lowest terms; to twenty-fourths.

When is a fraction expressed in its lowest terms? How may a fraction be reduced to higher terms?

4. Reduce 4 to eighths; $7\frac{3}{4}$ to fourths.

What is an integer? a mixed number? How may integers and mixed numbers be reduced to fractions?

5. Reduce $\frac{24}{8}$ to an integer; $\frac{16}{5}$ to a mixed number.

What is an improper fraction? a proper fraction?

How may an improper fraction be reduced to an integer or a mixed number? What is the value of a fraction?

6. Add $\frac{3}{8}$ and $\frac{7}{8}$. Subtract $\frac{2}{6}$ from $\frac{5}{6}$.

What are similar fractions? What must be done to fractions that are not similar before they can be added or subtracted?

7. Define reduction.

8. Multiply $\frac{2}{5}$ by 4; $\frac{3}{8}$ by 4.

How do you multiply a fraction by an integer?

9. Divide $\frac{4}{5}$ by 2; $\frac{3}{4}$ by 2.

Tell how to divide a fraction by an integer.

10. Multiply 6 by $\frac{2}{3}$. Divide 6 by $\frac{2}{3}$.

How do you multiply an integer by a fraction?

Tell how to divide an integer by a fraction. What is the reciprocal of a fraction?

EXERCISES

239. **1.** Reduce to lowest terms: $\frac{6}{8}$; $\frac{9}{12}$; $\frac{8}{10}$; $\frac{10}{12}$; $\frac{8}{14}$; $\frac{12}{16}$; $\frac{15}{20}$; $\frac{20}{24}$; $\frac{24}{32}$.

2. Reduce to an improper fraction: $4\frac{1}{2}$; $6\frac{2}{3}$; $9\frac{1}{4}$; $7\frac{5}{6}$; $8\frac{4}{5}$.

3. Reduce to an integer or a mixed number: $\frac{10}{2}$; $\frac{12}{5}$; $\frac{14}{4}$; $\frac{16}{8}$; $\frac{15}{3}$; $\frac{18}{5}$; $\frac{22}{6}$; $\frac{28}{8}$.

4. Reduce to fractions having the least common denominator: $\frac{3}{8}$ and $\frac{1}{4}$; $\frac{2}{3}$ and $\frac{3}{4}$; $\frac{5}{6}$ and $\frac{5}{8}$; $\frac{11}{12}$ and $\frac{7}{8}$.

Give answers promptly:

5. $\frac{3}{4}+\frac{2}{3}$	**13.** $\frac{3}{4}\div 4$	**21.** $\frac{4}{5}+\frac{3}{2}$	**29.** $8\times 2\frac{1}{2}$
6. $\frac{5}{6}-\frac{1}{2}$	**14.** $8\div\frac{1}{5}$	**22.** $\frac{8}{3}-\frac{3}{4}$	**30.** $6\frac{3}{4}\times 4$
7. $\frac{2}{3}$ of 9	**15.** $\frac{3}{5}+\frac{1}{4}$	**23.** $\frac{7}{4}\times 6$	**31.** $3\frac{1}{3}\div 2$
8. $\frac{3}{4}$ of 7	**16.** $4\div\frac{2}{3}$	**24.** $\frac{5}{2}\div 4$	**32.** $6\div 1\frac{1}{3}$
9. $5\times\frac{3}{8}$	**17.** $\frac{5}{8}-\frac{1}{8}$	**25.** $8\times\frac{5}{8}$	**33.** $3\frac{1}{2}-1\frac{7}{8}$
10. $\frac{4}{5}\times 6$	**18.** $8\times\frac{3}{4}$	**26.** $9\times\frac{6}{5}$	**34.** $1\frac{4}{5}+6\frac{3}{4}$
11. $\frac{6}{7}\div 2$	**19.** $6\times\frac{4}{5}$	**27.** $\frac{12}{5}\div 4$	**35.** $5\frac{1}{4}+2\frac{3}{8}$
12. $\frac{1}{8}\div 8$	**20.** $\frac{5}{6}\div 2$	**28.** $\frac{11}{4}\div 3$	**36.** $4\frac{2}{3}-3\frac{1}{2}$

37. There are 30 days in April and we received the daily paper on $\frac{5}{6}$ of them. How many papers did we receive?

38. If a newsboy made $\$\frac{1}{10}$ profit by selling some papers for $\$\frac{2}{5}$, how much did he pay for them?

39. Jane has gone to school $\frac{2}{3}$ as many years as Roy. She has attended for 4 years. How long has Roy attended?

40. When you study $4\frac{3}{4}$ hours a day in school and $2\frac{3}{8}$ hours at home, how long do you study each day?

41. How wide must I cut a strip of silk to make a ruffle $5\frac{1}{4}$ in. wide with a hem of $1\frac{3}{8}$ in. on one side and $\frac{3}{8}$ in. on the other?

42. How many yards of silk, 21 in. wide, will be needed to make this ruffle $5\frac{7}{8}$ yards long, allowing $\frac{1}{8}$ yd. for the seams?

43. When a steamship had slowed down to $\frac{1}{4}$ speed, it was going at the rate of 5 miles an hour. What was its rate at full speed?

44. When my pencil was new, it was $7\frac{1}{2}$ inches long. After I had used it a week, it was only $5\frac{3}{4}$ inches long. How much of it did I use in a week?

45. A certain locality had $4\frac{1}{3}$ inches of rainfall in one month and $2\frac{5}{6}$ inches the next. How much was the rainfall for both months?

46. A door is 8 feet high and its height is $2\frac{2}{3}$ times its width. How wide is it?

47. What is the area of the top of a table that is 10 feet long and $3\frac{1}{2}$ feet wide?

48. If a town has $9\frac{4}{5}$ miles of asphalt pavement and $5\frac{1}{2}$ miles of macadam, how much more asphalt than macadam has it?

WRITTEN EXERCISES

240. Reduce to lowest terms:

1. $\frac{44}{64}$	**3.** $\frac{84}{132}$	**5.** $\frac{125}{375}$	**7.** $\frac{216}{576}$	**9.** $\frac{330}{792}$
2. $\frac{36}{96}$	**4.** $\frac{72}{240}$	**6.** $\frac{192}{448}$	**8.** $\frac{288}{648}$	**10.** $\frac{625}{1000}$

11. Reduce to an improper fraction: $7\frac{5}{12}$; $8\frac{3}{10}$; $14\frac{3}{4}$; $19\frac{2}{3}$; $27\frac{4}{5}$; $32\frac{7}{8}$; $40\frac{5}{6}$; $51\frac{3}{5}$.

12. Reduce to a whole or a mixed number: $\frac{76}{2}$; $\frac{64}{3}$; $\frac{95}{4}$; $\frac{138}{6}$; $\frac{156}{8}$; $\frac{246}{10}$; $\frac{345}{12}$; $\frac{464}{16}$; $\frac{760}{24}$.

Do as the signs indicate:

13. $\frac{1}{3}+\frac{3}{4}+\frac{5}{8}$	**15.** $\frac{7}{8}-\frac{2}{3}+\frac{1}{4}$	**17.** $1\frac{2}{3}+3\frac{1}{4}-\frac{7}{8}$
14. $\frac{5}{6}+\frac{2}{3}+\frac{23}{24}$	**16.** $\frac{9}{16}+\frac{5}{8}-\frac{1}{2}$	**18.** $3\frac{1}{6}-1\frac{3}{8}+\frac{17}{24}$

First add and then subtract:

19. $43\frac{3}{4}$	**20.** $75\frac{4}{5}$	**21.** $94\frac{1}{6}$	**22.** $126\frac{3}{10}$	**23.** $291\frac{7}{10}$
$27\frac{5}{8}$	$42\frac{1}{3}$	$38\frac{7}{8}$	$46\frac{2}{5}$	$145\frac{5}{6}$

Add:

24. $21\frac{1}{3}$	**25.** $45\frac{1}{4}$	**26.** $88\frac{6}{7}$	**27.** $195\frac{3}{8}$	**28.** $765\frac{3}{4}$
$46\frac{3}{4}$	$17\frac{5}{6}$	$45\frac{3}{4}$	$346\frac{4}{5}$	$291\frac{5}{8}$
$16\frac{7}{8}$	$91\frac{2}{3}$	$24\frac{1}{2}$	$562\frac{1}{4}$	$452\frac{11}{12}$

Find in the shortest way:

29. $96 \times \frac{5}{6}$

30. $84 \times \frac{7}{8}$

31. $\frac{2}{3} \div 24$

32. $36 \div \frac{3}{5}$

33. $\frac{3}{4}$ of 62

34. $\frac{7}{12} \times 76$

35. $\frac{15}{16} \div 10$

36. $419\frac{1}{6} \div 5$

37. $2653 \div \frac{7}{8}$

38. $24 \times 563\frac{4}{5}$

39. $672\frac{3}{4} \div 36$

40. $548 \times 42\frac{2}{3}$

41. $860 \div 68\frac{4}{7}$

42. $\frac{5}{12}$ of 4298

43. $391\frac{7}{8} \times 124$

Find the cost of:

44. 25 plows at \$$6\frac{3}{4}$ each.

45. 18 horse rakes at \$$14\frac{9}{10}$ each.

46. 52 wagons at \$$46\frac{13}{20}$ each.

47. $16\frac{1}{2}$ tons of hay at \$13 per ton.

48. 612 bushels of corn at \$$\frac{5}{8}$ per bushel.

49. 1276 barrels of potatoes at \$$1\frac{7}{8}$ per barrel.

50. Find the annual cost of fuel to a manufacturer who uses 1800 tons of coal per year at \$$2\frac{3}{4}$ per ton.

51. A farmer put 1225 bushels of apples into barrels holding $2\frac{1}{2}$ bushels each. How many barrels were there?

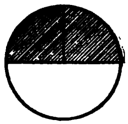
52. If a Hudson River steamboat goes 144 miles in $9\frac{3}{5}$ hours, what is its average rate per hour?

53. A farmer in the state of New York one year raised 948 bushels of grain on $16\frac{1}{2}$ acres of ground. What was the average yield per acre?

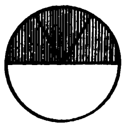
54. An automobile went 128 miles one day, $\frac{7}{8}$ as far the next, and $1\frac{1}{4}$ times as far the third day as the second. How far did it run in the three days?

Multiplication of Fractions

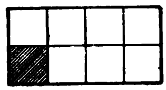
241. Multiplication of fractions by fractions.



1. What part of a circle is $\frac{1}{2}$ of $\frac{1}{2}$ of it?
George had $\frac{1}{2}$ of a dollar, and he gave $\frac{1}{2}$ of his money to his sister. What part of a dollar did he give her?

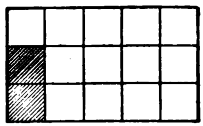


2. What part of a circle is $\frac{1}{3}$ of $\frac{1}{2}$ of it?
Emma cut $\frac{1}{2}$ of a pie into thirds and gave $\frac{1}{3}$ of $\frac{1}{2}$ to James. What part of the pie did she give him?



3. What part of a rectangle is $\frac{1}{2}$ of $\frac{1}{4}$ of it? $\frac{1}{4}$ of $\frac{1}{2}$ of it? $\frac{2}{4}$ of $\frac{1}{2}$ of it? $\frac{3}{4}$ of $\frac{1}{2}$ of it?

4. What part of a rectangle is $\frac{1}{3}$ of $\frac{1}{5}$ of it? $\frac{2}{3}$ of $\frac{1}{5}$ of it? $\frac{2}{3}$ of $\frac{2}{5}$ of it? $\frac{2}{3}$ of $\frac{4}{5}$ of it?



5. How does $\frac{1}{3}$ of $\frac{1}{5}$ compare in value with $\frac{1\times 1}{3\times 5}$? $\frac{2}{3}$ of $\frac{1}{5}$ with $\frac{2\times 1}{3\times 5}$? $\frac{2}{3}$ of $\frac{2}{5}$ with $\frac{2\times 2}{3\times 5}$? $\frac{2}{3}$ of $\frac{4}{5}$ with $\frac{2\times 4}{3\times 5}$?

6. Find the value of $\frac{1}{2}$ of $\frac{1}{3}$; $\frac{1}{2}$ of $\frac{2}{3}$; $\frac{1}{2}$ of $\frac{3}{4}$; $\frac{1}{3}$ of $\frac{1}{4}$; $\frac{1}{4}$ of $\frac{1}{3}$; $\frac{2}{3}$ of $\frac{1}{4}$; $\frac{1}{4}$ of $\frac{2}{3}$; $\frac{1}{3}$ of $\frac{3}{4}$; $\frac{3}{4}$ of $\frac{1}{3}$; $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{3}{4}$ of $\frac{2}{3}$.

242. Finding a fractional part of a fraction is called **multiplying a fraction by a fraction.**

The word "of" between two fractions signifies multiplication, and when the sign × is used between two fractions, it may be read "of."

Such expressions are sometimes called **compound fractions.**

243. To multiply a fraction by a fraction, *multiply the numerators together for the numerator of the product and the denominators for the denominator of the product.*

EXERCISES

244. Find quickly:

1. $\frac{1}{2}$ of $\frac{1}{6}$	**6.** $\frac{4}{5} \times \frac{1}{3}$	**11.** $\frac{3}{4}$ of $\frac{1}{6}$	**16.** $\frac{4}{5} \times \frac{5}{6}$
2. $\frac{1}{3}$ of $\frac{1}{3}$	**7.** $\frac{1}{2} \times \frac{2}{7}$	**12.** $\frac{2}{3}$ of $\frac{3}{5}$	**17.** $\frac{5}{8} \times \frac{2}{5}$
3. $\frac{1}{6}$ of $\frac{1}{4}$	**8.** $\frac{1}{4} \times \frac{4}{5}$	**13.** $\frac{1}{3}$ of $\frac{6}{7}$	**18.** $\frac{3}{4} \times \frac{5}{6}$
4. $\frac{1}{2}$ of $\frac{4}{5}$	**9.** $\frac{3}{5} \times \frac{1}{2}$	**14.** $\frac{5}{6}$ of $\frac{2}{3}$	**19.** $\frac{3}{5} \times \frac{3}{4}$
5. $\frac{1}{3}$ of $\frac{3}{8}$	**10.** $\frac{2}{3} \times \frac{1}{6}$	**15.** $\frac{2}{3}$ of $\frac{7}{8}$	**20.** $\frac{3}{4} \times \frac{4}{7}$

21. What is the area of the cover of a book, if it is $\frac{1}{2}$ of a foot long and $\frac{1}{3}$ of a foot wide?

22. The teacher lives $\frac{1}{3}$ of a mile from school and Bessie, $\frac{2}{5}$ as far. How far does Bessie have to walk to school?

23. The champion runner of our school circled a $\frac{1}{3}$-mile track in a minute. What distance did he run in $\frac{2}{3}$ of a minute?

24. It takes a single horse $\frac{6}{7}$ of a day to plow a field that a two-horse team can plow in $\frac{2}{5}$ of the time. In what part of a day can the team plow it?

25. If your father's garden plot contains $\frac{3}{4}$ of an acre and he gives you $\frac{1}{5}$ of it for your garden, what part of an acre does your garden contain?

WRITTEN EXERCISES

245. **1.** Find $\frac{4}{5}$ of $\frac{7}{12}$, or multiply $\frac{7}{12}$ by $\frac{4}{5}$.

$$\frac{4}{5} \times \frac{7}{12} = \frac{\cancel{4} \times 7}{5 \times \underset{3}{\cancel{12}}} = \frac{7}{15}$$

To find $\frac{1}{5}$ of $\frac{7}{12}$ we divide $\frac{7}{12}$ by 5. You have learned that this may be done by multiplying the denominator by 5; then, $\frac{1}{5}$ of $\frac{7}{12} = \frac{7}{5 \times 12}$, and $\frac{4}{5}$ of $\frac{7}{12} = 4$ times $\frac{1}{5}$ of $\frac{7}{12}$, or $\frac{4 \times 7}{5 \times 12}$.

Or $$\frac{\cancel{4}}{5} \times \frac{7}{\underset{3}{\cancel{12}}} = \frac{7}{15}$$

Canceling and multiplying, the result is found to be $\frac{7}{15}$.

It is not necessary to rewrite the fractions. We may simply cancel as in the second process.

Find:

2. $\frac{4}{5}$ of $\frac{5}{8}$ **5.** $\frac{6}{7}$ of $\frac{14}{15}$ **8.** $\frac{27}{28} \times \frac{7}{9}$ **11.** $\frac{5}{12} \times \frac{18}{25}$

3. $\frac{5}{7}$ of $\frac{7}{9}$ **6.** $\frac{4}{5}$ of $\frac{15}{16}$ **9.** $\frac{15}{32} \times \frac{4}{5}$ **12.** $\frac{15}{16} \times \frac{32}{35}$

4. $\frac{9}{8}$ of $\frac{5}{6}$ **7.** $\frac{5}{6}$ of $\frac{24}{25}$ **10.** $\frac{21}{40} \times \frac{8}{7}$ **13.** $\frac{10}{34} \times \frac{17}{50}$

14. Find the value of $\frac{2}{3}$ of $3\frac{3}{4}$.

SUGGESTION. — Reduce the mixed number to an improper fraction.

Find the value of:

15. $\frac{5}{8}$ of $10\frac{4}{5}$ **17.** $15\frac{5}{6} \times \frac{3}{5}$ **19.** $5\frac{2}{3} \times 7\frac{4}{5}$

16. $\frac{3}{4}$ of $13\frac{3}{5}$ **18.** $20\frac{1}{4} \times \frac{2}{9}$ **20.** $6\frac{7}{8} \times 4\frac{3}{10}$

21. Find the value of $3\frac{1}{3} \times \frac{1}{12} \times 3 \times 1\frac{3}{5}$.

$$3\frac{1}{3} \times \frac{1}{12} \times 3 \times 1\frac{3}{5} = \frac{\overset{2}{\cancel{10}}}{\cancel{3}} \times \frac{1}{\underset{3}{\cancel{12}}} \times \frac{\cancel{3}}{1} \times \frac{\overset{2}{\cancel{8}}}{\cancel{5}} = \frac{4}{3} = 1\frac{1}{3}$$

Reducing the mixed numbers to improper fractions, regarding the integer 3 as $\frac{3}{1}$, and canceling, the product is $\frac{4}{3}$, or $1\frac{1}{3}$.

Find the value of:

22. $1\frac{4}{7} \times \frac{3}{4} \times 14$ **25.** $\frac{5}{6} \times \frac{1}{3} \times 12 \times 3\frac{3}{8}$

23. $6\frac{1}{4} \times 8 \times 4\frac{1}{5}$ **26.** $1\frac{1}{15} \times \frac{5}{12} \times 4\frac{7}{8} \times 5\frac{1}{3}$

24. $1\frac{1}{3} \times \frac{1}{10} \times 9 \times 5\frac{1}{6}$ **27.** $2\frac{1}{2} \times \frac{3}{8} \times 9\frac{3}{5} \times \frac{3}{4} \times 1\frac{1}{3}$

28. How many feet are there in $\frac{5}{6}$ of a rod? in $2\frac{3}{4}$ rods?

29. John bought a book for $\$1\frac{3}{4}$ and afterward sold it for $\frac{4}{5}$ of the cost. How much did he receive for it?

30. At $\$6\frac{3}{4}$ per ton, find the cost of $5\frac{1}{2}$ tons of coal; of $11\frac{3}{5}$ tons.

31. Roscoe could dig potatoes $\frac{2}{3}$ as fast as John. How many bushels could he dig while John was digging $42\frac{3}{4}$ bushels?

32. Find the area of a board walk that is $24\frac{3}{8}$ feet long and $3\frac{1}{2}$ feet wide.

33. During a feeding test, a lamb gained $40\frac{1}{2}$ pounds at an average cost of $2\frac{8}{9}$ cents per pound. Find the total cost.

34. How many cubic feet of coal will a bin hold, if it is $12\frac{1}{2}$ feet long, $8\frac{1}{4}$ feet wide, and $6\frac{2}{3}$ feet deep?

35. If butter fat will make $1\frac{1}{6}$ times its weight of butter, and milk is $\frac{1}{24}$ butter fat, how many pounds of butter can be made from 720 pounds of milk?

Division of Fractions

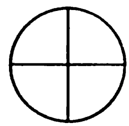
246. Division of fractions by fractions.

1. How many toys can be bought for $\$\frac{4}{5}$ at $\$\frac{1}{5}$ each? at $\$\frac{2}{5}$?

2. How many cups can be filled out of $\frac{6}{8}$ gal., if each holds $\frac{1}{8}$ gal.? $\frac{2}{8}$ gal.? $\frac{3}{8}$ gal.? As the divisor grows larger what happens to the quotient?

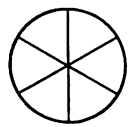
3. $\frac{4}{5} \div \frac{1}{5} = ?$ $\frac{4}{5} \div \frac{2}{5} = ?$ $\frac{6}{8} \div \frac{2}{8} = ?$ $\frac{6}{8} \div \frac{8}{8} = ?$

How may we divide one fraction by another when the fractions are similar?



4. How many times is $\frac{1}{4}$ contained in 1? in $\frac{1}{2}$?

What part of the first result is the second?



5. How many times is $\frac{1}{6}$ contained in 1? in $\frac{1}{2}$? in $\frac{1}{3}$? in $\frac{2}{3}$?

What part of the first result is the second? the third? the fourth?

6. Divide 1 by $\frac{5}{6}$ and from the result find $\frac{1}{2} \div \frac{5}{6}$; $\frac{1}{3} \div \frac{5}{6}$; $\frac{2}{3} \div \frac{5}{6}$.

7. Compare $\frac{1}{2} \div \frac{5}{6}$ with $\frac{1}{2} \times \frac{6}{5}$; $\frac{2}{3} \div \frac{5}{6}$ with $\frac{2}{3} \times \frac{6}{5}$; $\frac{4}{5} \div \frac{2}{5}$ with $\frac{4}{5}$ multiplied by the *reciprocal* of $\frac{2}{5}$.

247. To divide a fraction by a fraction, *change to similar fractions and divide the numerator of the dividend by the numerator of the divisor.* Or,

Multiply the dividend by the reciprocal of the divisor.

EXERCISES

248. Give quotients quickly:

1. $\frac{1}{2} \div \frac{1}{4}$	**6.** $\frac{1}{2} \div \frac{2}{3}$	**11.** $\frac{4}{5} \div \frac{1}{2}$	**16.** $\frac{7}{10} \div \frac{1}{2}$
2. $\frac{2}{3} \div \frac{1}{6}$	**7.** $\frac{5}{6} \div \frac{1}{2}$	**12.** $\frac{3}{4} \div \frac{2}{3}$	**17.** $\frac{9}{10} \div \frac{2}{5}$
3. $\frac{3}{4} \div \frac{1}{8}$	**8.** $\frac{7}{8} \div \frac{1}{4}$	**13.** $\frac{1}{3} \div \frac{2}{5}$	**18.** $\frac{5}{12} \div \frac{2}{3}$
4. $\frac{3}{8} \div \frac{1}{4}$	**9.** $\frac{5}{6} \div \frac{2}{3}$	**14.** $\frac{2}{3} \div \frac{3}{4}$	**19.** $\frac{7}{12} \div \frac{1}{4}$
5. $\frac{3}{4} \div \frac{1}{2}$	**10.** $\frac{3}{8} \div \frac{3}{4}$	**15.** $\frac{3}{4} \div \frac{5}{6}$	**20.** $\frac{11}{12} \div \frac{3}{4}$

21. Mary's mother gave her \$$\frac{3}{4}$ with which to buy ribbon at \$$\frac{1}{4}$ a yard. How many yards could she buy?

22. At \$$\frac{1}{8}$ per pound, how many pounds of copper can be bought for \$$\frac{1}{2}$?

23. How long will $\frac{2}{3}$ of a barrel of flour last a family that uses $\frac{1}{6}$ of a barrel a week?

24. A man took a bicycle trip, covering $\frac{1}{9}$ of the distance each hour. How long had he been riding when he had covered $\frac{2}{3}$ of the distance?

25. A coal dealer wishes to put $\frac{4}{5}$ of a ton of coal into bags that hold $\frac{1}{20}$ of a ton each. How many bags does he need?

WRITTEN EXERCISES

249. **1.** Divide $\frac{15}{16}$ by $\frac{5}{6}$.

$$\frac{15}{16} \div \frac{5}{6} = \frac{\overset{3}{\cancel{15}}}{\underset{8}{\cancel{16}}} \times \frac{\overset{3}{\cancel{6}}}{\cancel{5}} = \frac{9}{8} = 1\frac{1}{8}$$

Since $1 \div \frac{5}{6}$ is $\frac{6}{5}$, $\frac{15}{16} \div \frac{5}{6}$ is $\frac{15}{16}$ of $\frac{6}{5}$, or $\frac{15}{16} \times \frac{6}{5}$. Canceling and multiplying, the result is found to be $\frac{9}{8}$, or $1\frac{1}{8}$.

Divide:

2. $\frac{9}{10}$ by $\frac{3}{8}$	**5.** $\frac{17}{18}$ by $\frac{1}{6}$	**8.** $\frac{5}{6}$ by $\frac{3}{14}$	**11.** $\frac{17}{20}$ by $\frac{5}{12}$
3. $\frac{7}{12}$ by $\frac{4}{9}$	**6.** $\frac{14}{15}$ by $\frac{7}{8}$	**9.** $\frac{7}{8}$ by $\frac{5}{24}$	**12.** $\frac{23}{32}$ by $\frac{3}{16}$
4. $\frac{11}{16}$ by $\frac{3}{4}$	**7.** $\frac{24}{25}$ by $\frac{4}{5}$	**10.** $\frac{8}{9}$ by $\frac{4}{35}$	**13.** $\frac{35}{48}$ by $\frac{7}{24}$

14. Divide $4\frac{3}{4}$ by $\frac{5}{8}$.

SUGGESTION. — Reduce the mixed number to an improper fraction.

Divide:

15. $5\frac{1}{2}$ by $\frac{3}{4}$ **19.** $12\frac{1}{4}$ by $1\frac{2}{5}$ **23.** $28\frac{3}{4}$ by $12\frac{1}{2}$

16. $7\frac{2}{3}$ by $\frac{5}{6}$ **20.** $16\frac{1}{5}$ by $6\frac{3}{4}$ **24.** $44\frac{2}{5}$ by $13\frac{7}{8}$

17. $9\frac{4}{5}$ by $\frac{7}{8}$ **21.** $20\frac{5}{6}$ by $9\frac{3}{8}$ **25.** $270\frac{5}{6}$ by $31\frac{1}{4}$

18. $6\frac{3}{8}$ by $\frac{2}{3}$ **22.** $25\frac{2}{3}$ by $4\frac{5}{7}$ **26.** $462\frac{6}{7}$ by $64\frac{4}{5}$

27. Find the value of $1\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \div \frac{3}{8} \div 3 \div \frac{1}{6}$.

$$1\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \div \frac{3}{8} \div 3 \div \frac{1}{6} = \frac{\cancel{5}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{5}} \times \frac{8}{3} \times \frac{1}{\cancel{3}} \times \frac{\overset{2}{\cancel{6}}}{1} = \frac{16}{3} = 5\frac{1}{3}$$

After reducing all integers and mixed numbers to improper fractions, we take the reciprocals of the fractions that are divisors and write them as multipliers. Canceling and reducing, the result is $5\frac{1}{3}$.

Find the value of:

28. $\frac{3}{8} \times \frac{5}{6} \div \frac{2}{3} \div \frac{3}{4}$ **31.** $\frac{7}{10} \times \frac{5}{8} \div 2\frac{1}{2} \div 4 \div \frac{1}{5}$

29. $\frac{1}{6} \times 7 \div \frac{4}{5} \div \frac{7}{9}$ **32.** $\frac{5}{12} \div 1\frac{1}{3} \div \frac{5}{9} \times \frac{1}{8} \times \frac{5}{6}$

30. $1\frac{2}{3} \div \frac{5}{6} \times \frac{1}{2} \div \frac{6}{7}$ **33.** $\frac{14}{15} \times \frac{2}{7} \div \frac{1}{8} \times \frac{5}{8} \div \frac{3}{4}$

34. Divide $\frac{1}{2}$ of $\frac{3}{8}$ of $\frac{5}{6}$ by $\frac{1}{4}$ of $\frac{5}{12}$.

SUGGESTION. $\frac{1}{2}$ of $\frac{3}{8}$ of $\frac{5}{6}$ divided by $\frac{1}{4}$ of $\frac{5}{12} = \frac{1}{2} \times \frac{3}{8} \times \frac{5}{6} \times \frac{4}{1} \times \frac{12}{5}$.

Divide:

35. $\frac{2}{3}$ of $\frac{5}{7}$ of $\frac{21}{22}$ by $\frac{3}{8}$ of $\frac{2}{11}$ of $\frac{4}{5}$

36. $\frac{5}{6}$ of $\frac{13}{14}$ of $\frac{7}{8}$ by $\frac{1}{3}$ of $\frac{2}{5}$ of $\frac{3}{4}$ of 26

37. $\frac{7}{12}$ of $2\frac{3}{4}$ by $\frac{1}{2}$ of $\frac{7}{10}$ of $\frac{1}{8}$ of $3\frac{2}{3}$

38. $\frac{3}{4}$ of $\frac{15}{28}$ of $4\frac{1}{5}$ by $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{9}{10}$ of $6\frac{1}{4}$

39. $\frac{7}{8}$ of $\frac{11}{12}$ of $\frac{4}{5}$ of 75 by $\frac{1}{4}$ of $\frac{9}{16}$ of $\frac{5}{6}$ of $\frac{11}{24}$ of 32

40. How much candy can you buy for $\$1\frac{1}{2}$ at $\$\frac{2}{5}$ per pound?

41. A street vender has $2\frac{3}{4}$ pecks of chestnuts. How many times will they fill a measure that holds $\frac{1}{8}$ of a peck?

42. How many times can $5\frac{1}{4}$ gallons of vinegar be drawn from a barrel that contains $31\frac{1}{2}$ gallons?

43. A farmer received \$$238\frac{3}{4}$ for some corn at \$$\frac{5}{8}$ per bushel. How many bushels did he sell?

44. A certain kind of barbed wire weighs $1\frac{3}{8}$ lb. per rod. How much does it weigh per foot?

45. If it takes $18\frac{5}{6}$ yards of canvas to make a tent, how many tents can be made out of $150\frac{2}{3}$ yards?

46. If it requires $44\frac{1}{3}$ cubic yards of earth to level the lawn in front of Dr. Robinson's house, how many loads of $1\frac{7}{12}$ cubic yards will be needed?

47. If a corn-fed hog gains $11\frac{3}{4}$ pounds for each bushel of corn that he eats, how many bushels will increase his weight $70\frac{1}{2}$ pounds?

48. An experiment in feeding wheat showed that a hog gained $13\frac{1}{2}$ pounds for each bushel of wheat fed. How many bushels of wheat were required to increase his weight from $225\frac{3}{4}$ pounds to $397\frac{7}{8}$ pounds?

250. Simplifying complex fractions.

You have learned that fractions indicate division, the numerator being the dividend and the denominator the divisor.

We may indicate the division of a fraction by an integer, of an integer by a fraction, or of a fraction by a fraction, in *fractional form*, by writing the dividend above a line and the divisor below.

We may write, $\frac{2}{3} \div 6$ like this, $\frac{\frac{2}{3}}{6}$; $8 \div \frac{3}{4}$ like this, $\frac{8}{\frac{3}{4}}$; and $\frac{1}{2} \div \frac{4}{5}$ like this, $\frac{\frac{1}{2}}{\frac{4}{5}}$.

Such indicated expressions of division are sometimes called **complex fractions.**

When the indicated division is performed, the complex fraction is said to be **simplified.**

WRITTEN EXERCISES

251. **1.** Simplify $\dfrac{\frac{3}{4}}{\frac{5}{6}}$.

SOLUTION. $$\frac{\frac{3}{4}}{\frac{5}{6}} = \frac{3}{4} \div \frac{5}{6} = \frac{3}{\underset{2}{\cancel{4}}} \times \frac{\overset{3}{\cancel{6}}}{5} = \frac{9}{10}$$

Simplify:

2. $\dfrac{\frac{5}{8}}{\frac{3}{4}}$ **6.** $\dfrac{8}{\frac{4}{5}}$ **10.** $\dfrac{1\frac{3}{4}}{1\frac{1}{2}}$ **14.** $\dfrac{4\frac{7}{8}}{19\frac{1}{2}}$ **18.** $\dfrac{3\frac{1}{4}+\frac{2}{3}}{6\frac{3}{4}-\frac{7}{8}}$

3. $\dfrac{\frac{6}{7}}{\frac{2}{5}}$ **7.** $\dfrac{\frac{3}{8}}{6}$ **11.** $\dfrac{2\frac{1}{2}}{3\frac{1}{3}}$ **15.** $\dfrac{8\frac{1}{3}}{33\frac{1}{3}}$ **19.** $\dfrac{\frac{2}{3}\text{ of }8}{6\frac{2}{3}}$

4. $\dfrac{\frac{7}{8}}{\frac{7}{12}}$ **8.** $\dfrac{6}{3\frac{1}{3}}$ **12.** $\dfrac{3\frac{3}{4}}{6\frac{2}{3}}$ **16.** $\dfrac{6\frac{1}{4}}{37\frac{1}{2}}$ **20.** $\dfrac{2\frac{9}{10}}{\frac{1}{2}\text{ of }\frac{4}{5}}$

5. $\dfrac{\frac{5}{6}}{\frac{15}{16}}$ **9.** $\dfrac{5\frac{1}{2}}{4}$ **13.** $\dfrac{5\frac{1}{5}}{5\frac{4}{7}}$ **17.** $\dfrac{62\frac{1}{2}}{66\frac{2}{3}}$ **21.** $\frac{7}{8}$ of $\dfrac{5}{4\frac{3}{8}}$

252. Finding what part one number is of another.

1. What part of 4 is 3? of 6 is 2? of 12 is 8?

2. What part of \$8 is \$4? of 3 pints is 2 pints? of 4 fifths is 2 fifths? of $\frac{4}{5}$ is $\frac{2}{5}$? of $\frac{5}{6}$ is $\frac{1}{6}$? of $\frac{6}{7}$ is $\frac{4}{7}$?

3. What part of 1 pound is 8 ounces? of 4 yards, or 12 feet, is 6 feet? of $\frac{1}{2}$, or $\frac{2}{4}$, is $\frac{1}{4}$? of $\frac{5}{8}$ is $\frac{1}{4}$, or $\frac{2}{8}$?

4. Tell how to find what part one number is of another.

WRITTEN EXERCISES

253. **1.** What part of 96 is 64? of $3\frac{1}{2}$ is $\frac{7}{8}$?

$$\frac{64}{96} = \frac{2}{3} \qquad \frac{\frac{7}{8}}{3\frac{1}{2}} = \frac{7}{8} \div 3\frac{1}{2} = \frac{7}{8} \div \frac{7}{2} = \frac{\cancel{7}}{\underset{4}{\cancel{8}}} \times \frac{\cancel{2}}{\cancel{7}} = \frac{1}{4}$$

2. What part of 144 is 80? of $\frac{2}{3}$ is $\frac{5}{12}$? of 12 is $\frac{3}{8}$?

What part of

3. 225 is 75?	8. $\frac{5}{6}$ is $\frac{3}{8}$?	13. $8\frac{1}{3}$ is $6\frac{2}{3}$?
4. 896 is 280?	9. $\frac{8}{9}$ is $\frac{4}{5}$?	14. $5\frac{5}{8}$ is $3\frac{3}{4}$?
5. 1000 is 875?	10. $\frac{15}{16}$ is $\frac{5}{8}$?	15. $12\frac{3}{4}$ is $6\frac{4}{5}$?
6. 1728 is 576?	11. 25 is $2\frac{1}{2}$?	16. $31\frac{1}{4}$ is $9\frac{3}{8}$?
7. 2240 is 1344?	12. 75 is $6\frac{1}{4}$?	17. $66\frac{2}{3}$ is $37\frac{1}{2}$?

18. When an acre of land yields $18\frac{3}{4}$ bushels of grain, what part of an acre will yield $12\frac{1}{2}$ bushels?

19. If $26\frac{2}{3}$ pounds of cream produced $6\frac{2}{3}$ pounds of butter fat, what part of the cream was butter fat?

20. The celery from an acre of land sold for \$$218\frac{3}{4}$ and the cost of raising it was \$$87\frac{1}{2}$. What part of the selling price was the cost?

21. The height of the Statue of Liberty is about 150 feet, and the length of the forefinger holding the torch $8\frac{1}{3}$ feet. What part of the whole height is the length of the forefinger?

22. A public park has an area of $325\frac{2}{5}$ acres, of which $108\frac{7}{15}$ acres are occupied by a lake. What fractional part of the park does the lake occupy?

254. Comparison of fractions.

1. Compare $\frac{2}{3}$ with $\frac{1}{3}$ in this way: $\frac{2}{3} \div \frac{1}{3} = 2$; that is, $\frac{2}{3}$ is 2 times $\frac{1}{3}$.

In the same way compare $\frac{4}{5}$ with $\frac{2}{5}$; $\frac{1}{2}$ with $\frac{1}{4}$; $\frac{1}{2}$ with $\frac{1}{8}$; $\frac{1}{3}$ with $\frac{1}{6}$; $\frac{2}{3}$ with $\frac{1}{6}$; $\frac{3}{4}$ with $\frac{1}{2}$.

2. Compare $\frac{1}{3}$ with $\frac{2}{3}$ in this way: $\frac{1}{3} \div \frac{2}{3} = \frac{1}{2}$; that is, $\frac{1}{3}$ is $\frac{1}{2}$ of $\frac{2}{3}$.

In the same way compare $\frac{2}{5}$ with $\frac{4}{5}$; $\frac{1}{4}$ with $\frac{1}{2}$; $\frac{1}{8}$ with $\frac{1}{2}$; $\frac{1}{6}$ with $\frac{1}{3}$; $\frac{1}{6}$ with $\frac{2}{3}$; $\frac{1}{2}$ with $\frac{3}{4}$.

3. Compare $\frac{1}{2}$ with $\frac{1}{6}$; $\frac{1}{6}$ with $\frac{1}{2}$; $\frac{5}{6}$ with $\frac{2}{3}$; $\frac{2}{3}$ with $\frac{5}{6}$; $\frac{1}{2}$ with $\frac{1}{3}$; $\frac{1}{3}$ with $\frac{1}{2}$; $1\frac{1}{4}$ with $\frac{1}{4}$; $\frac{1}{4}$ with $1\frac{1}{4}$.

EXERCISES

255. Compare:

1. $\frac{3}{4}$ with $\frac{3}{8}$	**5.** $\frac{1}{2}$ with $\frac{2}{5}$	**9.** $1\frac{1}{2}$ with $\frac{1}{2}$	**13.** $\frac{2}{3}$ with $1\frac{2}{3}$
2. $\frac{3}{8}$ with $\frac{3}{4}$	**6.** $\frac{1}{4}$ with $\frac{1}{6}$	**10.** $2\frac{3}{4}$ with $\frac{1}{4}$	**14.** $\frac{5}{8}$ with $1\frac{1}{4}$
3. $\frac{1}{3}$ with $\frac{1}{2}$	**7.** $\frac{3}{4}$ with $\frac{5}{6}$	**11.** $1\frac{1}{3}$ with $\frac{2}{3}$	**15.** $\frac{3}{4}$ with $2\frac{1}{2}$
4. $\frac{1}{2}$ with $\frac{2}{3}$	**8.** $\frac{5}{8}$ with $\frac{1}{4}$	**12.** $2\frac{2}{3}$ with $\frac{1}{3}$	**16.** $\frac{5}{6}$ with $3\frac{1}{3}$

17. Compare $1\frac{1}{2}$ with $\frac{1}{4}$. When $\frac{1}{4}$ of a dozen bananas costs 5 cents, find the cost of $1\frac{1}{2}$ dozen.

18. Compare $\frac{1}{4}$ with $1\frac{1}{4}$. When 15 quarts of strawberries cost $\$1\frac{1}{4}$, how many quarts can be bought for $\$\frac{1}{4}$?

19. If my watch gains $\frac{1}{6}$ of a minute in 3 days, in how many days will it gain $1\frac{1}{3}$ minutes?

20. If 24 bunches of celery cost $\$2\frac{2}{5}$, how many bunches can be bought for $\$\frac{3}{5}$?

21. If 3 pounds of live cocoons yield $\frac{1}{4}$ of a pound of silk, how many pounds of cocoons are required for $\frac{7}{8}$ of a pound of silk?

22. John walked $2\frac{1}{4}$ miles in $\frac{3}{4}$ of an hour. At that rate, how long would it take him to walk $1\frac{1}{2}$ miles?

WRITTEN EXERCISES

256. Compare:

1. $\frac{9}{10}$ with $\frac{3}{4}$	**5.** 24 with $6\frac{2}{3}$	**9.** $87\frac{1}{2}$ with $8\frac{1}{3}$
2. $\frac{5}{12}$ with $\frac{2}{3}$	**6.** 88 with $5\frac{3}{5}$	**10.** $15\frac{3}{4}$ with $94\frac{1}{2}$
3. $\frac{15}{16}$ with $\frac{5}{6}$	**7.** $8\frac{1}{4}$ with 66	**11.** $46\frac{2}{3}$ with $6\frac{5}{12}$
4. $\frac{11}{18}$ with $\frac{2}{3}$	**8.** $6\frac{1}{6}$ with 74	**12.** $91\frac{1}{3}$ with $22\frac{5}{6}$

13. Compare $\frac{2}{3}$ with $\frac{11}{12}$. If $\frac{11}{12}$ of a barrel of flour makes 231 loaves of bread, how many loaves will $\frac{2}{3}$ of a barrel make?

14. When 18 barrels of cabbages can be bought for $\$22\frac{1}{2}$, how many barrels can be bought for $\$7\frac{1}{2}$?

15. Compare $3\frac{3}{4}$ with $\frac{3}{5}$. If a woman receives \$$\frac{3}{5}$ for picking 40 quarts of strawberries, how many quarts must she pick to earn \$$3\frac{3}{4}$?

16. A farmer raised $6\frac{2}{3}$ tons of hay on 4 acres. If his whole crop was $86\frac{1}{4}$ tons, how many acres of hay had he?

17. If an automobile runs 8 miles while a train runs $26\frac{1}{4}$ miles, how far does it go while the train runs $7\frac{7}{8}$ miles?

18. When the cost of transporting 90 cans of milk from Medina, Ohio, to Cleveland is \$$13\frac{1}{2}$, how many can be transported for \$$1\frac{1}{5}$?

19. If it takes $3\frac{3}{4}$ bushels of seed to sow $4\frac{1}{2}$ acres of land, how many bushels are needed to sow $28\frac{4}{5}$ acres?

20. An orchard yielded $427\frac{1}{2}$ barrels of apples, averaging $31\frac{2}{3}$ barrels to 10 trees. Find the number of trees in the orchard.

MISCELLANEOUS EXERCISES

257. **1.** Find the cost of $4\frac{3}{8}$ yards of linen at \$$\frac{2}{5}$ per yard.

2. At a certain gold mine it costs \$$\frac{3}{5}$ a ton to mine the ore. How many tons of gold ore can be mined for \$$61\frac{1}{2}$? Find the cost of crushing this quantity of ore at \$$\frac{9}{10}$ per ton.

3. A train running from Boston to Providence averages $6\frac{1}{8}$ miles every 7 minutes. What is its rate per hour? How long does it take the train to go from one city to the other, the distance being 49 miles?

4. The coal used one year to run the trains of a Texas railroad was estimated to be equivalent to 910,938 barrels of oil, reckoning 1 ton of coal to $3\frac{1}{2}$ barrels of oil. How much coal was used?

5. A woman in Ceylon picks 16 pounds of tea leaves in one day from full-grown plants, but only $7\frac{1}{2}$ pounds from young plants. At $\frac{3}{4}$¢ per pound, how much more does she earn per day by picking from old plants?

6. It took $6\frac{1}{2}$ yards of cloth at $\$\frac{1}{2}$ per yard for Elsie's dress, and $11\frac{1}{4}$ yards at $\$\frac{3}{8}$ per yard for her mother's. How much less did the cloth for Elsie's dress cost than for her mother's?

7. A boy bought 90 cocoanuts at $\$\frac{1}{2}$ per dozen, and sold them at $\$\frac{1}{20}$ apiece. How much money did he gain?

8. A grocer bought a bunch of bananas for $\$1\frac{1}{4}$. He sold $3\frac{1}{3}$ dozen from the top at $\$\frac{3}{20}$ per dozen, and the rest, $6\frac{1}{4}$ dozen, at $\$\frac{1}{5}$ per dozen. How much did he gain?

9. If a 224-pound bag of salt costs $\$1\frac{3}{5}$, how much will a 56-pound bag cost at the same rate?

10. When 16 pounds of salt cost $\$\frac{1}{10}$, how many pounds are there in a bag that costs $\$1\frac{2}{5}$?

11. The cost of oiling a stretch of highway in Pennsylvania was \$65,000, of which the state paid $\$43,333\frac{1}{3}$. What part of the cost was paid by the state?

12. Mr. Thayer owned $\frac{3}{4}$ of a section of land. He sold $\frac{5}{8}$ of his land to Mr. Hall, who gave $\frac{2}{5}$ of his part to a son. What part of the whole section did Mr. Hall's son receive? How many acres?

13. In constructing $9\frac{3}{16}$ miles of railroad track 2640 ties were used per mile. Find the whole number of ties used and their cost at $\$\frac{4}{5}$ each.

14. If $57\frac{1}{6}$ bushels of seed are required for a rice field of $24\frac{1}{2}$ acres in South Carolina, and $\frac{5}{6}$ of a bushel for $\frac{3}{8}$ of an acre in Japan, how much seed is required per acre in each place? how much more per acre in South Carolina than in Japan?

15. A grain elevator had a bin $7\frac{1}{8}$ ft. square and 80 ft. deep. How many bushels did it hold, allowing $1\frac{1}{4}$ cu. ft. to the bushel?

16. One side of a tight board fence 45 yd. long and $1\frac{7}{8}$ yd. high was painted two coats. It took $18\frac{3}{4}$ lb. of paint for the first coat and $13\frac{1}{2}$ lb. for the second. How many square yards did a pound of paint cover for the first coat? for the second?

DECIMAL FRACTIONS

Notation and Numeration of Decimals

258. The orders of decimals below thousandths are ten-thousandths, hundred-thousandths, millionths, etc., as shown in the following table.

The orders below millionths are ten-millionths, hundred-millionths, billionths, ten-billionths, etc. They are seldom used.

Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
3	6	5	.	7	4	3	2	9	8
	4	0	.	0	0	2	4		
	8	8	.	3	0	6	9	6	

1. 365.743298 is read "365 *and* 743,298 *millionths.*"
   40.0024 is read "40 *and* 24 *ten-thousandths.*"
   88.30696 is read "88 *and* 30,696 *hundred-thousandths.*"
2. What decimal place is occupied by

Thousandths?	Ten-thousandths?	Hundred-thousandths?
Millionths?	Ten-millionths?	Hundred-millionths?

3. How many decimal figures are required to express thousandths? ten-thousandths? hundred-thousandths? millionths? ten-millionths? hundred-millionths?
4. 1, .1, .01, .001, .0001, .00001, .000001.
   What part is each decimal of the number on its left?
   How many tenths are there in 1? hundredths in .1? etc.

In the Arabic or decimal system of notation, a unit of any order is $\frac{1}{10}$ of the next higher or left-hand unit, and 10 *times the next lower or right-hand unit.*

In reading a decimal, it should be read as an integer, and the denomination of the right-hand figure should be added.

In reading a mixed decimal the word "and" is used between the integral and decimal parts, and not elsewhere.

EXERCISES

259. Read:

1. .44	8. .2563	15. 6.7561
2. .044	9. .02563	16. 4.2837
3. .144	10. .00492	17. 10.0361
4. .0144	11. .06007	18. 78.260005
5. .0072	12. .36091	19. 34.035462
6. .4503	13. .548273	20. 217.38527
7. .7902	14. .048273	21. 3654.0728

WRITTEN EXERCISES

260. Express in figures:

1. 2 hundreds and 25 thousandths.
2. 20 units and 733 thousandths.
3. 625 units and 85 ten-thousandths.
4. 16 thousands, 382 units, and 95 millionths.
5. 485 millions, 7 thousands, and 17 thousandths.
6. 1 million, 1 thousand, 1 unit, and 1 hundred-thousandth.
7. Seventy-five and twenty-one thousandths.
8. Ten and three thousand one hundred six millionths.
9. Ninety-six and four hundred ninety ten-thousandths.

10. Six hundred sixty-six thousand six hundred sixty-six millionths.

11. Six hundred sixty-six thousand and six hundred sixty-six millionths.

12. Four hundred seventeen thousand two hundred six millionths.

13. Four hundred seventy-seven thousand and two hundred sixty-nine millionths.

14. Ninety-six and thirty-two ten-thousandths.

15. Two hundred sixty and three hundred fifteen hundred-thousandths.

Write in words:

16. 401	18. .401	20. .0366	22. 2000.002
17. 400.001	19. 401,000	21. 300.0066	23. 2002.002

Reduction of Decimals

261. Reduction of decimals to common fractions.

WRITTEN EXERCISES

1. Reduce .0625 to a common fraction.

SOLUTION. $.0625 = \frac{625}{10000} = \frac{1}{16}$.

Reduce to common fractions in their lowest terms:

2. .625	9. .03125	16. .53125
3. .1875	10. .09375	17. .65625
4. .3125	11. .15625	18. .78125
5. .4375	12. .21875	19. .96875
6. .5625	13. .28125	20. .890625
7. .6875	14. .34375	21. .609375
8. .8125	15. .40625	22. .484375

262. Reduction of common fractions to decimals.

WRITTEN EXERCISES

1. Reduce $\frac{4}{7}$ to a six-place decimal.

$$7\overline{)4.000000}$$
$$.571429^{-}$$

$\frac{1}{7}$ of 40 tenths = 5 tenths and 5 tenths remaining; $\frac{1}{7}$ of 5 tenths or of 50 hundredths = 7 hundredths and 1 hundredth remaining; and so on. The last division gives $\frac{1}{7}$ of 60 millionths = $8\frac{4}{7}$ millionths. Since $8\frac{4}{7}$ millionths is nearer 9 millionths than 8 millionths, 9 is written in the quotient rather than 8; but a small minus sign is written after the 9 to show that the true quotient is a little less than .571429.

In the following, no results need be carried beyond six decimal places.

2. Reduce $\frac{4}{27}$ to a six-place decimal.

By Long Division

$$\begin{array}{r} .148148^{+} \\ 27\overline{)4.0000} \\ \underline{2\ 7} \\ 1\ 30 \\ \underline{1\ 08} \\ 220 \\ \underline{216} \\ 40 \end{array}$$

40, etc., as from the beginning.

By Short Division

$$\begin{array}{r|l} 3 & 4.000000 \\ \hline 3 & 1.333333^{+} \\ \hline 3 & .444444^{+} \\ \hline & .148148^{+} \end{array}$$

The small plus signs show that the true quotients are a little larger than those set down, but less than $\frac{1}{2}$ millionth larger.

After finding the first three figures of the quotient by long division, it is found that the new dividend is like the original dividend. Hence, the next three figures of the quotient will be like the first three; and in fact the same set of figures will recur, however far the division is carried.

The short division process is advised when the divisor can be separated readily into factors.

Reduce to decimals, not beyond six places:

3. $\frac{1}{4}$	10. $\frac{5}{6}$	17. $\frac{5}{12}$	24. $\frac{9}{32}$
4. $\frac{1}{8}$	11. $\frac{5}{8}$	18. $\frac{5}{16}$	25. $\frac{13}{32}$
5. $\frac{3}{8}$	12. $\frac{3}{7}$	19. $\frac{7}{16}$	26. $\frac{25}{32}$
6. $\frac{1}{6}$	13. $\frac{4}{5}$	20. $\frac{9}{16}$	27. $\frac{11}{12}$
7. $\frac{3}{4}$	14. $\frac{5}{9}$	21. $\frac{11}{16}$	28. $\frac{4}{15}$
8. $\frac{7}{8}$	15. $\frac{5}{7}$	22. $\frac{13}{16}$	29. $\frac{9}{64}$
9. $\frac{3}{16}$	16. $\frac{2}{11}$	23. $\frac{5}{32}$	30. $\frac{33}{64}$

Addition and Subtraction of Decimals

WRITTEN EXERCISES

263. The following have been added and tested in 7 minutes. Practice until you can do as well.

1.	2.	3.	4.
98.046	3.8649	3.1416	5.2064
48.792	8.0095	4.8	6.7187
25.4	.8008	2.75	4.6562
3.96	32.84	4.853	3.4687
85.244	1.7854	.9244	9.28125
38.008	86.4139	1.3065	45.05625

5.	6.	7.	8.
11.60625	.03794	.222462	45.6425
5.21875	.03826	.225729	38.8448
.04375	.05023	.227726	40.0505
2.53125	.06199	.248495	68.2734
4.59375	.07368	.270805	9.9999
16.00625	.09644	.299573	18.2184

Subtract:

9. 4.2532 3.8216	10. 8.4605 4.2841	11. 7.5225 3.1895
12. 25.6250 7.1849	13. 88.3300 6.4727	14. 3.42 0.8625
15. 7 6.48156	16. 10 3.141593	17. 5 0.006995

Subtract from 1:

18. .0625	20. .4375	22. .40625	24. .071875
19. .3099	21. .1827	23. .53667	25. .222414

26. Syracuse is situated on a railroad 439.44 miles long between New York and Buffalo, and is 148.72 miles from Buffalo. How far is Syracuse from New York?

27. The German mark is worth \$.2385, the French franc \$.193, and the English shilling \$.2434. How much more is the shilling worth than the franc? than the mark?

How much less than a quarter of a dollar is each of these foreign coins worth?

28. A nail 5 inches long is driven through a board so that it projects 2.419 inches on one side and 1.706 inches on the other. How thick is the board?

29. If a cubic foot of coal weighs 82.9628 lb. and of pine 41.0132 lb., how much more does 1 cu. ft. of coal weigh than 2 cu. ft. of pine?

30. Recently the total mileage of the Canadian Pacific Railway was 8667.5 miles. This was 2973.53 miles shorter than that of the Southern Railway and the Baltimore and Ohio combined, but 1468.82 miles longer than the Southern Railway alone. How long was each road?

31. The fastest time made by ships in crossing the Atlantic Ocean for each decade since 1850 has been: 217.75 hr., 190.05 hr., 178.883 hr., 139.3 hr., and 127.383 hr. Find the time gained each decade.

32. If the average weight of a boy is 61.28 lb. at 10 years of age, 64.89 lb. at 11 years, 72.55 lb. at 12 years, 78.32 lb. at 13 years, 87.41 lb. at 14 years, and 103.29 lb. at 15 years, find the average gain in weight for each year.

33. Some onions were raised on a plot of ground at a cost of $17.44 for growing them, $1.13 for gathering, and $10.95 for trimming and bunching. If they were sold for $131.51, how much profit was made?

Multiplication of Decimals

264. 1. Find the product of:

.2 and 3; .02 and 3; .002 and 3.

What is the product of tenths and units? of hundredths and units? of thousandths and units?

2. Since $.2 = \frac{2}{10}$ and $.3 = \frac{3}{10}$, what is the product of .2 and .3? of .02 and .3? of .002 and .3?

What is the product of tenths and tenths? of hundredths and tenths? of thousandths and tenths?

3. What is the product of hundredths and hundredths? of hundredths and thousandths?

4. Multiply:

.2	.2	.02	.002	.002	.02	.002
3	.3	.3	3	.3	.03	.03

5. How may the number of decimal places in the product be obtained from the number of decimal places in the multiplicand and multiplier?

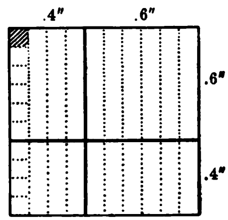
265. ***The number of decimal places in the product is equal to the number of decimal places in both multiplicand and multiplier.***

EXERCISES

266. Answer quickly:

1. .2 of 3	**5.** $4 \times .7$	**9.** $.05 \times .11$
2. .2 of .3	**6.** $.4 \times .7$	**10.** $.5 \times .025$
3. .2 of .03	**7.** $.04 \times .9$	**11.** $.015 \times .03$
4. .2 of .003	**8.** $.004 \times .3$	**12.** $.13 \times .0002$

13. What part of a square inch is .1 of .1 of it? .4 of .1 of it? .6 of .1 of it? .9 of .1 of it?



14. How many hundredths of a square inch are there in a square, each side of which is .4 in. long? in a rectangle .4 in. by .6 in.?

15. Find the area of a rectangle .6 in. by .4 in.; .6 in. by .6 in.

16. Find the area of a rectangle 1.2 in. by .5 in.; 7 ft. by .3 ft.; 1.5 ft. by .4 ft.; 1.1 rd. by .9 rd.

WRITTEN EXERCISES

267. **1.** Multiply .63 by .48.

$$\begin{array}{r} .63 \\ .48 \\ \hline 504 \\ 252 \\ \hline .3024 \end{array}$$

The product of 63 and 48 is 3024.

Since the multiplicand .63 has two decimal places and the multiplier .48 two decimal places, the product must contain $2 + 2$, or *four*, decimal places.

Therefore, the product is .3024.

Multiply:

2. $.49 \times .56$

3. $.84 \times .36$

4. $.971 \times .78$

5. $.842 \times .36$

6. $.523 \times .345$

7. $.871 \times .413$

8. $.649 \times .507$

9. $.987 \times .403$

10.
$$\begin{array}{r} .32 \\ \underline{.26} \\ 192 \\ \underline{64} \\ .0832 \end{array}$$

11.
$$\begin{array}{r} .34 \\ \underline{.45} \\ 170 \\ \underline{136} \\ .153\cancel{0} \end{array}$$

12.
$$\begin{array}{r} 60.29 \\ \underline{.095} \\ 30145 \\ \underline{54261} \\ 5.72755 \end{array}$$

Decimal ciphers are *prefixed*, when necessary, to give the product the proper number of decimal places.

Decimal ciphers on the right of the product are *omitted.*

Multiply :

13. .71 × .14

14. .29 × .28

15. .46 × 3.54

16. .67 × 10.22

17. .421 × 8.45

18. .545 × 3.05

19. .625 × 2.64

20. .875 × 8.56

21. 2.165 × 4.306

22. 5.008 × 7.24

23. 9.243 × 10.07

24. 1.008 × .375

25. 2.041 × .046

26. 1.001 × .008

27. 3.564 × 18.245

28. 8.069 × 100.004

29. If the railroad fare between Chicago and San Francisco is $62.50, and the fare in 1880 was 1.856 times as much, what was the fare then?

30. There are 30.8952 miles of asphalt pavement in a certain city. If the total length of paved streets is 2.75 times as much, how many miles of paved streets are there?

31. If it costs $5.94 to irrigate one acre of ground in Colorado, how much does it cost to irrigate 87.5 acres?

32. A California farmer raises 27.5 acres of sugar beets that average 9.81 tons to the acre. If he sells his beets for $4.40 a ton, how much money does he receive?

33. Find the cost at $.12 per bushel of enough cotton seed to plant 245 acres of land, if each acre requires 1.75 bushels.

Division of Decimals

268. Divide:

1. $2\overline{)8}$ 2 tenths$\overline{)8 \text{ tenths}}$ $.2\overline{).8}$ $.02\overline{).08}$ $.002\overline{).008}$

2. Compare the quotient obtained by dividing .8 by .2 with that obtained by dividing 8 by 2.

Compare $.02\overline{).08}$ with $2\overline{)8.}$

Compare $.002\overline{).008}$ with $2\overline{)8.}$

3. If the divisor is multiplied by 10, by what must the dividend be multiplied to keep the quotient the same? if the divisor is multiplied by 100? by 1000?

Multiplying both dividend and divisor by the same number does not change the quotient.

4. When the divisor is a decimal, we make the division easier by multiplying both dividend and divisor by a number that will change the *divisor* to an *integer*.

5. Change $.2\overline{).84}$ to $2\overline{)8.4}$ and then divide.

6. Change $.02\overline{).184}$ to $2\overline{)18.4}$ and then divide.

7. How should the dividend and the divisor be prepared for division when the divisor contains one decimal figure? two decimal figures? any number of decimal figures?

269. *The divisor may be changed to an integer, without changing the value of the quotient, by moving the decimal points in both dividend and divisor toward the right as many places as there are decimal figures in the divisor.*

When the dividend and divisor have been prepared for division by changing the divisor to an integer, the decimal point of the quotient should be placed vertically above or below that of the dividend.

EXERCISES

270. Divide:

1. $.3\overline{).6}$	**7.** $1.2\overline{)8.4}$	**13.** $.004\overline{).024}$
2. $.4\overline{).8}$	**8.** $1.1\overline{)1.32}$	**14.** $.003\overline{)1.236}$
3. $.2\overline{)1.2}$	**9.** $.09\overline{).729}$	**15.** $.008\overline{)1.688}$
4. $.3\overline{)1.5}$	**10.** $.07\overline{).042}$	**16.** $.012\overline{)1.440}$
5. $.6\overline{)4.2}$	**11.** $.11\overline{).099}$	**17.** $.011\overline{).0099}$
6. $.8\overline{)9.6}$	**12.** $.08\overline{).848}$	**18.** $.012\overline{).000096}$

19. Divide by .5: .25, .125, .0625, 20.5.

20. If 1.2 inches of a candle burn in one hour, in how many hours will 7.2 inches burn?

21. If a railroad train travels .9 of a mile in one minute, how many minutes does it take to travel 10.8 miles?

22. If Henry expends \$.05 a day for car fare, how long will \$1.50 last him?

23. If one pound of grapes makes .3 of a pound of raisins, how many pounds of grapes are required to make 10.5 pounds of raisins?

24. A blacksmith bought an anvil for \$9, paying \$.10 per pound for it. How much did the anvil weigh?

WRITTEN EXERCISES

271. 1. Divide .1296 by .048.

$$.1296 \div .048$$

$$\begin{array}{r} 2.7 \\ 48\overline{)129.6} \\ \underline{96} \\ 336 \\ \underline{336} \end{array}$$

Since the divisor expresses thousandths, to change the divisor to an integer *without changing the quotient*, both dividend and divisor are multiplied by 1000 by moving the decimal point in each three places toward the right.

Divide:

2. 8.75 by 3.5
3. 15.18 by 4.6
4. 31.68 by 4.4
5. 50.40 by 7.5
6. 57.60 by 12.8
7. 54.78 by 6.6
8. 3.268 by .76
9. 3.612 by .43
10. 10.962 by 8.7
11. 4.872 by 1.45
12. 8.906 by 3.65
13. 42.602 by 8.95
14. 48.300 by 6.44
15. 10.897 by 4.25

16.

$22.496 \div 3.2$

```
     7.03
32)224.96
   224
      96
      96
```

17.

$.011 \div 4.4$

```
    .0025
44).1100
     88
     220
     220
```

18.

$5 \div .15625$

```
            32
15625)500000
       46875
       31250
       31250
```

When the dividend contains fewer decimal figures than the divisor, the deficiency should be made up, before moving the decimal point, by annexing decimal ciphers. Thus, exercise 18, $5 \div .15625 = 5.00000 \div .15625$.

19. .021 by 2.8
20. .0198 by 3.6
21. .0162 by 4.5
22. .0403 by .65
23. .6345 by .75
24. .0546 by .65
25. .02275 by .625
26. .03262 by 93.2
27. .02346 by .68
28. 5.000 by .625
29. 1365 by .8125
30. 15 by .46875
31. 29 by .453125
32. 161 by .71875
33. 15.3 by .53125
34. 794.64 by 32.25
35. 91.6243 by 18.38
36. 66.112 by .01033
37. .009604 by .0056
38. 2352.9122 by 36.664
39. $47,787.50 by $76.46
40. $17,253.55 by $42.84

272. 1. Find the value of $.4 \times .6$; of $\frac{.4 \times .6}{.12}$.

2. How will the value of the dividend be affected if *one* of the factors, as .4, is multiplied by 10? if *both* factors are multiplied by 10?

3. If both factors of the dividend are multiplied by 10, by what number must the divisor be multiplied to prevent the quotient from being changed?

4. How, then, does $\frac{.4 \times .6}{.12}$ compare in value with $\frac{4 \times 6}{12}$?

WRITTEN EXERCISES

273. 1. Divide $7.5 \times 1.8 \times .33$ by $9.9 \times .25 \times 12$.

$$\frac{\overset{3}{\cancel{75}} \times \overset{3}{\cancel{18}} \times \cancel{33}}{\underset{3}{\cancel{99}} \times \cancel{25} \times \underset{2}{\cancel{12}} \times 10} = \frac{3}{20} = .15$$

Omitting decimal points from the dividend multiplies it by $10 \times 10 \times 100$, or by 10,000. Omitting decimal points from the divisor multiplies it by 10×100, or by 1000. Consequently, to free both dividend and divisor of decimals *without changing the quotient*, after omitting decimal points we must multiply the divisor by 10. By cancellation the quotient is then found to be $\frac{3}{20}$, or .15.

Divide, using cancellation:

2. 4.2×1.6 by .56
3. 9.5×1.3 by .38
4. $.44 \times 2.1$ by 1.32
5. 4.8×2.8 by .032
6. 2.88 by $.8 \times 2.4$
7. .168 by $.35 \times 1.2$
8. 8.1 by 3.6×1.8
9. 4.5 by 7.2×1.25
10. $4.2 \times 2.5 \times 7.2$ by $3.5 \times .75 \times 24$
11. $1.5 \times .48 \times 1.05$ by $.45 \times 2.5 \times .16$
12. $.216 \times 7.5 \times 1.68$ by $.315 \times .48 \times .96$
13. $5.5 \times 1.12 \times 7.8 \times .54$ by $.024 \times 2.6 \times .99$
14. $.9 \times .022 \times 2.5 \times 25.2$ by $.63 \times 37.5 \times 4.8$
15. $11.52 \times .105 \times .1728$ by $.048 \times 22.4 \times 7.5 \times .072$

WRITTEN EXERCISES

274. **1.** If a farmer sells potatoes for $.65 per bushel and receives $1145.30 for them, how many bushels does he sell?

2. A planter in Alabama pays $519.65 per year for the rent of his farm, at $4.75 per acre. How many acres does he hire?

3. A man paid $27.50 per month for the rent of a house, and after paying $467.50 moved out. How many months did he occupy the house?

4. A chest of tea was sold for $28.31. If the price per pound was $.38, how many pounds did the chest contain?

5. How many pounds are there in a sack of coffee that sells at $.135 a pound and brings $17.82?

6. At $2.85 a box, how many boxes of oranges may a wholesale dealer buy for $2485.20?

7. When 4.8 bushels of wheat make a barrel of flour, how many barrels will 648 bushels of wheat make?

8. A miller in Minnesota sold one day's product of flour for $8002.80, at $3.42 per barrel. How many barrels did the mill turn out that day?

9. If the ruble of Russia is worth $.515 in our money, to how many rubles is $638.60 equivalent?

10. The area of a field is 350.2 square rods, and one side is 13.6 rods long. Find the length of the other side.

11. A wall map is 24.3 inches long, and has an area of 675.54 square inches. Find its width.

12. The cost of drilling a well at $.875 per foot was $1333.50. How deep was the well?

13. A train went from Kendallville, Ind., to Toledo, Ohio, a distance of 91 miles, in 1.25 hours. At what rate per hour did the train run?

MISCELLANEOUS EXERCISES

275. 1. The strawberries grown on 4.5 acres of land were sold for \$2377.35. What was the income per acre?

2. A cow gave 5738.35 lb. of milk in a year. How many quarts of milk did she give, if 1 quart weighs 2.15 lb.?

3. The cost of building a new road 21.7 miles long was \$6983.06. What was the cost per mile?

4. A California road 25 miles long was sprinkled with petroleum, 175.5 barrels to the mile. Find the cost of the petroleum at 70¢ per barrel.

5. A farmer drew his produce to market, a distance of 6.75 miles, at a cost of \$3.51 per load. What was the cost per mile of drawing a load?

6. If it requires a flow of 3.77 gallons of water per minute to irrigate 1 acre of land, how many acres can be irrigated by a flow of 207.35 gallons per minute?

7. If .92 of an iceberg is under water, how many cubic feet of ice are there in an iceberg that has 196,880 cubic feet beneath the surface?

8. In the United States the average cost of living in 1860 was \$115.191 per person, and in 1904, \$97.192. How much less on the average did a year's living for a family of 5 persons cost in 1904 than in 1860?

9. It is 89.885 miles from Albion to Utopia, and 101.215 miles from Utopia to Carthage. Find the cost, at \$.13 per mile, of the coal consumed by a locomotive in making a run from Albion to Carthage.

10. One year an electric company operated 128.18 miles of its own track and 24.28 miles of track leased from another company. The operating expenses of the road were \$334,421.01. Find the operating expenses per mile of track.

REVIEW PROBLEMS IN INDUSTRIES

276. A farmer in Tennessee had $6\frac{1}{2}$ acres of land devoted to peanut raising. Find the cost of:

1. Seed, 2 bushels per acre, at $1.02 per bushel.

2. Preparing the ground, $1.90 per acre.

3. Planting, $1.10 per acre.

4. Cultivating and caring for the crop, $4.72 per acre.

5. Harvesting, $16.50 per acre.

6. What was the total expense and what was the expense per acre?

7. The field produced $354\frac{1}{4}$ bushels of peanuts. What was the average yield per acre?

8. How much was received for the crop, at 96¢ per bushel?

9. Find the profit on the crop.

10. In addition to the cost, $340.08, the buyer paid $50.70 to ship the peanuts to Chicago. He then sold them at $1.24 per bushel. Find his gain.

11. The buyer in Chicago sold 215 bushels @ $1.44 and the rest @ $1.36. How much did he gain, the cost being $439.27?

12. One year Virginia produced 3,713,347 bu. of peanuts; N.C., 3,460,439 bu.; Ga., 1,435,775 bu.; Ala., 1,021,708 bu.; Fla., 967,927 bu.; and Tenn., 747,668 bu. How many more bushels did Virginia produce than each of the other states?

One year two boys and two girls rented a lot next to their house for a garden. Part of the lot had been used as a garden before, and contained some berry bushes.

Some of the vegetables paid only for the cost of raising, but the following proved to be profitable:

CROP	SEED	YIELD
Beets	1 ounce @ 10¢	24 bunches
Peas	2 pints @ $12\frac{1}{2}$¢	$5\frac{1}{2}$ pecks
Tomatoes	2 packets @ 5¢	$6\frac{3}{4}$ bushels
Lima beans	$1\frac{1}{2}$ pints @ 10¢	$1\frac{1}{2}$ bushels
Currants	....	43 quarts
Gooseberries	....	21 quarts
String beans	2 pints @ 15¢	2 bushels 1 peck
Lettuce	3 packets @ 5¢	160 heads
Sweet corn	$2\frac{1}{2}$ pints @ 12¢	330 ears

13. How much did the seed cost?

Find the proceeds from:

14. Beets at 5¢ per bunch.

15. Peas at 40¢ per pk.

16. Tomatoes at 20¢ per pk.

17. Lima beans at 8¢ per qt.

18. Currants at 11¢ per qt.

19. Gooseberries at 12¢ per qt.

20. Find the sum received for lettuce, if $\frac{1}{2}$ of the crop sold at 5¢ per head, $\frac{3}{8}$ of the crop at 6¢ per head, and the rest at 2 heads for 15¢.

21. The string beans were grown by the children in two lots,

the first of which was $\frac{1}{3}$ of the whole yield, and brought 15¢ per quart; the second, picked in September, sold at 10¢ per quart. How much was received for string beans?

22. There were two crops of corn, one early and one late. The early crop yielded 132 ears, which sold at 20¢ per dozen, while the late crop sold at 16¢ per dozen. How much did the corn bring?

23. The tomato seed they had planted in a box in late winter, and had thus raised 90 young plants, 24 of which they set out for themselves. They sold the rest at 30¢ per dozen. How much did their tomato plants bring?

24. What were the total receipts for vegetables and plants?

25. One corner of the garden was devoted to raising flowers. A packet of aster seed produced 40 healthy plants which yielded 6 fine flowers apiece. At 25¢ per dozen flowers, how much money did the asters bring?

26. The first sweet peas were picked July 14, the last October 6. If the average number of stems picked per day was 36, how many were obtained during the season?

27. These flowers were sold at 18¢ per bunch of 36 stems. Find the proceeds from sweet peas.

28. The girls also planted some choice dahlia bulbs, and from the mature plants secured that summer 672 dahlias. Find the sum obtained by selling the dahlias at 20¢ per dozen.

29. What were the total receipts from flowers? from the whole garden?

30. The expenses of the garden were: cost of vegetable seed as found in exercise 13; 4 packets of flower seed @ 10¢; 8 dahlia bulbs @ $2.85 per dozen; $4.27 for fertilizer; $8.50 for rent. Find the total cost.

31. Find the total profits from the garden, and each child's share, if the profits were divided equally.

A gardener in Michigah set out $\frac{3}{4}$ of an acre of strawberries.

Expense of Cultivation

Preparing the ground	$2	25
Plants, 7500 at $2 per thousand		
Setting the plants	2	75
Hoeing, etc.	8	50
Weeding, etc.	18	75
Straw, $3\frac{3}{4}$ tons at $4 per ton		
Spreading the straw	1	50
Fertilizer	15	

Expense of Harvesting and Marketing

150 bushel crates at $15 per hundred		
4800 quart baskets at $3 per thousand		
Picking 4800 quarts @ $1\frac{1}{2}$¢		
Shipping and selling, 54¢ per crate		

32. Find the cost of the plants; of the straw; of cultivation.

33. How much did the crates cost? the baskets?

34. Compute the cost of picking; of shipping and selling.

35. Find the expense of harvesting and marketing.

36. What was the gardener's total expense?

Receipts

2 bushel crates @ $5.12		
12 " " @ $4.16		
36 " " @ $3.80		
76 " " @ $2.48		
24 " " @ $1.94		

37. Compute the gardener's receipts for his crop.

38. How much did he gain?

39. Find the average price received for the crop per crate; per quart; the highest price per quart; the lowest.

40. What was the yield in quarts per acre?

PART II

PRELIMINARY REVIEW

277. 1. The following table shows the number of days' attendance in the graded schools of a certain city.

Find the total attendance for each month; then find the total attendance in each school; then find the total attendance in all the schools during the year, by adding the totals for each month, and also by adding the totals for each school.

School	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June	Totals for each school
Lincoln	4824	5236	5361	4550	5555	4482	5810	4965	4946	4461	
Greeley	9042	9864	9430	8681	9998	8585	9787	8960	9862	7751	
Park	3977	4885	4202	3872	4724	3636	4427	4016	4215	3366	
Douglass	5055	5482	5225	4838	5752	4500	5499	5005	5176	4858	
Irving	5987	6548	6367	5749	6900	5484	6518	6000	6288	5244	
White	8101	8736	8452	7686	9275	7810	8797	8050	8486	6787	
Whittier	5166	5490	5278	4900	5804	4516	5500	5096	5230	4375	
Longfellow	8104	8821	8330	7651	9352	7558	8890	8172	8472	6785	
Totals for each month											

2. Find the average monthly attendance for each school. How much greater was it for the Greeley School than for the Longfellow School?

3. Find the average attendance at all the schools for each month. How much greater was it for January than for December?

4. Find the average daily attendance for all the city schools, regarding 200 days as a school year.

Add and test in 4 minutes or less:

5.	**6.**	**7.**	**8.**
428.356	362.44	428.114	504.405
29.438	171.86	32.56	367.242
51.097	93.55	9.29	596.385
34.253	47.286	8.47	89.755
8.756	9.099	82.625	7.4836
11.482	12.764	75.075	5.2948
15.943	346.201	54.343	8.3006
506.042	798.45	281.983	11.5998

Subtract rapidly:

9.	**10.**	**11.**	**12.**
34.645	52.000	100.000	1000.00
5.256	7.248	5.272	524.38

13.	**14.**	**15.**	**16.**
75.05	3.864	10	25
3.4642	.2457	1.4265	6.4878

Multiply:

17. 650 by 4.2
18. 3.87 by .4
19. 5.65 by .24
20. 3.87 by 6.2
21. .875 by 8.09
22. .4964 by .55
23. .0284 by 47.5
24. 16.288 by 5.305
25. 19.082 by 156.7
26. 586.48 by .0105

Factor the following numbers:

27. 102
28. 201
29. 405
30. 504
31. 248
32. 729
33. 1728
34. 2160
35. 1452
36. 1296
37. 1050
38. 3650
39. 85,050
40. 15,876
41. 20,736
42. 41,088

43. What is an even number? an odd number?
Which of the numbers in exercises 27–42 are even? odd?

Reduce to lowest terms:

44. $\frac{45}{60}$ 47. $\frac{80}{96}$ 50. $\frac{15}{75}$ 53. $\frac{95}{380}$

45. $\frac{26}{39}$ 48. $\frac{75}{125}$ 51. $\frac{49}{56}$ 54. $\frac{875}{1200}$

46. $\frac{39}{65}$ 49. $\frac{44}{110}$ 52. $\frac{48}{112}$ 55. $\frac{132}{5280}$

Find the sum and the difference of:

56. $\frac{3}{8}$ and $\frac{5}{16}$ 60. $\frac{4}{5}$ and $\frac{7}{8}$ 64. $3\frac{5}{8}$ and $2\frac{3}{4}$

57. $\frac{7}{8}$ and $\frac{11}{16}$ 61. $\frac{15}{16}$ and $\frac{9}{10}$ 65. $9\frac{2}{3}$ and $5\frac{5}{7}$

58. $\frac{3}{4}$ and $\frac{2}{3}$ 62. $1\frac{5}{6}$ and $\frac{11}{12}$ 66. $7\frac{16}{25}$ and $4\frac{17}{20}$

59. $\frac{5}{8}$ and $\frac{8}{9}$ 63. $2\frac{1}{2}$ and $1\frac{7}{12}$ 67. $4\frac{38}{75}$ and $3\frac{7}{50}$

68. What may be done to the terms of a fraction without changing its value?

69. What are similar fractions?

70. What must be done to fractions that are not similar before they can be added or subtracted?

Perform the operations indicated:

71. $2\frac{1}{4} + 3\frac{5}{8} - 1\frac{13}{16}$ 74. $4\frac{3}{4} - 1\frac{2}{3} + \frac{9}{15} + 2\frac{2}{5}$

72. $5\frac{1}{2} - 2\frac{7}{10} + 15\frac{4}{5}$ 75. $6\frac{1}{2} - 4\frac{1}{3} + 8\frac{3}{4} - 9\frac{5}{6} + \frac{2}{5}$

73. $9\frac{1}{4} - 2\frac{5}{6} - 3\frac{2}{3}$ 76. $20 - 1\frac{1}{6} - 2\frac{3}{5} - 4\frac{7}{8} - 9\frac{3}{4}$

Reduce to improper fractions:

77. $3\frac{9}{10}$ 79. $4\frac{11}{12}$ 81. $33\frac{1}{3}$ 83. $87\frac{1}{2}$

78. $6\frac{7}{8}$ 80. $15\frac{4}{25}$ 82. $66\frac{2}{3}$ 84. $83\frac{1}{3}$

Multiply or divide as indicated:

85. $1\frac{5}{7} \times 4\frac{3}{8}$ 90. $16\frac{4}{5} \times 12\frac{1}{2}$ 95. $266\frac{2}{3} \div 33\frac{1}{3}$

86. $5\frac{1}{15} \times 2\frac{13}{16}$ 91. $56\frac{7}{8} \div \frac{7}{12}$ 96. $500 \div 16\frac{2}{3}$

87. $3\frac{6}{25} \times 2\frac{7}{9}$ 92. $18\frac{2}{3} \div 4\frac{28}{33}$ 97. $350 \div 87\frac{1}{2}$

88. $4\frac{2}{3} \div 3\frac{1}{21}$ 93. $14\frac{2}{5} \times 10\frac{5}{12}$ 98. $625 \div 37\frac{1}{2}$

89. $14\frac{7}{8} \div 3\frac{1}{2}$ 94. $90 \div 23\frac{7}{16}$ 99. $1000 \div 62\frac{1}{2}$

100. Tell why $\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3}$. Why is the divisor inverted?

101. Compare $\frac{7}{8}$ with $\frac{7}{10}$ in two ways, as follows:

(*a*) Find how much greater or less $\frac{7}{8}$ is than $\frac{7}{10}$.

(*b*) Find how many times $\frac{7}{8}$ contains $\frac{7}{10}$.

102. Compare $\frac{7}{12}$ with $\frac{7}{10}$ in two ways.

103. The standard size adopted for common brick is $8\frac{1}{4}$ in. by 4 in. by $2\frac{1}{4}$ in. Find the volume of a brick of standard size.

104. How much greater or less in volume is each of the following than a brick of the standard size:

Milwaukee brick, $8\frac{1}{2}$ in. by $4\frac{1}{8}$ in. by $2\frac{3}{8}$ in.?
Maine brick, $7\frac{1}{2}$ in. by $3\frac{3}{8}$ in. by $2\frac{3}{8}$ in.?
North River brick, 8 in. by $3\frac{1}{2}$ in. by $2\frac{1}{4}$ in.?

105. To how many standard bricks are 1000 Milwaukee bricks equivalent in volume? 1000 North River bricks?

Multiply:

106. .77 by .24
107. .38 by 5.6
108. .72 by .025
109. 9.6 by .288
110. .0125 by 4.4
111. .0065 by .72
112. .0327 by .09
113. 1.625 by 7.6
114. 3840 by .875
115. 43.11 by $.66\frac{2}{3}$
116. 105.602 by $.12\frac{1}{2}$
117. 360.044 by $.37\frac{1}{2}$

Divide:

118. .86 by 17.2
119. .7755 by .75
120. .2292 by .048
121. 219.45 by 8.75
122. .15795 by 3.25
123. .02163 by 3.09
124. .485322 by .94
125. 15.54378 by 4.2
126. 4.59459 by .7854
127. 25.76112 by 3.1416

Reduce to a common fraction in its lowest terms:

128. .3125 **129.** .00875 **130.** .01025 **131.** .000225

Reduce to a decimal to the nearest thousandth:

132. $\frac{4}{9}$ **133.** $\frac{7}{9}$ **134.** $\frac{11}{12}$ **135.** $\frac{29}{17}$ **136.** $\frac{527}{288}$

278. 1. If a flour mill turns out 30 barrels of flour per hour and runs continuously from 7 A.M. on Monday to 12 M. on Saturday, how many barrels will it turn out in a year of 52 weeks?

2. A man's gas meter registered 28,470 cubic feet on Jan. 1 and 35,670 cubic feet on Apr. 1. Find the cost of his gas for the quarter, at 90¢ per 1000 cubic feet.

3. Find the freight charges on a car load of steel shipped from Pittsburg to Montreal, weight 48,200 pounds, at $20\frac{1}{2}$¢ per 100 pounds.

4. The average weight of a newspaper having a daily circulation of 240,000, half of which are sent by mail, is $2\frac{7}{8}$ ounces. Find the cost of postage, at 1¢ per pound, for a day; for a year of 313 week days.

5. The "Twentieth Century Limited" made a run of 133.4 miles between Toledo, Ohio, and Elkhart, Ind., in 1 hr. 54 min. How many miles per hour did the train run?

6. On one occasion a short run of 7.29 miles was made by an American train in 4 minutes. At what rate per hour did the train run?

7. A tank 6 feet long, $2\frac{1}{2}$ feet wide, and $2\frac{2}{3}$ feet deep (inside dimensions) has its sides, ends, and bottom lined with sheet lead weighing $3\frac{1}{2}$ pounds per square foot. Find the cost of the lead at 6¢ per pound.

8. Find the cost of draining a meadow by laying 3960 feet of tile at $10 per thousand feet, and digging and filling 3960 feet of ditch at $33\frac{1}{3}$¢ per rod.

9. The meadow contained $62\frac{1}{2}$ acres. Before it was drained it was worth $2125, and after it was drained it was valued at $3900. How much per acre was its value increased by drainage?

10. Of three cargoes of bananas unloading at Pier No. 1, the first contained 22,360 bunches from the British West Indies, the second 19,250 bunches from Cuba, and the third 20,780 bunches from Santo Domingo. How many bunches were there in the three cargoes?

11. A fruit auctioneer sold 26 car loads of oranges, 360 boxes to the car, in 2 hours 10 minutes. How many boxes did he sell per minute?

12. The auctioneer's receipts from this shipment of oranges were $21,060. How much, on the average, did he receive for one box?

13. The same auctioneer once sold 21,000 crates of pineapples, or 630,000 pineapples, in $1\frac{1}{2}$ hours, realizing $45,000. Find the average number of pineapples in a crate, the average price received for a crate, and the average number of crates sold per minute.

14. Find the cost of 150 barrels of limes at $4.75 per barrel.

15. If a refrigerator box contains 80 quarts of Florida strawberries, how many quarts of strawberries are there in a car load containing 125 such refrigerator boxes?

16. If it requires 200 pounds of ice to keep the strawberries in each refrigerator box cool during shipment, how many pounds of ice are required for the whole car load?

17. Find the cost of the ice at $2.15 per ton.

18. Find the amount received for the 10,000 quarts of strawberries in the Northern market, at 35¢ per quart.

19. When California cherries are packed, 10 pounds are put into a box. One year a California orchard produced 2201 boxes of cherries that were sold at $6\frac{1}{2}$¢ per pound, 6365 boxes that were sold at $5\frac{1}{2}$¢ per pound, and 32,219 pounds that were sold at 7¢ per pound. Find the value of the whole crop.

20. A fruit grower shipped 1600 ten-pound boxes of cherries to New York, where they were sold at auction at 90¢ per box. Picking and packing cost 1¢ per pound, freight and refrigeration 22¢ per box, and other expenses amounted to \$25. Find the amount received less expenses.

21. He had been offered $5\frac{3}{8}$¢ per pound on the trees. Did he gain or lose by shipping the cherries, and how much, if the sum received for them, less expenses, was \$903?

22. The freight and refrigeration charges on 640 boxes of cherries shipped from San Francisco to Chicago amounted to \$121.60. Find the cost of shipping per box.

23. Find the freight charges on a car load of oranges weighing 44,000 pounds, shipped from Los Angeles to Boston at \$1.25 per 100 pounds.

24. Find the value of 4200 crates of No. 30 pineapples (that is, 30 in a crate), at \$1.25 per crate in Jacksonville; at \$2.10 per crate, wholesale, in New York; at 15¢ apiece retail.

25. When a box of No. 172 oranges (that is, 172 in a box) sells for \$4.30, what is the equivalent price per dozen?

26. I have 240 fowls for which I am offered $12\frac{1}{2}$¢ per pound, or \$4.75 per dozen. If the average weight is $3\frac{1}{4}$ pounds, which is the better offer, and how much?

27. The engine for a sugar mill weighed 61,460 pounds, the sugar mill 64,750 pounds, the connecting machinery 28,800 pounds, the boiler 19,200 pounds, and various fittings 6750 pounds. The whole was shipped to me, and I was charged $22\frac{1}{2}$¢ per 100 pounds freight. How much did the freight cost me?

28. If it costs $3\frac{1}{2}$¢ a rivet to drive rivets by ordinary hammers and only $1\frac{1}{4}$¢ by machinery, how much will be saved by using the latter method in building a steamboat whose frame contains 200,000 rivets?

29. A telephone line was constructed between two villages that were $5\frac{1}{2}$ miles apart. Find the cost of surveying the line at 96¢ per mile.

30. Poles were set 132 feet apart, beginning 66 feet from either end of the line. Find the number needed; their cost at $1.25 each.

31. It cost 19¢ to dig the hole for each pole. What was the cost of digging the 220 holes?

32. Every pole was fitted with 2 oak brackets, each mounted with a glass insulator and fastened with 2 spikes. Find the total cost of fitting the poles at 5¢ each for labor, $12 per M for brackets, $21 per M for insulators, and $\frac{3}{8}$¢ each for spikes.

33. What was the cost of setting the 220 poles at 17¢ each?

34. Two galvanized iron wires, each weighing 165 pounds per mile, were strung the entire distance of $5\frac{1}{2}$ miles. Find, to the nearest cent, the cost of the wire at $4\frac{3}{4}$¢ per pound.

35. Find the cost of trimming trees and stringing the wires at $6 per mile. Find the total cost of constructing the line.

36. The population of the village at one end of the line was 6517, and at the other 6731. Soon after the completion of the line, 1 person out of every 32 was a subscriber. How many subscribers were there in both villages?

37. At $14.25 for each telephone, how much did the company expend for the 414 instruments?

38. Find the annual income from subscribers, if each paid $2.50 per month.

39. In one year the total number of calls by subscribers was 776,250, costing $12,420. What was the average cost per call?

40. The manager of a city telephone company made 100 calls on the telephone to find how long it took the operators at the exchange to make connections for customers. The connections for the 100 calls were made in 4 min. 7 sec. Find the average time of making one connection.

DENOMINATE NUMBERS

279. To ascertain the quantity of anything, or to *measure* it, is to find how many times it contains some established unit called the **unit of measure.**

Thus, to measure the corn in a bin is to find how many times the whole quantity of corn in the bin contains some unit measure, as 1 bushel, or 1 hundredweight.

280. A concrete number in which the unit of measure is established by law or custom is called a **denominate number.**

16 bushels is a denominate number; also 16 bushels 3 pecks.

281. A denominate number that is composed of units of one denomination only is called a **simple denominate number.**

16 bushels is a simple denominate number.

282. A denominate number that is composed of units of two or more denominations that are related to each other is called a **compound denominate number.**

16 bushels 3 pecks is a compound denominate number.

Tables of denominate numbers will be found in the Appendix

Reduction

283. Easy reductions.

EXERCISES

1. Reduce 72 inches to feet; to yards.
2. Reduce $1\frac{1}{2}$ yards to feet; to inches.
3. How many inches are there in 1 yd. 1 ft.?

Reduce:

4. $\frac{5}{8}$ lb. to ounces.
5. 1.4 T. to pounds.
6. .3 mile to rods.
7. $2\frac{1}{3}$ sq. yd. to sq. ft.
8. 18 cu. ft. to cu. yd.
9. $\frac{1}{4}$ day to minutes.
10. .75 gal. to pints.
11. $.12\frac{1}{2}$ bu. to quarts.
12. $5\frac{1}{2}$ hr. to minutes.
13. $7\frac{1}{2}°$ to minutes.

WRITTEN EXERCISES

284. **1.** Reduce 5 hr. 15 min. 12 sec. to seconds.

$$\begin{array}{r} 5 \text{ hr.} \\ \underline{60} \\ 300 \\ \underline{+15} \\ 315 \text{ min.} \\ \underline{60} \\ 18900 \\ \underline{+12} \\ 18912 \text{ sec.} \end{array}$$

Since there are 60 minutes in an hour, in 5 hours there are 5 × 60 minutes, or 300 minutes, and in 5 hr. 15 min. there are 300 minutes + 15 minutes, or 315 minutes.

Since there are 60 seconds in a minute, in 5 hr. 15 min. 12 sec., or in 315 min. 12 sec., there are 315 × 60 seconds, and 12 seconds more, or 18,912 seconds.

2. Reduce 18,912 seconds to hours, minutes, and seconds.

$$\begin{array}{r|l} 60 & \underline{18912} \\ 60 & \underline{\ 315,} + 12 \text{ sec.} \\ & \quad 5, + 15 \text{ min.} \end{array}$$

5 hr. 15 min. 12 sec.

Since 60 sec. = 1 min., 18,912 sec. = 315 min. and 12 sec. over.

Since 60 min. = 1 hr., 315 min. = 5 hr. and 15 min. over.

Hence 18,912 sec. = 5 hr. 15 min. 12 sec.

Reduce:

3. 4 wk. 3 da. to days.

4. 7 lb. 5 oz. (av.) to ounces.

5. 112 bu. 3 pk. to pecks.

6. 220 ft. 10 in. to inches.

7. 5 mi. 275 rd. to rods.

8. 14 yr. 7 mo. to months.

9. 3 yd. 2 ft. to inches.

10. 25 tons 12 cwt. to pounds.

11. 15 minutes to seconds.

12. 45 gal. 2 qt. to pints.

Reduce to compound denominate numbers:

13. 174 in.

14. 47 pt. (liquid).

15. 85 qt. (dry).

16. 21,929 sec.

17. 1924 oz. (av.).

18. 18,645 rd.

19. 7232 lb.

20. $33\frac{1}{2}$ pk.

21. $17\frac{3}{8}$ gal.

22. $85\frac{1}{4}$ min.

23. 1000 sq. in.

24. 1525 sq. ft.

25. 25,000 cu. in.

26. 12,300 cu. ft.

27. 104,965 cu. in.

28. The highest mountain in the world, Mt. Everest, is 29,002 feet high. What is its height in miles and feet?

29. I bought a 5-gallon can of maple sirup for $5. At what price per pint must I sell it to gain $1.40?

30. If it takes a pint of ice cream to serve 3 persons, how many persons will 4 gal. 3 qt. serve?

31. A dealer paid $4.75 for a crate containing 1 bu. 2 pk. of strawberries put up in quart boxes. If he sold them at 15¢ per box, how much did he gain?

32. When a train is running 45 miles an hour, what is its rate in feet per second?

33. When an automobile is running 44 feet per second, what is its rate in miles per hour?

34. The regulation military pace is a step 30 in. long, 2 steps per second. If a company of soldiers should keep this pace for 3 hr. 40 min., how many miles would they march?

35. The Park Row building, which contains 29 stories, is 309 feet high. Express in feet and inches the average height of a story.

285. More difficult reductions.

WRITTEN EXERCISES

1. What part of a bushel is 1 pk. 2 qt.?

SOLUTION

1 pk. 2 qt. = 10 qt.
1 bu. = 32 qt.
1 pk. 2 qt. = $\frac{10}{32}$ bu. = $\frac{5}{16}$ bu.

2. What decimal part of an hour is $40\frac{1}{2}$ min.?

SOLUTION

$$40\tfrac{1}{2} \text{ min.} = \frac{40.5}{60} \text{ hr.} = .675 \text{ hr.}$$

3. Reduce $2^\circ\ 33.'7$ to seconds of arc.

SOLUTION

$$2^\circ = 2 \times 3600'' = 7200''$$
$$33.'7 = 33.7 \times 60'' = 2022''$$
$$2^\circ\ 33.'7 = 9222''$$

What part

4. Of a day is $4\frac{1}{2}$ hr.?
5. Of a yard is 1 ft. $1\frac{1}{2}$ in.?
6. Of a gallon is 2 qt. $1\frac{1}{2}$ pt.?
7. Of a bushel is 2 pk. 3 qt.?
8. Of a long ton is a short ton?
9. Of a cubic yard is 20 cu. ft. 432 cu. in.?

What decimal part

10. Of a mile is 1293.6 ft.?
11. Of a ton is 7 cwt. 64 lb.?
12. Of an hour is 1 min. 48 sec.?
13. Of a week is 2 days 10.8 hours?
14. Of a right angle is $4^\circ\ 30'$?
15. Of a square mile is 230.4 acres?

Reduce:

16. 28.5 yd. to inches.
17. 3.1875 bu. to quarts.
18. 2.428 tons to pounds.
19. .96875 of a mile to feet.
20. $\frac{5}{16}$ gal. to quarts and pints.
21. $39'\ 22\frac{1}{2}''$ to a fraction of a degree.
22. $\frac{7}{8}$ of a right angle to degrees and minutes.
23. $\frac{5}{8}$ cu. yd. to cubic feet and cubic inches.
24. .9 of a day to a compound denominate number.

Addition and Subtraction

WRITTEN EXERCISES

286. **1.** Add 12 ft. 6 in. and 14 ft. 9 in.

ft.	in.
12	6
14	9
27	3

The sum of the inches is 15 in., or 1 ft. 3 in.; consequently 3 is written in the column for inches, and 1 is reserved to be united with the numbers in the column for feet.

The sum of the feet is 27 ft.

Hence, the entire sum is 27 ft. 3 in.

Add the following:

2.

ft.	in.
8	4
9	10

3.

yd.	in.
4	24
5	21

4.

gal.	qt.
16	$3\frac{1}{2}$
55	$2\frac{1}{2}$

5.

mi.	rd.
22	100
10	300

6.

lb.	oz.
8	4
	13

7.

hr.	min.
6	40
8	50

8.

wk.	da.
17	4
12	3

9.

mo.	da.
4	12
3	14

10.

hr.	min.	sec.
1	20	45
	39	15

11.

ft.	in.
15	9
6	10
12	5

12.

yr.	mo.
3	8
	11
5	2

13.

T.	cwt.	lb.
3	4	75
	9	45
2	0	250

14.

cu. yd.	cu. ft.
148	10
352	15
85	24
124	5

15.

bu.	pk.	qt.
20	1	4
35	0	2
15	7	0
120	3	2

16.

44°	14′	48″
36	27	16
5	0	24
4	18	12

17. Subtract 5 lb. 7 oz. from 21 lb. 2 oz.

lb.	oz.
21	2
5	7
15	11

Since 7 oz. cannot be subtracted from 2 oz., 1 lb. taken from the 21 lb. is reduced to ounces and united with the 2 oz., giving 18 oz. 18 oz. – 7 oz. = 11 oz., which is written in the remainder. Next subtracting 5 lb. from the 20 lb. left in the minuend, the number of pounds in the remainder is found to be 15.

Hence, the remainder is 15 lb. 11 oz.

Subtract:

	bu.	pk.		ft.	in.		yd.	in.
18.	25	2	**19.**	100	$2\frac{2}{5}$	**20.**	12	18
	12	3		50	4		9	27

	hr.	min.					T.	cwt.	lb.
21.	24		**22.**	180°		**23.**	5	8	40
	16	50		44°	45′		2	3	160

	mi.	rd.		mi.	ft.		hr.	min.	sec.
24.	4	80	**25.**	12	1000	**26.**	8	45	18
	3	120		8	4500		6	30	56

27. How long was it from Apr. 10, 1899, to Feb. 2, 1906?

yr.	mo.	da.
1906	2	2
1899	4	10
6	9	22

The later date is written as the minuend and the earlier date as the subtrahend, writing the number of the month instead of its name.

Subtract as in denominate numbers, considering 30 days as a month and 12 months as a year. The remainder is the difference in time as accurately as it can be expressed in years, months, and days.

Subtract:

	yr.	mo.	da.		yr.	mo.	da.		yr.	mo.	da.
28.	1907	1	1	**29.**	1906	12	25	**30.**	1910	3	2
	1904	10	24		1901	7	27		1905	8	6

31. How many years, months, and days old are you?

32. Find the age of each of the following men at the time when he became President of the United States:

Name	Born	Became President
George Washington	Feb. 22, 1732	Apr. 30, 1789
Abraham Lincoln	Feb. 12, 1809	Mar. 4, 1861
Ulysses S. Grant	Apr. 27, 1822	Mar. 4, 1869
Theodore Roosevelt	Oct. 27, 1858	Sept. 14, 1901

33. Add 25 ft. 6 in.; 32 ft. 4 in.; 28 ft. 8 in.; 56 ft. 10 in.; 18 ft. $11\frac{1}{2}$ in.; 23 ft. $4\frac{1}{2}$ in.

34. Add 5 hr. 3 min.; 16 hr. 10 min.; 4 hr. 45 min.; 7 hr. 59 min.; 6 hr. 17 min.; 14 hr. 11 min.

35. Subtract 22 bu. 3 pk. from 32 bu. $1\frac{1}{2}$ pk.

36. Subtract 6 dollars, 12 cents, 5 mills from 10 dollars.

37. Add 48 A. 37.5 sq. rd.; 72 A. 49.2 sq. rd.; 95 A. 85 sq. rd.; 56 A. 122.3 sq. rd.; 40 A. 140 sq. rd.

38. Add 88 rd. 7 ft.; 92 rd. $11\frac{1}{2}$ ft.; 16 rd. 14 ft.

39. A man left home at 9:30 A.M. on Thursday and returned at 5:45 P.M. on the Wednesday following. How long was he away from home?

40. Some boys gathered 10 bushels of walnuts and sold all of them but 1 bu. 3 pk. Find the quantity sold.

41. The most northerly points reached by several of the more successful polar expeditions were as follows:

Commander	Nationality	Year	North Latitude
Greeley	American	1882	83° 24′
Nansen	Norwegian	1896	86° 14′
Duke of Abruzzi	Italian	1900	86° 34′
Peary	American	1906	87° 6′

How near the north pole (90° north latitude) did each expedition reach? Express the distance in miles, taking $1' = 1.15$ mi.

Multiplication

WRITTEN EXERCISES

287. **1.** Multiply 16 bu. 3 pk. by 7.

bu.	pk.
16	3
	7
117	1

7 times 3 pk. = 21 pk. = 5 bu. 1 pk.
7 times 16 bu. = 112 bu., and 112 bu. + 5 bu. = 117 bu.
The product is 117 bu. 1 pk.

Multiply:

2. 9 ft. 8 in. by 6

3. 7 gal. 3 qt. by 5

4. 2 hr. 9 min. by 8

5. 6 bu. 3 pk. by 4

6. $16° \ 35' \ 20''$ by 4

7. $11° \ 0' \ 16''$ by 15

8. 24 lb. 9 oz. by 20

9. 4 yr. 11 mo. by 11

10. Find the area, in square feet, of a rectangle 5 ft. 5 in. long and 2 ft. 6 in. wide.

$$\begin{array}{r} 65 \\ \underline{30} \\ 12\,\underline{)1950}\cdots\text{ sq. in.} \\ 12\,\underline{)\ 162.5} \\ 13.542^{-}\cdots\text{ sq. ft.}\end{array}$$

Since 5 ft. 5 in. = 65 in., and 2 ft. 6 in. = 30 in., the number of square inches in the area is 30×65, or 1950 sq. in.

Since there are 144 sq. in. in a square foot, the area in square feet may be obtained by dividing 1950 by 144, or by its factors 12 and 12, as shown.

The area is found to be 13.542 sq. ft., to the nearest third decimal place.

Find the area of each of the following, carrying inexact results to the nearest third decimal place:

	Figure	Base	Altitude	Area
11.	Rectangle	6 ft. 3 in.	4 ft. 7 in.	—— sq. ft.
12.	Rectangle	75 ft. $2\frac{1}{2}$ in.	32 ft. $4\frac{1}{2}$ in.	—— sq. ft.
13.	Triangle	44 ft. 1 in.	10 ft. 4 in.	—— sq. ft.
14.	Triangle	60 ft. 5 in.	33 ft. 11 in.	—— sq. ft.
15.	Square	18 ft. 6 in.	18 ft. 6 in.	—— sq. ft.
16.	Parallelogram	24 rd. 2 ft.	22 rd.	—— sq. ft.

17. Find the volume, in cubic feet, of a rectangular solid whose dimensions are 24 ft. 7 in., 12 ft. 11 in., and 5 ft. 4 in.

SUGGESTION. — First find the volume in cubic inches. Then, since there are 1728 cu. in. in a cubic foot, divide the number of cubic inches by 1728, or by $12 \times 12 \times 12$.

Find the volume, in cubic feet, of the following solids:

	LENGTH	BREADTH	THICKNESS
18.	19 ft. 4 in.	12 ft. 7 in.	6 ft. 2 in.
19.	28 ft. $3\frac{1}{2}$ in.	20 ft. 4 in.	5 ft.
20.	52 ft. $7\frac{1}{2}$ in.	33 ft. 6 in.	5 ft. 10 in.

Division

WRITTEN EXERCISES

288. **1.** Divide 33 hr. 21 min. 45 sec. by 15.

$$\begin{array}{r|rrr} & \text{hr.} & \text{min.} & \text{sec.} \\ 15 & 33 & 21 & 45 \\ \hline & 2 & 13 & 27 \end{array}$$

33 hr. ÷ 15 = 2 hr. and 3 hr. remainder.
3 hr. 21 min. = 201 min.
201 min. ÷ 15 = 13 min. and 6 min. remainder.
6 min. 45 sec. = 405 sec.
405 sec. ÷ 15 = 27 sec.

2. How many times is 2 hr. 13 min. 27 sec. contained in 33 hr. 21 min. 45 sec.?

SOLUTION

Reducing the denominate numbers to a *common denomination*, as seconds,

2 hr. 13 min. 27 sec. = 8007 sec.
33 hr. 21 min. 45 sec. = 120,105 sec.

Dividing 120,105 sec. by 8007 sec., the quotient is 15.
Hence 2 hr. 13 min. 27 sec. is contained 15 times in 33 hr. 21 min. 45 sec.

3. How many times is 8 ft. 6 in. contained in 144 ft. 6 in.?

SOLUTION

Reducing to a common denomination, in this case to feet,

8 ft. 6 in. = 8.5 ft. and 144 ft. 6 in. = 144.5 ft.

Dividing, the quotient is found to be 17.

Divide:

4. 40 gal. 1 qt. by 7
5. 81 lb. 9 oz. (av.) by 5
6. 53 ft. 6 in. by 6
7. 92 yd. 27 in. by 9
8. 202° 2′ by 11
9. 8 T. 250 lb. by 25
10. 102 mi. 120 rd. by 14
11. 280 sq. ft. 120 sq. in. by 30
12. 49 lb. 8 oz. (av.) by 5 lb. 8 oz.
13. 239 ft. 7 in. by 9 ft. 7 in.
14. 107° 36′ 6″ by 15° 22′ 18″.

15. If the circumference of an automobile wheel is 8 ft. 3 in., how many revolutions will the wheel make in going a mile?

16. The distance around a square farm is 3 mi. 240 rd. Find the length of each side, in rods; the area in acres.

17. If a horse eats $1\frac{1}{2}$ pecks of oats per day, how long will $31\frac{1}{2}$ bushels of oats last him?

18. How long will 30 hundredweight of oats last him, if a bushel of oats weighs 32 pounds?

19. The area of a rectangle is 47 sq. ft. 36 sq. in. and the base is 8 ft. 9 in. Find the altitude.

SOLUTION

47 sq. ft. 36 sq. in. = 47.25 sq. ft.
8 ft. 9 in. = 8.75 ft.

Since 47.25 is the product of the number of feet in the base and the number of feet in the altitude, and the base is 8.75 ft., the number of feet in the altitude is 47.25 ÷ 8.75, or 5.4.

Hence, the altitude is 5.4 ft.

Find the missing dimension in each case:

20. Rectangle, area $38\frac{1}{4}$ sq. ft., base $8\frac{1}{2}$ ft.

21. Rectangle, area 1 acre, altitude $10\frac{2}{3}$ rd.

22. Triangular lot, area 10 acres, base $53\frac{1}{3}$ rd.

MISCELLANEOUS EXERCISES

289. 1. A locomotive weighed 158,750 lb. Find the weight in tons and pounds.

2. A house was built on a lot 86 ft. 3 in. wide. If the width of the house was $\frac{1}{3}$ that of the lot, how wide was the house?

3. A stationer bought 9 half-gross boxes of pencils at $2.25 per box and sold the pencils at 5¢ apiece. Find his gain.

4. How much is paid for making 6 Persian rugs, each containing $4\frac{1}{3}$ sq. yd., if the workmen receive $3 per sq. yd.?

5. What is the average weight of the members of a football team, if the 11 men weigh together 1 ton 35 lb.?

6. At every beat the heart pumps 6 ounces of purified blood into the system. If a man's heart beats 70 times per minute, how many tons of purified blood leave the heart in one day?

7. If 1 lb. of India tea makes $7\frac{1}{2}$ gal. of liquid tea of a certain flavor and color, how much liquid tea will 4 oz. make?

8. If $721\frac{1}{2}$ lb. of wool are clipped from 74 sheep, what is the average clip per sheep?

9. An athlete ran 18 times around a track $293\frac{1}{3}$ yd. in circumference. How many miles did he run?

10. A stove dealer received by freight, in one shipment, several lots of stoves, weighing respectively: 1 T. 14 cwt. 64 lb., 3 T. 49 lb., 2 T. 9 cwt., 5 T. 18 cwt. 87 lb. Find the freight charges at $.25 per hundredweight.

11. When a grocer had sold $3\frac{7}{8}$ gal. of strained honey from a 5-gallon can at 36¢ per quart, he had received enough money to pay for the whole can. How much did he gain on the can?

12. A pumping engine at the waterworks worked from Mar. 1 to Sept. 1 without stopping day or night, and consumed 642 T. 1056 lb. of coal. How many pounds did it consume per hour?

PERCENTAGE

290. 1. If Margaret is given 200 words to spell and she misses 5 words out of every *hundred*, how many words does she miss?

2. A certain school has 400 pupils. One stormy day 7 per hundred, or 7 *per cent*, of the pupils were absent. How many were absent?

3. If a man's yearly salary is $800 and he pays $\frac{12}{100}$, or 12 per cent, of it for house rent, how much is his rent?

4. Helen's book contains 300 pages. She read .15, or 15 per cent, of it one evening. How many pages did she read?

5. Find .06, or 6 per cent, of $500; 2 per cent of $700; 4 per cent of $600; 8 per cent of $900.

291. Per cent means *per hundred*, or **hundredths**, and the sign for it is %.

5 per cent, 5%, $\frac{5}{100}$, and .05 *represent the same thing.*

WRITTEN EXERCISES

292. Express as a common fraction:

1. 1%	4. 11%	7. 27%	10. 63%
2. 7%	5. 19%	8. 33%	11. 81%
3. 9%	6. 17%	9. 51%	12. 77%

Express as a decimal:

13. 15%	15. 36%	17. 4%	19. 10%
14. 24%	16. 45%	18. 6%	20. 30%

Express as per cent with the sign:

21. $\frac{9}{100}$	24. .48	27. .08	30. .60
22. $\frac{18}{100}$	25. .75	28. .20	31. .09
23. $\frac{41}{100}$	26. .55	29. .02	32. .90

293. Finding a per cent of a number.

1. Find $\frac{1}{3}$ of 12; $\frac{3}{4}$ of 16; $\frac{2}{5}$ of 20; $\frac{5}{6}$ of 24; $\frac{7}{8}$ of 32; $\frac{3}{10}$ of 80.

How do you find a fractional part of a number?

2. How many are $\frac{1}{100}$ of 300? 1% of 300? $\frac{7}{100}$ of 500? 7% of 500? $\frac{11}{100}$ of 600? 11% of 600?

3. Find .4 of 200; .04 of 200; 4% of 200; .08 of 500; 8% of 500; .12 of 300; 12% of 300.

4. How many are 3% of 700? 6% of 400? 2% of 1200?

5. Find 5% of \$300; 4% of \$400; 7% of \$900.

EXERCISES

294. How many are

1. $\frac{3}{100}$ of 400?	4. .07 of 700?	7. 6% of 900?
2. $\frac{9}{100}$ of 800?	5. .16 of 300?	8. 5% of 1000?
3. $\frac{13}{100}$ of 200?	6. .21 of 400?	9. 8% of 1200?

Find:

10. 10% of \$30	12. 11% of \$500	14. 25% of 400 bu.
11. 20% of \$80	13. 14% of \$300	15. 40% of 150 lb.

16. Mr. Livingston had \$100 and paid 22% of it for an overcoat. How much did the overcoat cost?

17. A farmer had 300 chickens, of which 15% were Plymouth Rocks. How many Plymouth Rocks had he?

18. There are 800 books in a library, of which 12% are historical. How many historical books are in the library?

19. In an orchard of 1100 trees 3% were plum trees. How many plum trees were there?

20. There are 200 children in a school, and 53% of them are girls. How many girls are there in the school?

21. A certain state senate has 50 members. When 30% of them are Democrats, how many Democratic senators are there?

WRITTEN EXERCISES

295. 1. Find 24% of $47.25.

$$\begin{array}{r} \$47.25 \\ .24 \\ \hline 18900 \\ 9450 \\ \hline \$11.34\cancel{0}\cancel{0} \end{array}$$

Since 24% of a number is .24 of it, 24% of $47.25 is .24 of $47.25.

Multiplying $47.25 by .24, the result is $11.34.

Hence 24% of $47.25 = $11.34.

Find to the nearest cent:

2. 14% of $375	6. 36% of $37.50	10. 12% of $4500
3. 26% of $562	7. 85% of $84.26	11. 44% of $7084
4. 5% of $83.40	8. 6% of $264.50	12. 75% of $342.34
5. 7% of $60.86	9. 8% of $409.25	13. 92% of $620.72

14. Only 93% of a bushel of clover seed was good seed. If a bushel weighs 60 pounds, how many pounds of good seed were there?

15. How much did a grocer pay for a tub of butter that sold for $15, if it cost 88% of the sum for which it sold?

16. Of the 1380 persons on board an Atlantic steamship, the crew was 40%. How many were in the crew?

17. A vessel that has 3255 miles to go covers 14% of the distance in one day. Find the distance traveled in a day.

18. A crop of sweet potatoes sold for $743.75. If the expense of raising them was 48% of this, how much did the crop cost?

19. The wheat necessary to make a barrel of flour that sold at $4.75 was purchased for 68% of the value of the flour. How much was paid for the wheat?

20. A dealer was obliged to sell a lot of pineapples for which he had paid $115.50, at 92% of the cost. For how much did he sell them?

WRITTEN EXERCISES

296. **1.** Express decimally and as a common fraction in its lowest terms: 25%; 5%; $33\frac{1}{3}\%$; 125%; $\frac{1}{2}\%$.

SOLUTIONS.

$$25\% = .25;\ \text{also}\ 25\% = \tfrac{25}{100} = \tfrac{1}{4}.$$
$$5\% = .05;\ \text{also}\ 5\% = \tfrac{5}{100} = \tfrac{1}{20}.$$
$$33\tfrac{1}{3}\% = .33\tfrac{1}{3};\ \text{also}\ 33\tfrac{1}{3}\% = \frac{33\frac{1}{3}}{100} = \tfrac{100}{300} = \tfrac{1}{3}.$$
$$125\% = 1.25;\ \text{also}\ 125\% = \tfrac{125}{100} = \tfrac{5}{4}.$$
$$\tfrac{1}{2}\% = .00\tfrac{1}{2};\ \text{also}\ \tfrac{1}{2}\% = \frac{\frac{1}{2}}{100} = \tfrac{1}{200}.$$

It is seen that $\frac{1}{2}\%$ means $\frac{1}{2}$ of 1%. What does $\frac{1}{4}\%$ mean? $\frac{2}{5}\%$? $\frac{3}{8}\%$?

Express decimally as hundredths:

2. $22\frac{1}{2}\%$	**5.** $3\frac{1}{3}\%$	**8.** $\frac{1}{5}\%$	**11.** 106%
3. $35\frac{3}{4}\%$	**6.** $6\frac{1}{4}\%$	**9.** $\frac{3}{4}\%$	**12.** 120%
4. $46\frac{2}{3}\%$	**7.** $8\frac{1}{3}\%$	**10.** $\frac{5}{8}\%$	**13.** 150%

Express as a common fraction in its lowest terms:

14. 20%	**19.** $12\frac{1}{2}\%$	**24.** $3\frac{1}{3}\%$	**29.** $83\frac{1}{3}\%$
15. 40%	**20.** $37\frac{1}{2}\%$	**25.** $6\frac{2}{3}\%$	**30.** $112\frac{1}{2}\%$
16. 50%	**21.** $62\frac{1}{2}\%$	**26.** $6\frac{1}{4}\%$	**31.** $133\frac{1}{3}\%$
17. 60%	**22.** $87\frac{1}{2}\%$	**27.** $8\frac{1}{3}\%$	**32.** $137\frac{1}{2}\%$
18. 75%	**23.** $66\frac{2}{3}\%$	**28.** $16\frac{2}{3}\%$	**33.** $266\frac{2}{3}\%$

34. To what per cent is $\frac{3}{4}$ equivalent? $\frac{1}{25}$? $\frac{2}{3}$? $1\frac{1}{5}$?

SOLUTIONS.

$$\tfrac{3}{4} = .75 = 75\% \qquad \tfrac{2}{3} = .66\tfrac{2}{3} = 66\tfrac{2}{3}\%$$
$$\tfrac{1}{25} = .04 = 4\% \qquad 1\tfrac{1}{5} = 1.20 = 120\%$$

Find the per cent equivalent of:

35. $\frac{1}{2}$	**38.** $\frac{1}{3}$	**41.** $\frac{2}{3}$	**44.** $1\frac{3}{4}$	**47.** $\frac{5}{12}$
36. $\frac{1}{4}$	**39.** $\frac{3}{8}$	**42.** $\frac{5}{6}$	**45.** $1\frac{5}{8}$	**48.** $\frac{11}{16}$
37. $\frac{4}{5}$	**40.** $\frac{1}{6}$	**43.** $\frac{7}{8}$	**46.** $2\frac{1}{3}$	**49.** $\frac{32}{125}$

297. In many exercises, when the *per cent* has an easy *fractional equivalent*, the work can be shortened by using the fraction.

Thus, in finding $33\frac{1}{3}\%$ of 36 it is much briefer and easier to do it by finding $\frac{1}{3}$ of 36 than by multiplying 36 by $.33\frac{1}{3}$.

The following *table of equivalents* should be *memorized*, and applied whenever possible:

TABLE

$50\% = \frac{1}{2}$	$60\% = \frac{3}{5}$	$87\frac{1}{2}\% = \frac{7}{8}$
$25\% = \frac{1}{4}$	$80\% = \frac{4}{5}$	$33\frac{1}{3}\% = \frac{1}{3}$
$75\% = \frac{3}{4}$	$12\frac{1}{2}\% = \frac{1}{8}$	$66\frac{2}{3}\% = \frac{2}{3}$
$20\% = \frac{1}{5}$	$37\frac{1}{2}\% = \frac{3}{8}$	$16\frac{2}{3}\% = \frac{1}{6}$
$40\% = \frac{2}{5}$	$62\frac{1}{2}\% = \frac{5}{8}$	$83\frac{1}{3}\% = \frac{5}{6}$

EXERCISES

298. Find:

1. 50% of 18	6. $12\frac{1}{2}\%$ of 56	11. 75% of 80
2. 25% of 28	7. $33\frac{1}{3}\%$ of 45	12. $62\frac{1}{2}\%$ of 64
3. 20% of 25	8. $16\frac{2}{3}\%$ of 60	13. $66\frac{2}{3}\%$ of 48
4. 40% of 60	9. $37\frac{1}{2}\%$ of 72	14. $83\frac{1}{3}\%$ of 66
5. 75% of 48	10. $66\frac{2}{3}\%$ of 36	15. $87\frac{1}{2}\%$ of 96

16. What is 100% of 25? of 32? of 46? of any number? Find 200% of 25; 300% of 16; 500% of 12.

17. The Simplon tunnel in Switzerland is 12 miles long, and the St. Gothard is 75% as long. How long is the St. Gothard tunnel?

18. A man's salary was increased to \$24 per week. If his former salary was $87\frac{1}{2}\%$ of this amount, how much had he been receiving?

19. In a football game one team scored 18 points and the other $33\frac{1}{3}\%$ of that number. What was the score?

20. A lamb that weighed 90 pounds when sold had gained $16\frac{2}{3}\%$ of its weight in the previous two months. How much had it gained in that time?

21. A certain ore yielded $66\frac{2}{3}\%$ of iron. How many tons of iron were obtained from 48 tons of the ore?

22. If an electric railway connecting two cities is 24 miles long and $37\frac{1}{2}\%$ of its length is within city limits, how many miles are within city limits?

WRITTEN EXERCISES

299. 1. Find $12\frac{1}{2}\%$ of \$275.20; $\frac{1}{8}\%$ of \$275.20.

$$8\overline{)\$275.20}\qquad\qquad 8\overline{)\$2.7520}$$
$$\$\ 34.40\qquad\qquad\qquad \$\ .344\cancel{0}$$

Since $12\frac{1}{2}\% = \frac{1}{8}$, the shortest way to find $12\frac{1}{2}\%$ of \$275.20 is to find $\frac{1}{8}$ of \$275.20, which is \$34.40.

Hence, $12\frac{1}{2}\%$ of \$275.20 = \$34.40.

Since $\frac{1}{8}\%$ means $\frac{1}{8}$ of 1%, we first find 1% of \$275.20 by moving the decimal point two places to the left, and then find $\frac{1}{8}$ of \$2.7520 = \$.344.

Find:

2. 25% of \$93.84	**7.** $\frac{1}{4}\%$ of \$36,928	**12.** 120% of \$624.50
3. $33\frac{1}{3}\%$ of \$46.08	**8.** $\frac{1}{8}\%$ of \$87,891	**13.** 250% of \$316.48
4. $37\frac{1}{2}\%$ of \$91.36	**9.** $\frac{3}{8}\%$ of \$40,000	**14.** 175% of \$546.84
5. $16\frac{2}{3}\%$ of \$79.74	**10.** $\frac{1}{6}\%$ of \$39,762	**15.** $162\frac{1}{2}\%$ of \$74.24
6. $66\frac{2}{3}\%$ of \$84.42	**11.** $\frac{2}{3}\%$ of \$87,531	**16.** $166\frac{2}{3}\%$ of \$97.89

17. If a codfish trawl line contains 750 hooks, and the average catch fills 80% of the hooks, how many fish are taken each time?

18. The cost of building roads in Pennsylvania is divided as follows: state, $66\frac{2}{3}\%$; county, $16\frac{2}{3}\%$; town, $16\frac{2}{3}\%$. How much did each contribute toward a road that cost \$5685.30?

19. If artificial ice is made at a cost of \$1.60 per ton and retailed for $262\frac{1}{2}\%$ of the cost, what is the selling price?

20. There are 1528 people in a certain village in Ohio. If $87\frac{1}{2}\%$ of the inhabitants are native born, how many are native born?

21. If the income one year from a certain publication was \$14,568, of which $37\frac{1}{2}\%$ came from advertisements and $33\frac{1}{3}\%$ from sales and subscriptions, what was the income from each of these departments?

22. An acre of land produced 18,756 pounds of sugar beets. How many pounds of sugar did these beets contain, if a test showed them to be $16\frac{2}{3}\%$ sugar?

Only 75% of the sugar could be extracted. How many pounds of sugar did the acre yield?

MISCELLANEOUS EXERCISES

300. Find:

1. 6% of 243
2. $3\frac{3}{4}\%$ of 368
3. 9% of 785
4. $\frac{5}{8}\%$ of 904
5. $6\frac{1}{4}\%$ of 682
6. 25% of 760
7. $13\frac{1}{2}\%$ of 984
8. 44% of 841
9. $16\frac{2}{3}\%$ of 549
10. $87\frac{1}{2}\%$ of 968
11. 105% of \$213.81
12. 110% of \$496.28
13. $137\frac{1}{2}\%$ of \$620.80
14. 225% of \$391.04
15. $308\frac{1}{3}\%$ of \$845.37

16. A diver's suit weighs 125 pounds. How much does the helmet weigh, if it is 64% of the entire weight?

17. The total yield of some date palms was 128 bunches of dates. Find the average yield per tree, if it was $6\frac{1}{4}\%$ of the total yield.

One of these bunches weighed 45 pounds, but the average weight was 40% of this. What was the average weight per bunch?

18. It cost Mr. James \$196.50 to raise and market his cherries. If he paid 30% of this for picking them, how much did the picking cost?

If he sold the cherries for $283\frac{1}{3}\%$ of their cost, how much did he receive for them?

19. A car load of lambs weighing 25,000 pounds lost $7\frac{1}{2}\%$ of their weight in the course of transportation to market. Find the whole loss of weight.

20. Find the output of a starch factory that uses 1200 bushels of potatoes per day, if the potatoes weigh 60 pounds per bushel and yield 16% of their weight in starch.

301. Finding what per cent one number is of another.

1. What part of 100 is 31? What per cent of 100 is 31? What per cent of 100 is 5? 20? 25? 50? 75? $33\frac{1}{3}$?

2. What part of 10 is 5? How many hundredths of 10 is 5? What per cent of 10 is 5?

3. What part of 12 is 3? How many hundredths of 12 is 3? What per cent of 12 is 3? What per cent of 12 is 6? 9?

4. What part of 15 is 5? What per cent of a number is $\frac{1}{3}$ of it? What per cent of 15 is 5? 10?

5. What part of $\frac{4}{5}$ is $\frac{2}{5}$? What per cent of $\frac{4}{5}$ is $\frac{2}{5}$?

EXERCISES

302. What per cent of

1. 18 is 9?	**6.** \$36 is \$9?	**11.** $\frac{2}{8}$ is $\frac{1}{8}$?
2. 28 is 7?	**7.** \$48 is \$12?	**12.** $\frac{5}{8}$ is $\frac{3}{8}$?
3. 50 is 5?	**8.** 72 mi. is 9 mi.?	**13.** $\frac{3}{4}$ is $\frac{1}{2}$?
4. 60 is 12?	**9.** 25 bu. is 10 bu.?	**14.** $\frac{2}{8}$ is $\frac{1}{4}$?
5. 75 is 25?	**10.** 60 lb. is 15 lb.?	**15.** $\frac{3}{4}$ is $\frac{1}{8}$?

16. A quart is what per cent of a gallon?

17. What per cent of a pound is 12 ounces?

18. What per cent of an hour is 20 minutes?

19. Mrs. Hicks used 50 yards of thread from a 200-yard spool. What per cent of the spool of thread did she use?

20. Mr. Carter had \$24 in his pocketbook and spent \$6 for a ton of coal. What per cent of his money did he spend?

21. If 5 out of every 50 tons of anthracite coal mined in Pennsylvania are used about the mines, what per cent of the output is used there?

22. From an Alabama farm worked on shares 36 bales of cotton were secured, of which 12 went to the owner. What per cent of the cotton did the owner receive?

23. Of the 40 passengers on an electric car running between two cities, 30 rode the entire distance. What per cent of the passengers rode all the way?

24. A freight car and its load weighed 60 tons. If the load weighed 40 tons, what per cent of the total weight was the weight of the load?

WRITTEN EXERCISES

303. **1.** What per cent of \$288 is \$14.40?

SOLUTION. — Since \$14.40 is $\frac{14.40}{288}$ (= \$14.40 ÷ \$288 = .05) of \$288, \$14.40 is 5 *hundredths*, or 5%, of \$288.

2. What per cent of \$8.64 is \$3.24?

SOLUTION. \$3.24 ÷ \$8.64 = .375; that is, \$3.24 is $37\frac{1}{2}$% of \$8.64.

What per cent of

3. \$150 is \$72?	**8.** \$2160 is \$54?	**13.** $\frac{5}{6}$ is $\frac{5}{8}$?
4. \$380 is \$95?	**9.** \$77.25 is \$6.18?	**14.** $\frac{15}{16}$ is $\frac{9}{10}$?
5. \$728 is \$455?	**10.** \$48.32 is \$3.02?	**15.** .64 is .12?
6. \$5.82 is \$1.94?	**11.** \$50.94 is \$84.90?	**16.** .48 is .08?
7. \$2.50 is \$4.05?	**12.** \$85.36 is \$10.67?	**17.** 96% is 30%?

What per cent of

18. 1 mile is 52 rods?	**20.** 1 ton is 910 pounds?
19. 1 section is 100 acres?	**21.** 1 day is 1260 minutes?

22. If the income from an acre of strawberries was \$495 and the cost of production was \$198, what per cent of the income was the cost?

23. What per cent of the 800 paper mills of this country does New York State contain, if it contains 180 paper mills?

24. In a certain school there are 752 pupils, of whom 282 are in the high school department. What per cent of the pupils are in the high school?

25. In crossing the Atlantic a vessel used 3570 of the 4760 tons of coal on board. What per cent of the supply was used?

26. A man who owed $20,000 could pay only $4900. What per cent of his debts could he pay?

27. Of 6225 baskets of fruit shipped from Delaware to New York, 2739 were spoiled by a delay. What per cent of the fruit was spoiled?

28. A certain mining town contains 4160 inhabitants, of whom 3640 are miners. What per cent of the population are miners?

29. In a recent year there were 210,000 miles of ocean cable in the world, of which 40,600 were in the Atlantic Ocean. What per cent of the cable was in the Atlantic?

30. A 20-dollar gold coin was so badly worn that it had lost 10 cents in value. What per cent of its weight had been worn away?

31. In a year when there were 270,000 Indians in the United States, 60,000 of them could speak English. What per cent of the Indian population could speak English?

32. The United States one year produced 20,600,000 barrels of salt, of which Michigan furnished 7,725,000 barrels. What per cent of the whole did Michigan produce?

33. A manufacturing business valued at $225,000 is owned by three partners: Hale, Chesley, & Hoit. If Mr. Hale's share is $140,625, Mr. Chesley's $56,250, and Mr. Hoit's $28,125, what per cent of the business does each man own?

304. Finding a number when a per cent of it is given.

1. If $\frac{3}{4}$ of a number is 9, what is $\frac{1}{4}$ of it? What is the number?

2. If $\frac{7}{100}$ of a number is 21, what is $\frac{1}{100}$ of it? $\frac{100}{100}$ of it? If 7% of a number is 21, what is 1% of it? 100% of it?

3. If 15% of a number is 30, what is 1% of it? What is the number? How does this result compare with $30 \div .15$?

In what two ways, then, may we find a number, if 15% of it is given?

4. 6% of a number is 24. Find the number in two ways.

5. If $33\frac{1}{3}$%, or $\frac{1}{3}$, of a number is 4, what is the number? If 75%, or $\frac{3}{4}$, of a number is 15, what is the number?

EXERCISES

305. Find the number of which

1. 12 is 1%	6. 12 is 20%	11. 10 is $\frac{1}{2}$%
2. 15 is 5%	7. 22 is 25%	12. 13 is $33\frac{1}{3}$%
3. 27 is 9%	8. 41 is 50%	13. 20 is $16\frac{2}{3}$%
4. 60 is 30%	9. 80 is 40%	14. 25 is $12\frac{1}{2}$%
5. 36 is 12%	10. 27 is 75%	15. 50 is $66\frac{2}{3}$%

16. If 50% of a man's working day is $4\frac{1}{2}$ hours, how many hours per day does he work?

17. A baseball team lost 8 games, or 25% of the total number of games played. How many games were played?

18. When a ring is 16 carats fine, it is $66\frac{2}{3}$% pure gold. If the ring were pure gold, how many carats fine would it be?

19. How many sweet potato plants does it take to yield a barrel of sweet potatoes, if 25 plants yield 20% of a barrel?

20. How much mail and express matter passes through the Grand Central Station, New York, each day, if $33\frac{1}{3}$% of it, or 300 tons, is mail matter alone?

21. If a fast freight goes 20% of its journey from New Orleans to Louisville in 10 hours, how long does the trip take?

22. If it takes 25% of a man's salary to pay his rent, which is \$15 a month, what is his monthly salary?

23. When a grocer had sold 40% of a case of eggs, 12 dozen had been sold. How many dozen eggs did the case hold?

24. A fruit dealer sold $37\frac{1}{2}$% of a box of grapefruit. How many did the box contain at first, if he sold 24?

25. Find the total weight of a bale of sponges from Florida, if the covering weighs 2 pounds, or 4% of the total weight.

26. It requires 120 tons of wood pulp to make the paper necessary to supply a certain newspaper for one day. If this is 80% of the weight of the paper, how many tons of paper are used per day?

WRITTEN EXERCISES

306. 1. Find the number of which 84 is 24%.

SOLUTION. 24% of the number = 84;
that is, .24 of the number = 84.
Hence the number = 84 ÷ .24 = 350.

2. Find the number of which 273 is $87\frac{1}{2}$%.

SOLUTION. Since $87\frac{1}{2}\% = \frac{7}{8}$, 273 is $\frac{7}{8}$ of the number.
Since $\frac{7}{8}$ of the number = 273,
$\frac{1}{8}$ of the number = $\frac{1}{7}$ of 273, or 39.
Hence, the number = 8 times 39, or 312.

Or, to shorten the work, the number $= \frac{8}{\cancel{7}}$ of $\overset{39}{\cancel{273}} = 312$.

Find the number of which

3. 225 is 15%	**8.** \$24.84 is 6%	**13.** \$ 1.14 is $\frac{3}{4}$%
4. 176 is 20%	**9.** \$36.95 is 4%	**14.** \$ 4.20 is $\frac{5}{8}$%
5. 384 is 25%	**10.** \$43.47 is 42%	**15.** \$79.62 is $37\frac{1}{2}$%
6. 504 is 28%	**11.** \$93.60 is 64%	**16.** \$87.95 is $166\frac{2}{3}$%
7. 880 is 55%	**12.** \$68.22 is 75%	**17.** \$90.97 is $183\frac{1}{3}$%

18. A manufacturer received 500 pounds of yarn, which was 8% of his order. How much yarn had he ordered?

19. How much steel was used in constructing the Chicago government building, if 12% of it, or 1200 tons, was used in the dome?

20. In six hours 6000 tons of coal were loaded on a vessel. If this was 80% of her load, how many tons did she carry?

21. A quantity of rice lost 20% of its weight in being hulled at the mill. If the hulls weighed 4752 pounds, what was the weight of the rough rice?

22. When 6875 tons of ice had been put into an ice house on Lake Erie, $62\frac{1}{2}$% of its capacity was occupied. Find the capacity of the ice house in tons.

23. If the shrinkage of a quantity of wheat in drying was 2% and amounted to 775 pounds, how much did the wheat weigh before drying?

24. If 32% of the people in a certain city are voters, and there are 40,176 voters, find the population of the city.

25. It is estimated that by skimming cream from milk only 80% of the butter is obtained. If the modern method of separating extracts all of the butter, what should be the yield of a cow that by the old method produced 180 pounds of butter per year?

26. A grocer bought 65 dozen ears of green corn at 12¢ per dozen. If this was 75% of the sum for which he sold the corn, how much did he receive for it?

27. A barrel of flour weighing 196 pounds sold at the grocery for $3\frac{1}{2}$¢ per pound, which was 140% of the cost. Find the cost.

28. How many feet of barbed wire are there on a spool, if 12 feet is $\frac{1}{5}$% of it?

29. If the productive area of coal in the United States is 154,220 square miles, or 55% of the entire coal area, what is the extent of the coal fields in this country in square miles?

307. Summary.

You have learned how to find: a per cent of a number; what per cent one number is of another; and a number when a per cent of it is given.

308. The number of which a per cent is found is called the **base**.

309. The *number* of hundredths found is called the **rate**, or **rate per cent.**

310. The result obtained by finding a per cent of the base is called the **percentage**.

Thus, in 5% of \$120 = \$6, \$120 is the *base*, 5% is the *rate*, and \$6 is the *percentage*.

\$120 + \$6, or \$126, is the **amount**; \$120 − \$6, or \$114, is the **difference.**

311. We may now express in brief form the principles learned in the preceding pages:

The **percentage** *equals the base multiplied by the rate.*
The **rate** *equals the percentage divided by the base.*
The **base** *equals the percentage divided by the rate.*

WRITTEN EXERCISES

312. **1.** Given the base 245, and the rate 8%; find the percentage.

2. If the percentage is \$5.50 and the base \$22, find the rate.

3. Find the base, if the percentage is \$75 and the rate 4%.

Fill blanks:

	BASE	RATE	PERCENTAGE
4.	\$249.36	$16\frac{2}{3}$%	———
5.	———	78%	\$412.62
6.	\$694.80	———	\$260.55
7.	\$848.25	106%	———

8. If \$275 will pay 55% of Mr. Hall's debts, how much does he owe?

9. Find the number of bushels of wheat in a schooner's cargo of 27,600 bushels of grain, if $46\frac{2}{3}$% of it is wheat.

10. What per cent of the work of painting a house did a painter do in $6\frac{5}{6}$ days, if he completed the work in $27\frac{1}{3}$ days?

11. If 28 shirts is $\frac{2}{5}$% of the weekly output of a shirt factory, how many shirts are made there each week?

12. A wholesale dealer bought 8270 barrels of sweet potatoes at \$1.25 per barrel. If he sold 60% of them @ \$1.75, 30% @ \$2.25, and the remainder @ \$1, how much did he gain?

13. This diagram, 4 inches long, represents the value of all the boots and shoes made in the United States in a certain year.

The part that represents the value produced by Massachusetts is 1.79 in. long. What per cent of the total value did Massachusetts produce that year?

14. New York's part of the diagram is .39 in. long; New Hampshire's, .37 in. long; Ohio's, .27 in. long; Pennsylvania's, .21 in. long; Maine's, .19 in. long; Illinois's .16 in. long.

What per cent did each of these states produce?

15. What per cent of the total value of the boots and shoes was produced in all the other states of the United States?

16. If 64% of the farms in the United States one year were worked by their owners, and there were 5,737,350 farms, how many were worked by their owners?

17. In a certain year 1,146,992 cords of spruce wood were used in the manufacture of wood pulp. If this was 76% of all the wood used for pulp that year, how many cords were used?

Commercial Discount

313. 1. To stimulate trade a merchant reduced his prices 5% on purchases amounting to $20 or more. What was the reduction in the price of a rug marked $40?

2. If a manufacturer makes a deduction of 2% from his prices, for cash payment, what deduction for cash would he make on a bill of goods amounting to $300?

314. Any deduction made from a fixed price or amount is called a **discount.**

315. Discounts are usually reckoned as some per cent of the price or amount. The per cent of discount is called the **rate of discount.**

316. Many articles of merchandise have fixed prices which are published in catalogues and price lists.

Such prices are known as **list prices.**

317. For various reasons, discounts are often given from the list price or the marked price and are frequently spoken of as so many "per cent off."

Thus, if an article listed or marked at $1.50 is sold at "10% off," the discount is 10% of $1.50, or $.15, and the selling price $1.50 – $.15, or $1.35.

318. The price after the discount has been taken off is called the **net price.**

EXERCISES

319. Find the discount when the list price and rate of discount are:

1. 20¢, 5%	5. $12, 50%	9. $18, $33\frac{1}{3}$%
2. 50¢, 6%	6. $25, 20%	10. $64, $12\frac{1}{2}$%
3. 25¢, 8%	7. $36, 25%	11. $48, $16\frac{2}{3}$%
4. 80¢, 10%	8. $24, 40%	12. $72, $37\frac{1}{2}$%

13. Find the discount on a mandolin that is listed at $25, if it is sold at 5% off for cash.

14. The list price of a wagon is $80, from which 10% discount is offered. Find the net price.

15. At a bargain sale some tapestry curtains were marked down 20%. If the regular price was $5 per pair, what was the selling price?

16. A bill of goods amounting to $600 was discounted 3% for cash. How much was the discount?

17. A graphophone catalogued at $50 was sold at 25% off. Find the discount.

18. How much does a lady pay for 4 yards of linen, if the price, $.50 per yard, is marked down 8%?

19. To make room for new stock a dealer sold oilcloth marked $.75 per yard at a discount of 20%. What was the selling price?

WRITTEN EXERCISES

320. Find the discount on articles listed as follows:

1. $2.25, 8% off	**4.** $14.75, 20% off
2. $3.80, 5% off	**5.** $35.48, 25% off
3. $5.50, 6% off	**6.** $54.75, $33\frac{1}{3}$% off

Find the net price, net cost, and total discount in each:

	Articles	Price	Rate of Discount
7.	12 tents	$28.00	3 %
8.	15 rowboats	35.00	10 %
9.	20 shotguns	32.00	$37\frac{1}{2}$%
10.	18 gun cases	3.00	$16\frac{2}{3}$%
11.	24 revolvers	7.50	$33\frac{1}{3}$%
12.	36 fishing rods	1.60	$12\frac{1}{2}$%
13.	30 hunting knives	1.75	20 %

14. Find the net price of a desk listed at \$32.50, discount 20%.

15. There was a discount of 50% on a set of Shakespeare's works the list price of which was \$41.50. Find the discount.

16. The list price of a set of Dickens was \$22.50. At a discount of $33\frac{1}{3}$%, for how much was the set sold?

17. If your father buys a set of books by Victor Hugo at \$16.25, discount 40%, how much does he pay?

18. A merchant sold go-carts at 30% off. What was the selling price of one marked \$18.50?

19. How much must a dealer pay for 9 cameras listed at \$25 each, if there is a discount of 15%?

20. Mr. Burton bought a suit of clothes from a dealer who was selling out, at 25% off. If the suit was marked \$27, how much did he pay for it?

21. Find the cash cost of 220 feet of garden hose at $7\frac{1}{2}$¢ per foot, discount for cash 10%.

22. Find the net cost of 3 cases of crayons, each holding 100 gross, at a discount of 20%, list price 5¢ per gross.

23. Silk mufflers listed at \$2.25 each are sold to a retailer at a discount of $16\frac{2}{3}$%. If the retailer buys 3 dozen, how much discount does he receive?

24. A manufacturer purchased 3600 yards of gingham at $6\frac{3}{4}$¢ per yard, and received 3% discount by paying within 10 days. What was the net cost of the goods?

25. A hardware merchant ordered 250 pounds of horseshoe nails at \$.22 per pound, discount 40%. How much was his bill? By paying cash, the merchant received a further discount of 5%. How much did he have to pay?

26. A merchant in Minneapolis bought a car load of damaged wheat weighing 42,000 lb. at $62\frac{1}{2}$% discount. If the original price was \$.72 per bushel of 60 lb., find the net cost.

REVIEW PROBLEMS IN INDUSTRIES

321. 1. A man in Illinois raised 72 acres of broom corn. If $4\frac{1}{2}$ bu. of seed were required, how much was sown per acre?

2. He bought the seed by weight. If a bushel of seed weighs 50 pounds, how many pounds were there?

3. Find the cost of the 225 lb. of seed at $7.40 per 100 lb.

4. When the corn was full grown, the broom-making material, or brush, was obtained by cutting off the tops and thrashing out the seed. Find the cost of thrashing the crop from 72 acres at $2.50 per acre.

5. To dry the brush it was placed on shelves in sheds 9 ft. 8 in. high, each shed containing a shelf for every 4 inches of its height. How many shelves were there in a shed?

6. The cured brush was bound into 120 bales whose united weight was 20.7 tons. Find the average weight of a bale.

7. How many pounds of cured brush were produced per acre?

8. Find the yield of each acre in bales. What per cent of an acre's yield was used in making one bale?

9. It cost $35 per ton to produce the crop and prepare the brush for market. What was the total expense?

10. The total cost being $724.50, how much in all did the owner gain, if he sold the baled brush at $4\frac{3}{4}$¢ per pound?

11. Broom corn is often sold by the ton. How much more or less per ton would he have made, if he had sold it at $90 per ton?

12. If one ton of broom corn makes 100 dozen brooms of ordinary size, how many dozen brooms were produced from the crop of 20.7 tons? How many brooms did each of the 120 bales make?

13. In a recent year Illinois furnished 30,320 tons of broom corn, or $66\frac{2}{3}\%$ of the total amount grown in the United States. Find the production of broom corn in this country that year.

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14. In growing mushrooms, spawn is the material that takes the place of seed. Spawn consists of mushroom fibers in rich dirt. How many pieces of spawn were needed to plant a bed 120 feet long, if 4 pieces were planted for every 8 inches of its length?

15. A house for mushrooms had on the sides 6 such beds, and in the middle 4 beds. Each middle bed held twice as much as one of the others. How many pieces of spawn had to be used?

16. If the spawn was bought in bricks, and each brick was broken into 12 pieces for planting, how many bricks were required for the 10,080 pieces?

17. How many pounds of spawn were planted, if each of the 840 bricks weighed $1\frac{1}{4}$ pounds? At \$157.50 for the spawn, find its cost per pound.

18. If the bricks had been broken into 9 pieces instead of into 12, how many more bricks would have been used?

19. What would have been the extra cost for the 280 additional bricks at 15¢ per pound?

20. The yield of mushrooms was $\frac{3}{8}$ of a pound for every square foot of bed. How many pounds of mushrooms were produced, if the total area of the beds was 5040 sq. ft.?

21. The choice mushrooms, $33\frac{1}{3}$% of the crop of 1890 lb., were put into 10-pound baskets. How many baskets were used?

22. If 3-pound baskets were used for 50% of the mushrooms, find the number of 3-pound baskets used.

23. How many 5-pound baskets did the remainder fill?

24. The proprietor of the Laurel House bought the choice ones, 630 lb., at 65¢ per pound. How much did he pay for them?

25. The 3-pound baskets, 315 in all, were sold on the market for \$378. What was the average price received for them per pound and per basket?

26. How much was received for the 315 pounds put up in 5-pound baskets at \$2.25 per basket?

27. What were the receipts for the entire crop?

The scarcity of oysters in natural beds and the increased demand for oysters has led to the transplanting and raising of oysters either on old beds or on artificially prepared beds called "oyster farms." Where the ocean bottom is not naturally rocky or gravelly, it is prepared for the reception of the young oysters, or "seed oysters," by strewing it with oyster shells.

28. An oyster farm of 250 acres in Chesapeake Bay was

strewn with 17,500 bushels of oyster shells. Find their cost at $3\frac{1}{2}$¢ per bushel.

29. Thirty-five barrels of seed oysters from the Potomac were planted per acre. How many barrels of seed were required for the farm of 250 acres?

30. Find the cost of the seed, 8750 barrels, at $1.25 per barrel.

31. A barrel contains 15,000 seed oysters. How many oysters were planted per acre? How many were planted on the whole farm?

32. The oysters matured in three years. During this time 32% were destroyed by starfish and other natural enemies, and 6% stolen by "oyster pirates." How many of the 131,250,000 were thus lost?

33. Of the remaining 81,375,000, 84% were gathered. How many oysters were gathered?

34. Of the choice oysters, $3\frac{3}{4}$ millions were sold at 11¢ a dozen, and $4\frac{1}{2}$ millions at 9¢ a dozen. How much was received for them?

35. There were 15,000 barrels of second-grade oysters, averaging 1400 to the barrel. How much was received for these at $37\frac{1}{2}$¢ per hundred?

36. The rest of the oysters, 39,105,000, were shucked and sold at the rate of $\frac{1}{4}$¢ each. How much was received for them?

37. What were the total receipts for this crop of oysters?

38. A retailer bought a barrel containing 1368 second-grade oysters for $6.84. He removed them from the shells and sold them at 40¢ a quart. How much did he gain, if the average number in a quart was 48?

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39. A rice grower in South Carolina sowed 225 bushels of rice seed, or $2\frac{1}{2}$ bushels per acre. How many acres did he sow?

40. During the season the field received a depth of 18.72 inches of water from irrigating canals and 9.35 inches of rainfall. If 15.68 inches evaporated, find the useful depth of water.

41. After drying the field for harvest, the planter used two reapers to cut the 90 acres of grain. If each reaper cut 11.25 acres per day, how long did it take to harvest the crop?

When the grain was thrashed there were 900 sacks of rough rice, weighing 162 pounds each. Find the number of:

42. Sacks per acre.

43. Bushels of 45 pounds each, per acre.

44. Bushels from the whole field.

45. For having his land irrigated the owner paid 2 sacks of rice per acre to the canal company. What per cent of his crop did he pay? What per cent did he have left?

46. Having paid 20 % of his crop of 3240 bushels for irrigation, how many bushels of rough rice did he have left?

47. This rice was sold at the mill for 85¢ per bushel. How much money was received for the 2592 bushels?

48. The expense, aside from irrigation, was $17.88 per acre. What was the expense for the 90 acres?

49. What was the profit on the crop?

50. How many pounds of rough rice were sold at the mill, each of the 2592 bushels weighing 45 pounds?

51. Every 162-pound sack of rough rice yielded 100 pounds of cleaned rice when milled. How many pounds of cleaned rice were obtained from the 116,640 pounds of rough rice?

52. How many hours did it take to mill this rice, if the capacity of the mill was 648 sacks per 12 hours?

53. How much less did the 72,000 pounds of cleaned rice weigh than the rough rice?

54. If the hulls removed from the rough rice weighed 23,328 pounds, what per cent of the rough rice consisted of hulls?

55. If the rest of the 44,640 pounds lost in milling consisted of bran and polish which sold, when mixed, at $15 per ton, find the sum received for the bran and polish.

56. If 23% of the cleaned rice was broken during milling, how many pounds of broken rice were there? how many pounds of whole rice?

57. At $5\frac{3}{4}$¢ per pound, what was the value of the 55,440 pounds of unbroken rice?

58. If the 16,560 pounds of broken rice sold for $414, at what price per pound did it sell?

59. If Louisiana and Texas one year produced 297,900,000 of the 331,000,000 pounds of rice raised in the United States, what per cent of the whole crop did these states produce?

60. At a cost of $1.05 per rod, find the expense of making $3\frac{1}{4}$ miles of irrigating canals for rice land.

61. A violet grower, in order to have a large number of cuttings from which to select, bought 14,550, and planted $\frac{1}{3}$ of them. How many cuttings did he plant?

62. His receipts during the season were distributed as follows:

October . . .	$187.50
November . . .	236.25
December . . .	337.50
January . . .	405.00
February . . .	375.00
March	337.50
April	281.25
May	90.00

What per cent of the total receipts were the receipts of each month?

63. The schedule of prices per 100 violets was:

Oct., $1	Dec., $2	Feb., $1	Apr., $.75
Nov., $1	Jan., $1.50	Mar., $.75	May, $.75

How many violets were sold during each month? Find the total number sold.

64. Since 218,250 violets were obtained from 4850 plants, what was the average number of violets from each plant?

65. All these flowers were sent to florists in distant places at an express charge of 8¢ per 100 violets. Find the entire expense of sending them by express.

66. If these charges, $174.60, were $33\frac{1}{3}$% of the entire expense, how much did it cost to cultivate and market the violets?

67. What was the gain on the season's crop of violets?

MEASURES AND EQUIVALENTS

Measures of Weight

322. Avoirdupois weight.

The pound composed of 16 ounces, used in weighing bulky articles, is called an **avoirdupois pound.**

In delicate weighing, a small unit of weight called a **grain,** originally the weight of a grain of wheat, is used.

1 grain = $\frac{1}{7000}$ of an avoirdupois pound

For complete tables of denominate numbers see the Appendix.

323. Troy weight.

In weighing gold, silver, platinum, and some kinds of jewels, a weight equal to 5760 grains, or $\frac{5760}{7000}$ of an avoirdupois pound, is used as a unit of weight.

This weight is called a **troy pound.**

$\frac{1}{12}$ of a troy pound, or 480 grains, is called a **troy ounce.**

$\frac{1}{20}$ of a troy ounce, or 24 grains, is called a **pennyweight.**

These related weights form the table of **troy weight.**

24 grains (gr.)	= 1 pennyweight,	pwt.
20 pennyweights	= 1 ounce,	oz.
12 ounces	= 1 pound,	lb.

In writing prescriptions physicians often use these units:

20 grains	= 1 scruple
3 scruples	= 1 dram
8 drams	= 1 ounce (troy)

Except in filling prescriptions, druggists buy and sell by avoirdupois weight.

WRITTEN EXERCISES

324. 1. Which is heavier, and how much, a pound of gold or a pound of cotton?

2. How many grains are there in an avoirdupois ounce?

3. What decimal part of a troy ounce is an avoirdupois ounce? Obtain the answer to the nearest thousandth.

4. A druggist bought 1 100-ounce tin of quinine, 3 50-ounce tins, 5 25-ounce tins, and 3 15-ounce tins. How many pounds and ounces of quinine did he buy?

5. A drug company wishes to make 4000 pills containing 5 grains of quinine each, 4000 containing 3 grains each, and 12,000 containing 2 grains each. How many pounds of quinine must be used? Find its cost at $26\frac{1}{2}$¢ per ounce.

6. A man sold 125 bales of cotton, weighing 30 T. 875 lb. net, at 8.45¢ per pound. How much did he receive?

7. Find the value of 12 T. 840 lb. of pig iron, at $18.40 per long ton.

8. When oats are selling at $1.10 per 100 lb., how much will be received from a sale of 5 T. 635 lb.?

9. How much is a pound of gold worth at the United States coinage rate, $20.67183 per ounce?

Find the value, to the nearest cent, of the following weights of silver, at 66¢ per ounce:

10. 5 lb. 8 oz.	12. 4 oz. 5 pwt.	14. 2 lb. 10 oz. 3 pwt.
11. 2 lb. 10 oz.	13. 11 oz. 4 pwt.	15. 1 lb. 16 pwt. $12\frac{1}{2}$ gr.

16. The United States silver dollar weighs $412\frac{1}{2}$ grains, and is $\frac{9}{10}$ pure silver. Find its bullion value, or the value of the silver in it, when silver is worth 64 cents per ounce.

17. How many silver spoons, each weighing 2 oz. 5 pwt., can be made from a silver bar weighing 6 lb. 4 oz. 10 pwt.?

18. *A double eagle weighs* $21\frac{1}{2}$ *pennyweights.* United States gold coins are $\frac{9}{10}$ pure gold and $\frac{1}{10}$ copper, by weight. How many grains of pure gold does a double eagle contain?

19. The value of a gold coin is the value of the pure gold in it. The value of a double eagle is \$20. How much, to the nearest hundredth of a cent, is 1 pennyweight of pure gold worth?

20. How many pennyweights of pure gold are worth \$1?

21. A Newark goldsmith bought 1 lb. 2 oz. of gold bullion at \$1.03 per pennyweight. How much did it cost him?

22. Gold coin in banks is often piled in bags containing \$5000 each, and is weighed instead of being counted out. How many pounds and ounces should such a bag contain?

23. If you should draw \$15,000 in gold coin from a bank, how many pounds and ounces of gold ought you to receive?

24. Gold coins are received by the government at their face value so long as the loss of weight due to wearing does not exceed $\frac{1}{2}\%$. What is the least weight of a double eagle that the government will receive as \$20?

25. How much is a double eagle worth when it has lost $\frac{3}{4}\%$ in weight, if it is then worth only its actual weight?

26. Find the weight of \$1,000,000 in United States gold coin fresh from the mint.

27. In shipping \$1,000,000 in new gold eagles from New York to London, the loss in weight due to wear amounted to 188 pwt. 3 gr. Find the loss in dollars.

What per cent of the million dollars was lost?

28. Seventeen tons of ore sent to the smelter contained .48 oz. of gold worth \$20 per ounce, and 25 oz. of silver worth 65¢ per ounce. Find the value of precious metal per ton of ore.

29. One year the total production of platinum in the United States was 7 lb. 10 oz., worth \$1814. Find its value per ounce.

Measures of Volume and Capacity

325. A pile of 4-foot firewood, 8 feet long and 4 feet high, is called a **cord.**

1 cord = 128 cu. ft.

326. One fourth of a pint, liquid measure, is called a **gill.**

This measure is little used.

For the more common measures of volume and capacity, see the tables.

327. Useful equivalents.

A **gallon** is equivalent to **231 cu. in.,** and a **bushel** to **2150.42 cu. in.** Since 2150.42 cu. in. is only about 10 cu. in. less than $1\frac{1}{4}$ cu. ft., it is sufficiently accurate for all practical purposes to take $1\frac{1}{4}$ cu. ft. as the equivalent of a bushel.

1 gallon	=	**231 cu. in.**
1 bushel	=	**$1\frac{1}{4}$ cu. ft.**

WRITTEN EXERCISES

328. Find the number of cords of 4-foot wood in:

1. A pile 120 ft. long and 4 ft. high.
2. A pile 76 ft. long and 6 ft. high.
3. A pile $\frac{1}{4}$ mi. long and 8 ft. high.
4. A load 8 ft. long and $4\frac{1}{2}$ ft. high.
5. How many cubic inches are there in a quart, liquid measure? in a quart, dry measure?
6. How many bushels of grain can be stored in an elevator bin 7 ft. square and 80 ft. high?
7. A vat at a cheese factory is 11 ft. long, 4 ft. wide, and $3\frac{1}{2}$ ft. deep. How many gallons of milk will it contain?

8. The inside dimensions of a car are: length, 36 ft.; width, 8 ft. 4 in.; height, 8 ft. Find its cubical contents.

9. If the car could be filled to the top with oats weighing 32 pounds to the bushel, how many pounds of oats would it contain, the cubical contents of the car being 2400 cubic feet?

10. The greatest load that the railroad company allows the car to carry is 80,000 lb., called its *capacity*. Since the weight of the oats when the car is filled to the top is 61,440 lb., how much less is that than the permitted capacity of the car?

11. If the car could be loaded to the top with wheat weighing 60 pounds to the bushel, how much would the load exceed the capacity allowed by the company?

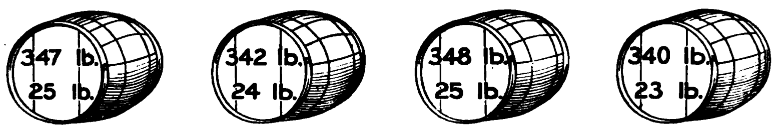
12. How many bushels of wheat would there be in the car, if the bottom were covered to a depth of 1 foot?

13. To what height is the car filled with wheat when the load is the greatest the car may carry, 80,000 pounds?

14. When empty, the car weighs 35,000 pounds. Find its weight when loaded with 1200 bushels of shelled corn weighing 56 pounds to the bushel.

15. Find its weight when loaded with 200 2-bushel bags of clover seed, 60 pounds to the bushel, 80 200-pound bags of bran, and 225 bushels of potatoes, 60 pounds to the bushel.

16. At a grocery store a boy saw these marks on the heads of some barrels of sugar:



The first weight is the weight of barrel and sugar, the second the weight of the empty barrel. Find the cost of the sugar in all the barrels at $4.25 per 100 pounds.

The abbreviation for barrel or barrels is bbl.

Find the weight and the cost of:

17. 16 bbl. flour, each 196 lb., at \$4.75 per barrel.
18. 20 bbl. salt, each 280 lb., at \$1.15 per barrel.
19. 120 bbl. pork, each 200 lb., at \$12.80 per barrel.
20. 2 bbl. dressed chickens, 275 lb., 280 lb., at $13\frac{1}{2}$¢ per pound.
21. 3 tierces lard, 250 lb., 290 lb., 260 lb., at \$7.55 per 100 lb.

Find the gain on each of the following:

	Article	Buying Price	Selling Price
22.	1 bbl. linseed oil, 49 gal.	38¢ per gal.	50¢ per gal.
23.	1 bbl. kerosene, 52 gal.	\$5.75	14¢ per gal.
24.	1 bbl. turpentine, 50 gal.	$67\frac{1}{2}$¢ per gal.	90¢ per gal.
25.	1 bbl. pine tar, 50 gal.	\$5.25	25¢ per gal.
26.	1 bbl. paint, 47 gal.	58¢ per gal.	75¢ per gal.
27.	1 bbl. molasses, 50 gal.	$28\frac{1}{2}$¢ per gal.	35¢ per gal.
28.	1 bbl. potatoes, $2\frac{3}{4}$ bu.	\$1.60	20¢ per pk.
29.	4 bags peanuts, each 4 bu.	\$1.40 per bu.	5¢ per qt.
30.	6 boxes dates, each 30 lb.	\$2.50 per box	10¢ per lb.
31.	7 cases eggs, each 30 doz.	\$6.45 per case	26¢ per doz.
32.	8 tubs butter, each 60 lb.	$22\frac{1}{2}$¢ per lb.	27¢ per lb.
33.	12 25-lb. boxes apricots	$9\frac{1}{2}$¢ per lb.	12¢ per lb.
34.	18 tons meal, 100-lb. bags	\$1.25 per bag	\$1.45 per bag
35.	12 tons bran, 200-lb. bags	\$19 per ton	\$1.10 per cwt.

329. Water in large quantities is measured by the barrel of $31\frac{1}{2}$ gallons, by the cubic foot, and by the thousand or the million gallons.

In all ordinary calculations the weight of a cubic foot of water is taken as $62\frac{1}{2}$ lb. or 1000 oz.

$$1 \text{ cu. ft.} = 1728 \text{ cu. in.} = \tfrac{1728}{231} \text{ gal.} = 7.48^{+} \text{ gal.}$$

Since 1 gal. = 231 cu. in. = $\frac{231}{1728}$ cu. ft., the weight of 1 gallon of water weighing $62\frac{1}{2}$ lb. per cu. ft. is $\frac{231}{1728}$ of $62\frac{1}{2}$ lb., or 8.355^{+} lb.

WRITTEN EXERCISES

330. Hereafter, unless the contrary is expressly stated, use the following measures and approximate equivalents:

1 barrel of water	**= $31\frac{1}{2}$ gallons**
1 cubic foot	**= $7\frac{1}{2}$ gallons**
1 cubic foot of water weighs $62\frac{1}{2}$ pounds	
1 gallon of water weighs $8\frac{1}{3}$ pounds	

1. Find the capacity in barrels of a rectangular cistern 6 feet square and $10\frac{1}{2}$ feet deep; of one 16 feet deep, $8\frac{3}{4}$ feet long, and $7\frac{1}{2}$ feet wide.

2. The tank in the tender of a locomotive held 7500 gallons. Find its capacity in cubic feet; in barrels.

3. A city uses $18\frac{3}{4}$ million gallons of water per day. Express this quantity of water in cubic feet.

4. A city had 8 slow sand filters for purifying its drinking water. Each filter discharged 275,000 cubic feet of water per day. How many gallons of water were filtered per day?

5. The area drained by the Hudson River above Albany is about 8240 square miles. A square mile is equal to 27,878,400 square feet. How many million gallons of water, to the nearest million, fall on this area during a year if the total rainfall is 40 inches?

6. The water of the Dead Sea is 1.24 times as heavy as fresh water. How much does a gallon of Dead Sea water weigh?

7. Milk is about 1.03 times as heavy as water. What is the approximate weight of milk in a vat containing 600 gallons of milk?

8. Ice is .92 as heavy as water. How much will a block of ice 2 ft. by 30 in. by 12 in. weigh?

9. A cubic foot of one kind of marble weighed $163\frac{3}{4}$ pounds, and of another kind $184\frac{5}{8}$ pounds. How many times as heavy as water was each kind?

10. Cork is .24 as heavy as water. How much does a cubic foot of cork weigh?

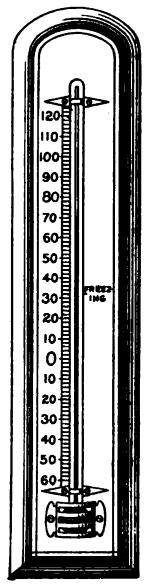
11. Find the weight of a granite corner stone 4 ft. square and 30 in. thick, if granite is 2.72 times as heavy as water.

12. Steel is about 7.84 times as heavy as water. Find the weight of a steel plate 90 in. by 56 in. and $\frac{3}{8}$ in. thick.

13. Lead is 11.35 times as heavy as water and tin 7.28 times as heavy as water. How much more does a cubic foot of lead weigh than a cubic foot of tin?

Measures of Temperature

331. On a Fahrenheit (F.) thermometer, the one in common use in America, the freezing point of water is marked 32, for 32 *degrees*, and the boiling point 212. For ordinary purposes, however, thermometers are not marked as far as the boiling point.



Since the difference between the freezing and boiling points of water, $212° - 32°$, is $180°$, one degree Fahrenheit is $\frac{1}{180}$ of this difference.

Temperatures below 0° are written with a minus sign, thus: $-1°$, $-2°$, $-10°$, etc.

EXERCISES

332. 1. At 5 A.M. the temperature was $-8°$ F. and at 2 P.M. it was 24° F. Find the rise in temperature.

2. At noon the temperature was 14° F. and at 10 P.M. it was $-3°$ F. What was the fall in temperature?

3. How far below the freezing point of water is 12° F.? −12° F.?

4. During an ascent of Pike's Peak the temperature fell from 60° F. to 31° F. How many degrees did it fall?

5. When the temperature in Montreal is − 14° F. and in New Orleans 58° F., what is the difference in temperature between the two places?

6. The temperature of a beef carcass was reduced from 96° F. to 30° F., the temperature of cold storage. How many degrees was the temperature reduced?

7. In freezing ice cream a mixture of ice and salt reduced the temperature from 50° F. to 28° F. How many degrees was the temperature reduced?

WRITTEN EXERCISES

333. 1. Cast iron melts at 2100° F. and steel at 2500° F. How much higher is the melting point of steel than that of cast iron?

2. Tin melts at 442° F. How many degrees above the melting point of ice (32° F.) is this? how many degrees above the boiling point of water?

Find the difference between the highest and lowest temperature in the following cities during a recent year (Fahrenheit scale):

City	Highest	Lowest	City	Highest	Lowest
3. Havre, Mont.	108°	−55°	8. Portland, Me.	97°	−17°
4. Cincinnati	105°	−17°	9. Portland, Ore.	102°	− 2°
5. Duluth	99°	−41°	10. Boston	102°	−13°
6. Mobile	102°	− 1°	11. San Diego	101°	32°
7. Key West	100°	41°	12. Santa Fe	97°	−13°

13. How much more did the temperature in Duluth vary than that in Key West?

Lumber Measure

334. The number of **board feet** in a board 1 inch (or less) in thickness is the number of square feet of surface in one side of the board.

A board 1 foot wide and 15 feet long contains 15 square feet, or 15 feet *board measure*, if the board is 1 inch or less in thickness.

For brevity we often use the word "foot" instead of board foot, and "thousand feet" or M instead of thousand board feet.

335. *When the thickness is greater than* 1 *inch, the number of feet board measure is obtained by multiplying the number of feet in length by the number of feet in width, and this product by the number of inches in thickness.*

Thus, a board 6 ft. long and 10 in. wide contains $(6 \times \frac{5}{6})$ ft., or 5 ft., if it is 1 in. thick or less than 1 in. thick; if it is 2 in. thick, it contains 2×5 ft., or 10 ft.; if $1\frac{1}{2}$ in. thick, $1\frac{1}{2} \times 5$ ft., or $7\frac{1}{2}$ ft.

The dimensions 6 ft. by 10 in. by 2 in. are commonly written in this way: $6' \times 10'' \times 2''$.

EXERCISES

336. Find the number of feet in boards 1 inch or less in thickness that have the following lengths and widths:

1. $4' \times 3''$	10. $9' \times 8''$	19. $18' \times 14''$
2. $16' \times 3''$	11. $24' \times 8''$	20. $18' \times 16''$
3. $22' \times 3''$	12. $7\frac{1}{2}' \times 8''$	21. $18' \times 18''$
4. $15' \times 4''$	13. $12' \times 10''$	22. $18' \times 20''$
5. $24' \times 4''$	14. $18' \times 10''$	23. $12' \times 22''$
6. $20' \times 4''$	15. $15' \times 10''$	24. $16' \times 24''$
7. $10' \times 6''$	16. $12' \times 12''$	25. $28' \times 24''$
8. $14' \times 6''$	17. $16' \times 12''$	26. $22' \times 30''$
9. $11' \times 6''$	18. $12' \times 14''$	27. $16' \times 36''$

28. How many board feet are there in a 2-inch plank 12 feet long and $1\frac{1}{4}$ feet wide?

WRITTEN EXERCISES

337. Find the number of feet in the following timbers:

Number of Pieces	Size	Length	Number of Feet
1. 24	2″ × 12″	12′	——
2. 15	2″ × 6″	14′	——
3. 250	2″ × 4″	10′	——
4. 112	3″ × 8″	16′	——
5. 64	2″ × 8″	14′	——
6. 80	2″ × 12″	12′	——
7. 125	4″ × 6″	18′	——
8. 75	6″ × 8″	18′	——
9. 48	4″ × 8″	12′	——
10. 84	7″ × 9″	11′	——
11. 112	6″ × 14″	14′	——
12. 96	8″ × 16″	20′	——

13. Find the cost of all the above timber at $24 per M.

Foot the following lumber bills at $21 per M:

In an expression like "sills 4″ × 8″ × 200′," 200′ is the *aggregate* length of sills; in "7 posts 4″ × 8″ × 22′," 22′ is the length of *each* post.

14. Sills 4″ × 8″ × 200′
Girder 8″ × 12″ × 30′
7 posts 4″ × 8″ × 22′
2 posts 4″ × 8″ × 18′
Beams 3″ × 8″ × 84′

15. Plates 4″ × 6″ × 100′
Ties 4″ × 6″ × 340′
Ridge 3″ × 12″ × 28′
45 beams 3″ × 8″ × 22′
24 rafters 2″ × 8″ × 20′

16. Sills 6″ × 8″ × 250′
Plates 4″ × 6″ × 250′
Ties 4″ × 6″ × 325′
Girders 8″ × 12″ × 46′
10 posts 4″ × 8″ × 24′

17. 70 joists 2″ × 10″ × 22′
20 joists 2″ × 10″ × 15′
8 joists 2″ × 10″ × 13′
Purlins 4″ × 6″ × 240′
Beams 3″ × 8″ × 200′

Find the cost of:

18. 440 studs, each $2'' \times 4'' \times 12'$, at \$20 per M.

19. 320 spruce rafters, $2'' \times 10'' \times 18'$, at \$22 per M.

20. 8 spruce girders, $8'' \times 12'' \times 28'$, at \$28 per M.

21. 800 hemlock boards, $1'' \times 8'' \times 12'$, at \$17 per M.

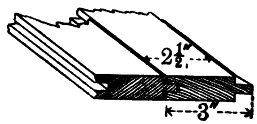
22. 640 lineal feet chestnut molding, at $2\frac{1}{2}$¢ per foot.

23. 6250 board feet maple flooring at \$36 per M.

24. 6 pine corner posts, $4'' \times 8'' \times 36'$, at \$30 per M.

25. At \$20 per thousand feet, what will be the cost of the boards for a fence 64 rods long and 5 boards high, each board being 6 in. wide and 1 in. thick?

26. How many pieces of Georgia pine flooring, $16'$ long and $2\frac{1}{2}''$ wide when laid, are required for the floor of a room $15' \times 16'$?



27. The boards are $\frac{7}{8}''$ thick. Since they were $3''$ wide before being tongued and grooved, they are sold as boards $3''$ wide. Find the cost of the flooring at \$54 per M.

28. The following materials were used in making a double floor:

38 spruce joists, $2'' \times 12'' \times 24'$, at \$25 per M
160 hemlock boards for lining, $1'' \times 7'' \times 12'$, at \$16 per M
38 strips, $2'' \times 2'' \times 24'$, at 2¢ per lineal foot
30 strips (for bridging), $2'' \times 3'' \times 15'$, at \$22 per M
64 planed boards, $1\frac{1}{4}'' \times 9'' \times 26'$, at \$30 per M
1 keg of nails, \$3.65

Find the total cost of the materials.

29. A plank of Oregon fir, sent to the New York State Commissioner of Education to be made into a table top, measured 24 ft. by 5 ft. by 7 in. How many feet of lumber did it contain?

30. It weighed 2050 lb. Find its weight per cubic foot.

Plastering, Painting, and Kalsomining

338. Plastering, painting, and kalsomining are often computed by the **square yard**.

Laths are 4 feet long and are sold in **bundles** of 50 or 100.

A bundle of 100 laths will cover 5 square yards of wall.

In the following exercises 1 bundle means 100 laths.

A fractional part of a bundle cannot be bought.

WRITTEN EXERCISES

339. 1. The cost of 100 square yards of plastering was estimated by a plasterer to be as follows:

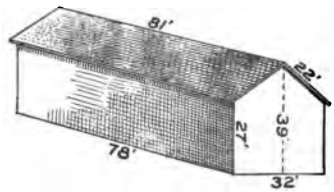
2000 lath at $2.25 per M	$ 4	50
Labor of putting on lath at 10 ¢ per bundle		
10 lb. of lath nails @ 6 ¢		
5 loads of sand @ 90 ¢		
4 barrels of common lime @ $1		
1 barrel of finishing lime @ $1.50	1	50
$1\frac{1}{2}$ bu. of plaster @ 25 ¢		
1 bu. of hair @ 50 ¢		50
Mason, $1\frac{1}{2}$ days @ $3.50		
Mason's helper, $1\frac{1}{2}$ days @ $2		
Laborer, $\frac{1}{2}$ day @ $1.50		
Use of screen for sand, etc.	1	
Cartage	2	20
Profit, 10% of the above		
Total cost of 100 sq. yd.	$	

Find the cost of plastering per square yard.

2. Find the cost, at 35 ¢ per square yard, of plastering the walls and ceiling of a room 16 ft. by 14 ft. and 9 ft. high, deducting the area of 2 windows each $3\frac{1}{2}$ ft. by 6 ft. and of 2 doors each 3 ft. by 7 ft.

3. Find the cost of painting the walls of the room mentioned in exercise 2 at 30 ¢ per square yard (deducting for windows and doors), and of tinting the ceiling at 10 ¢ per square yard.

4. Find the cost of painting the sides and ends of this barn, at $12\frac{1}{2}$¢ per square yard.



5. Find the cost of painting the roof of the barn at 16¢ per square yard.

Find the cost of plastering these rooms, each 10 ft. high, at 32¢ per sq. yd., computing area to the nearest .1 sq. yd. and deducting 1.2 sq. yd. for each door and .9 sq. yd. for each window:

6. Parlor 17′ 6″ × 16′, 3 doors, 4 windows.
7. Dining room 14′ 6″ × 14′, 4 doors, 3 windows.
8. Kitchen 11′ 1″ × 14′, 4 doors, 1 window.
9. 2 bedrooms, each 13′ 1″ × 10′, 2 doors, 1 window.
10. 1 bedroom 8′ 7″ × 9′, 1 door, 1 window.
11. Bath room 8′ 7″ × 5′ 8″, 1 door, 1 window.
12. Hall 3′ 7″ × 20′ 4″, 8 doors.
13. Hall 3′ 7″ × 15′ 9″, 3 doors, 1 window, 1 skylight 6′ × 8′.
14. How many bundles of lath must be purchased to cover the walls and ceiling of a room 18 ft. by 14 ft. and 9 ft. high, making no deductions for openings?
15. Find the cost of painting the front of an apartment house 28 feet wide and 54 feet high, at 20¢ per square yard, deducting the area of 14 windows each 4′ × 7′, and of one door 5′ × 9′.

Roofing

340. An area of 100 square feet is called a **square**.

Common shingles are about 18 inches long and average 4 inches in width. When laid "4 inches to the weather," the exposed surface of 1 shingle is (4×4) sq. in., or $\frac{1}{9}$ sq. ft.

It takes 900 shingles, then, to cover 1 square, but to allow for waste 1000 shingles per square are often estimated.

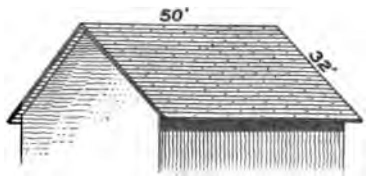
WRITTEN EXERCISES

341. 1. How many bunches of shingles, 250 shingles to the bunch, are required to shingle the roof of the barn shown on the preceding page, estimating 1000 shingles per square?

A fractional part of a bunch cannot be bought.

2. Find the cost of the shingles, 143 bunches, at $2.20 per M.

3. How much would it cost to shingle the roof shown here, at $3.20 per M for cedar shingles, $7.50 for shingle nails, and $16 for labor?



4. How much would it cost to cover the roof with galvanized sheet steel at $4.75 per square?

5. A cheaper roof may be made by covering the sheathing boards, shown in the picture, with sheathing paper, and covering this paper with felt roofing properly nailed and cemented. If the paper is 36 in. wide and is laid on lengthwise of the roof, with a lap of 4 in., how many strips will be required?

6. If 3 strips can be cut from each roll of sheathing paper, how much will the 24 strips of paper cost at 60¢ per roll?

7. Find the cost of the felt roofing at $87\frac{1}{2}$¢ per roll, if each roll covers one square.

8. Find the whole cost of this cheaper roof, including:

48 lb. roofing nails @ 4¢
32 lb. roofing caps @ $4\frac{1}{2}$¢
1 bbl. roofing cement, 50 gal., @ 13¢
18 gal. roofing cement @ 15¢
Labor, $11.20

9. Find the cost of the cheaper roof per square, to the nearest cent, the total cost being $56.56.

Papering and Carpeting

342. Wall paper is sold in **single rolls** 8 yards long, or in **double rolls** 16 yards long. It is usually 18 inches wide.

Fractional parts of a roll are not sold. In practice it is not always worth while, or even possible, to compute the exact cost of papering or carpeting rooms. What is sought is the approximate cost, making allowance for doors and windows, for matching patterns, etc.

EXERCISES

343. 1. Albert wishes to paper and carpet his room. It is 6 yards long and $4\frac{1}{2}$ yards wide. How many yards is it around the room? how many half yards?

2. How many strips of wall paper 18 inches wide will be required to paper the walls, if 2 strips are deducted for each of 2 windows and 2 strips for a door?

3. Since the ceiling of the room is 9 feet high, and the strips of wall paper need extend only from the baseboard to a few inches above the bottom of the border, Albert finds that after making allowances for matching the pattern he can cut 3 strips from a roll. How many rolls should he purchase, or how many double rolls?

4. How many lineal yards of border should he purchase, no allowance being made for matching the pattern?

5. If the ceiling paper is put on the long way of the room, it will require 1 roll for each strip. How many rolls of ceiling paper would then be required?

6. If the ceiling paper is put on the short way of the room, 3 strips can be cut from a double roll. How many double rolls would then be required?

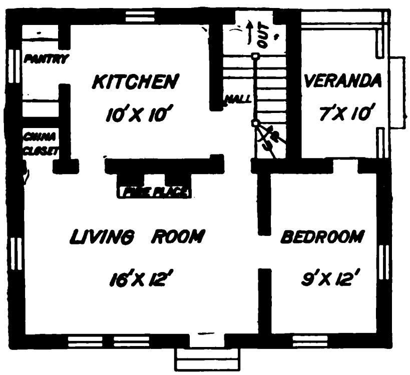
7. Draw a diagram of the floor of Albert's room. How many yards of carpet $\frac{3}{4}$ of a yard wide are required to cover the floor, no allowance being made for matching the pattern?

8. How many yards of matting 1 yard wide would be required to cover the floor, making no allowance for matching?

9. Making no deductions, how many square yards of linoleum $1\frac{1}{2}$ yards wide would be required? 2 yards wide?

WRITTEN EXERCISES

344 1. Find the cost of papering the walls and ceiling of the bedroom on the first floor of this cottage, at 50¢ per roll for wall paper, 40¢ per roll for ceiling paper, and 30¢ per lineal yard for the border, allowing 2 strips for each window and 3 strips for each door, putting on the ceiling paper the long way of the room, and supposing that 3 strips of wall paper can be cut from a roll.



2. Find the cost of covering the bedroom floor with carpet 27 in. wide, at $1.40 per lineal yard.

3. Find the cost of covering the kitchen floor with linoleum, one strip 2 yd. wide and another $1\frac{1}{2}$ yd. wide, at $1.10 per square yard.

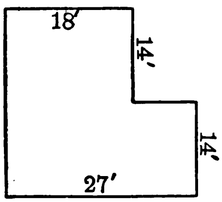
4. Find the cost of papering the living room just like the bedroom, deducting 2 strips for each window, 3 strips for each door, and 3 strips for the fireplace.

MISCELLANEOUS EXERCISES

345. **1.** The ceilings of the house shown on the preceding page are 9 ft. 3 in. high. How much will it cost to have the bedroom plastered, down to the floor, at 30¢ per square yard, deducting 1 square yard for each window and 2 square yards for each door?

2. Find the cost of plastering the kitchen, making similar deductions.

3. Find the cost of plastering the living room, deducting for windows and doors, but not for the fireplace.



4. This is a diagram of the cellar. Find the cost of paving the cellar floor with concrete to a depth of 4 in., at $4.25 per cubic yard.

5. The cellar was dug 6 ft. deep. Find the cost of excavation at 45¢ per cubic yard.

6. Find the cost of 25 squares of red roofing slate at $8.50 per square.

7. During six months the output of a slate quarry in New York was 784 squares of red slate and 1515 squares of green slate. Find the value of the output at an average price of $8.75 per square for red slate and $7 per square for green slate.

8. In a city 32,772 lineal feet of cement walks were constructed in a year by the city at a cost of $26,575, and 14,265 lineal feet by property owners at a cost of $10,000. Find the average cost per lineal yard, to the nearest cent, of the cement walks constructed that year.

9. How much more per lineal yard did it cost to construct cement walks at public expense than at private expense?

10. The floor of a warehouse 120 feet by 48 feet could sustain a weight of 150 pounds per square foot. How many tons could the entire floor support?

11. In a recent year the United States produced 24,317,910 square yards of carpet $\frac{3}{4}$ of a yard wide. How many lineal yards of this width were produced?

12. The two sloping sides of a slate roof were each $30' \times 20'$. Find the total weight of the slate on the roof, if it weighed $6\frac{3}{4}$ pounds per square foot.

13. The lawn in front of Mr. Child's house was 45 ft. long and 30 ft. wide, with a path 3 ft. wide running through it lengthwise. How many strips of sod 9 ft. long and 1 ft. wide were required to sod the lawn?

14. If 4000 strips of wood for golf sticks can be cut from $1333\frac{1}{3}$ feet of lumber, how many can be cut from 8000 feet of such lumber?

15. If one gallon of paint covers 650 square feet, how much paint is required to cover one side of a tight board fence 6 ft. 3 in. high and 78 ft. long?

16. A coal bin 12 ft. by 9 ft. and $5\frac{1}{3}$ ft. deep is filled with stove coal. How many tons of coal does it contain, if 1 ton occupies 36 cu. ft. of space?

17. Ice cut into blocks of a certain size, and well packed, is estimated to occupy 40 cu. ft. per ton. How many tons of ice can thus be packed into a space 24 ft. by 20 ft. by 18 ft.?

18. What is the cost of gravel per cubic yard, if $21.50 is paid for 15 loads of $1\frac{3}{8}$ cubic yards each?

19. Find the total cost of 85 cubic yards of sand at 27¢ per cubic yard, to be carted 800 feet at an additional cost of 1¢ per cubic yard for each 50 feet of distance.

20. How many bricks are contained in a wall 6 feet high, 36 feet long, and 15 inches thick, if there are 600 bricks per cubic yard?

21. At $7.50 per cubic yard, find the cost of a concrete wall 39 feet long, 5 feet high, and 18 inches thick.

22. If the total weight of this wall when dry was 38,025 pounds, how much did it weigh per cubic foot?

23. The excavation for a large reservoir was let out to contractors in sections. One section, 135 ft. by 102 ft., was dug 6 ft. deep. Find the cost at $19\frac{1}{2}$¢ per cubic yard.

24. If it took 4 trains of 10 cars each to carry this material away, what was the average car load?

25. A dredge in Boston harbor excavated 6 cubic yards of mud every 40 seconds, emptying the mud into a scow. How much mud did it load on the scow in $2\frac{1}{2}$ hours?

26. How long should it take a steam shovel to dig a trench 10 ft. wide, 18 ft. deep, and $\frac{3}{4}$ of a mile long, if it can remove 1200 cu. yd. of earth per day?

27. A certain hotel courtyard 75 ft. by 50 ft. has a rubber pavement. How much did the pavement cost, at $32.70 per square yard?

28. What is the total weight of this rubber pavement, if it weighs $15\frac{1}{4}$ pounds for each square foot of surface?

29. Find the expense of paving $\frac{5}{8}$ of a mile of a street 45 feet wide with asphalt, at a cost of $1.96 per square yard.

30. How much did it cost to construct $4\frac{1}{2}$ miles of macadam road 14 feet wide, at 50¢ per square yard?

31. If 60 cubic feet of water are required for sprinkling 850 square yards of asphalt pavement, how many cubic feet of water would be used in sprinkling a strip of pavement 36 feet wide and 5100 feet long? how many gallons?

PERCENTAGE

346. **1.** A boy who had 60 marbles lost 10% of them. How many marbles did he lose?

In this problem what number is the base? the rate? What name is given to the result?

How do you find the percentage when the base and rate are given?

2. Elmer had 40 cents. If he spent 8 cents, what per cent of his money did he spend?

When the base and percentage are given, how is the rate found?

3. A newsboy sold $66\frac{2}{3}\%$ of his papers. If he sold 24 papers, how many did he have at first?

Tell how to find the base, when the rate and percentage are given.

EXERCISES

347. **1.** Put this diagram on the blackboard, and as the teacher points to each number outside of the square, tell rapidly 50% of it.

16	40	64
32	50%	48
88	24	72

2. Change the rate and find 25% of each number; then $12\frac{1}{2}\%$; 75%; $37\frac{1}{2}\%$; $62\frac{1}{2}\%$; $87\frac{1}{2}\%$.

Find quickly:

3. 20% of 5

4. 40% of 20

5. 60% of 45

6. 80% of 60

7. $33\frac{1}{3}\%$ of \$1.50

8. $16\frac{2}{3}\%$ of \$2.40

9. $66\frac{2}{3}\%$ of \$3.18

10. $83\frac{1}{3}\%$ of \$6.42

11. 5% of \$120

12. $\frac{2}{5}\%$ of \$250

13. 6% of \$700

14. 8% of \$500

15. What per cent of 48 is 24? 16? 12? 8? 6? 32? 36? 40? 18? 30? 42?

16. 9 is what per cent of 12? of 36? of 72? of 15? of 30?

17. 6 is what per cent of 18? of 20? of 36? of 48? of 60?

18. 12 is what per cent of 24? of 16? of 18? of 32? of 72?

What per cent of

19. \$64 is \$8? **22.** 90 rd. is 60 rd.? **25.** \$1.20 is \$.12?

20. \$80 is \$20? **23.** 45 lb. is 18 lb.? **26.** \$1.50 is \$.25?

21. \$42 is \$14? **24.** 56 bu. is 35 bu.? **27.** \$4.00 is \$.50?

Find the number of which 12 is

28. 50 % **30.** $12\frac{1}{2}$ % **32.** $37\frac{1}{2}$ % **34.** 6 % **36.** 12 %

29. 20 % **31.** $16\frac{2}{3}$ % **33.** $66\frac{2}{3}$ % **35.** 8 % **37.** 24 %

Find the number of which 15 is

38. $33\frac{1}{3}$ % **39.** 40 % **40.** $37\frac{1}{2}$ % **41.** $62\frac{1}{2}$ % **42.** $83\frac{1}{3}$ %

43. 11 is 25 % of what number?

44. 24 is 60 % of what number?

45. 36 is 75 % of what number?

46. 48 is 80 % of what number?

47. 25 is $12\frac{1}{2}$ % of what number?

48. 40 is $16\frac{2}{3}$ % of what number?

49. 55 is $83\frac{1}{3}$ % of what number?

50. 63 is $87\frac{1}{2}$ % of what number?

51. How many inches is $16\frac{2}{3}$ % of a foot?

52. During a certain week John was absent from school 2 half days. What per cent of the time was he in attendance?

53. A man spends \$80 annually for his clothes, and this is $12\frac{1}{2}$ % of his income. What is his income?

54. A silver vase that weighs 30 ounces is 4 % pure silver. What weight of pure silver does it contain?

55. Ethel failed on 3 of the 15 questions in her examination. What per cent of the questions did she answer correctly?

56. After traveling 120 miles Mr. Joy had completed 40 % of his journey. How long was his journey?

57. If a piece of brass is composed of 2 parts of copper to 1 part of zinc, what per cent of the brass is copper?

SUGGESTION. — What per cent of 1 + 2, or 3, is 2?

58. Gold coins of the United States contain 9 parts of pure gold to 1 part of alloy. What per cent is pure gold?

59. I rent $66\frac{2}{3}$ % of my farm, or 220 acres. How many acres does the farm contain?

60. In a certain town 240 people, or 30 % of the inhabitants, are engaged in manufacturing. What is the population?

61. Of 450 crates of oranges shipped from Florida 9 were spoiled on the way. What per cent of the fruit was spoiled?

62. In a certain school 60 % of the pupils are girls. If there are 180 girls, how many pupils are there in the school?

63. Chickens lose 20 % of their weight in being dressed for market. A butcher buys 40 pounds of live chickens. How much will they weigh when dressed?

64. Mr. Mason bought 18 bushels of potatoes, and upon sorting them found that only 15 bushels were first class. What per cent of the potatoes were poor?

65. A grocer mixes 12 pounds of Java coffee with 36 pounds of Mocha. What per cent of the mixture is Java?

66. A local telephone company has 640 miles of line. If $12\frac{1}{2}$ % of it is in a certain county, how many miles of the line are in that county?

67. If the 400 books on travel in a library are 5 % of all the books, how many volumes does the library contain?

68. If the cost of operating a railroad one year was $360,000, and $66\frac{2}{3}$ % of this was expended by the freight department, what were the expenses of this department?

348. In expressing the rate, the fractional part of 1 per cent is frequently written in the decimal form.

Thus, $18\frac{7}{20}\%$ may be written 18.35%, which means the same as .1835.

349. The rate is often given approximately to the nearest tenth or hundredth of 1 per cent, according to the degree of accuracy desired.

Thus, since $\frac{13}{14} = .928+$, $5\frac{13}{14}\%$ (to the nearest tenth of 1%) is 5.9% and (to the nearest hundredth of 1%) 5.93%.

The decimal equivalents of these expressions are .059 and .0593, respectively.

WRITTEN EXERCISES

350. Give the decimal equivalent of:

1. 34.8%	**3.** 67.23%	**5.** 7.6%	**7.** 8.27%
2. 22.6%	**4.** 40.91%	**6.** 5.4%	**8.** 9.99%

Express as per cent with the sign:

9. .257	**11.** .8724	**13.** .026	**15.** .0844
10. .421	**12.** .5063	**14.** .082	**16.** .0619

Express with the sign to the nearest *tenth* of 1 per cent; also to the nearest *hundredth:*

17. $6\frac{5}{33}\%$	**19.** $12\frac{16}{27}\%$	**21.** $86\frac{17}{36}\%$	**23.** $36\frac{19}{48}\%$
18. $4\frac{19}{21}\%$	**20.** $45\frac{29}{52}\%$	**22.** $24\frac{28}{79}\%$	**24.** $90\frac{61}{82}\%$

MISCELLANEOUS EXERCISES

351. Find to the nearest cent:

1. $6\frac{1}{4}\%$ of \$28.60	**7.** $16\frac{2}{3}\%$ of \$376.32
2. $8\frac{1}{3}\%$ of \$41.25	**8.** $37\frac{1}{2}\%$ of \$849.74
3. 23% of \$30.86	**9.** 26.3% of \$1260.80
4. 78% of \$54.75	**10.** 3.98% of \$4529.12
5. 4.7% of \$92.66	**11.** 78.24% of \$7246.45
6. 8.9% of \$75.90	**12.** 96.05% of \$8036.28

What per cent of

13. 297 is 99? **16.** 1728 is 216? **19.** \$35 is \$1.68?

14. 8.25 is .66? **17.** 2240 is 784? **20.** \$48 is \$7.92?

15. 544 is 3.4? **18.** 4152 is 692? **21.** \$75 is \$8.40?

Find the number of which

22. \$306.72 is 3.6% **26.** \$2698.29 is $87\frac{1}{2}$%

23. \$465.60 is 75% **27.** \$4295.25 is 24.9%

24. \$507.15 is 23% **28.** \$6384.90 is $83\frac{1}{3}$%

25. \$812.16 is 4.7% **29.** \$7844.62 is 28.63%

What per cent (to the nearest hundredth) of

30. 79 is 27? **32.** 264 is 45? **34.** 46.49 is 28.6?

31. 42 is 69? **33.** 8.76 is 24? **35.** 629.4 is 397?

36. A sample of coal was found to be 73.19% carbon. How much carbon is there in a long ton of such coal?

37. If coal yields 65.9% of its weight in coke, how many pounds of coal will yield 5931 pounds of coke?

38. One year a National League baseball team won 106 of the 153 games played. What per cent of the games did the team win?

Give baseball per cents, or "averages," to the nearest tenth of 1%.

39. A player who led his team in batting came to bat 490 times during the season and made 171 hits. What was his batting average?

40. Find the fielding average of a player who accepted 899 chances and made 15 errors in one season.

41. On an English highway 6000 vehicles passed a certain point in 12 hours. $3\frac{2}{5}$% of them were drawn by horses, $7\frac{1}{2}$% were electric street cars, 9.9% motor vehicles, and $79\frac{1}{5}$% bicycles. How many vehicles of each kind were there?

42. In a year when there were 994,762 pensioners in the United States 274,447 were widows of soldiers. Find, to the nearest tenth, the per cent of pensioners that were widows.

43. If 22.9% of the cost of operating a mine in Alaska was for labor that cost $6434.90, find the cost of operating the mine.

44. One year there were 270,000 Indians in the United States, of whom 50,000 could read. Find, to the nearest tenth, the per cent of the Indian population that could read.

45. Find the number of bushels of wheat necessary to make 225 barrels of flour, if wheat yields 75% of its weight in flour.

46. In the manufacture of 432 lb. of plate glass, 151.2 lb. of sand, 146.88 lb. of limestone, 43.2 lb. of soda, 36.72 lb. of broken glass, and 54 lb. of other materials were used. What per cent of the whole was each substance?

47. Mr. Williams's income from his business one year was $2125, which was 25% of the value of his stock. The next year a fire damaged his stock to the extent of $1275. What per cent of the value of his stock did he lose by fire?

48. One year cotton goods to the value of $4,904,204 were imported into the Philippine Islands. 45.05% came from Great Britain, 6.52% from the United States, and 15.48% from Spain. What was the value of the cotton goods imported from each country?

49. The following table shows the production of certain furs for one year:

Kind of Fur	World	United States
Otter	33,640 skins	14,600 skins
Beaver	66,000 "	8,000 "
Fur seal	95,485 "	24,000 "
Mink	728,000 "	578,000 "
Muskrat	5,285,000 "	4,035,000 "

Find, to the nearest tenth for each kind of fur, the per cent produced by the United States.

352. Sum or difference of a number and some per cent of it.

1. How many are 6 increased by a number equal to $\frac{1}{3}$ of 6?
   6 plus $\frac{1}{3}$ of $6 = ?$ $\frac{4}{3}$ of $6 = ?$
2. How many are 6 decreased by a number equal to $\frac{1}{3}$ of 6?
   6 less $\frac{1}{3}$ of $6 = ?$ $\frac{2}{3}$ of $6 = ?$
3. How many are 12 increased by 25 % of itself?
   $100\% + 25\%$, or 125%, of $12 = ?$
4. How many are 12 decreased by 25 % of itself?
   $100\% - 25\%$, or 75%, of $12 = ?$
5. Find 15 increased by 20 %; decreased by $33\frac{1}{3}$ %.

EXERCISES

353. 1. The population of a certain town three years ago was 250. If it has increased 20 %, what is the population now?

2. One week Frank made \$3 selling papers. How much did he make the second week, if his profits decreased 25 %?

3. A train that was running 50 miles an hour increased its speed 4 %. How many miles an hour was it then running?

4. Mr. Thompson's flock of 240 sheep was decreased $12\frac{1}{2}$ % by an epidemic. How many sheep had he after the epidemic?

5. A canning factory using 6600 bu. tomatoes a season increased its capacity $16\frac{2}{3}$ %. How many bushels did it then use?

6. Formerly there were 400 fishermen in Digby, but their number has decreased 5 %. How many are there now?

7. During a certain month 200 girls were employed in a factory, and the next month 210. Find the per cent of increase.

SUGGESTION. — What per cent of 200 is 10?

8. One month Mr. Wilson burned 3000 ft. of gas in his house, and the next month 2400 ft. What was the per cent of decrease?

WRITTEN EXERCISES

354. 1. Mr. Charles invested $6540 in business, and a year later increased his capital $16\frac{2}{3}\%$. How much was his capital then?

2. Two years ago a lawyer's income was $4868, but last year it decreased $12\frac{1}{2}\%$. What was his income last year?

3. Flour that sold for $4.85 was advanced in price to $5.82. Find the per cent of advance.

4. A mason's helper working for $1.75 per day had his pay increased $14\frac{2}{7}\%$. How much did he then receive?

5. When the number of representatives in Congress from the state of New York was increased from 34 to 37, what was the per cent of increase, to the nearest tenth?

6. The income from the fisheries of Virginia one year was $3,179,498. The third year after, it had increased 45.09%. What was the income for the third year?

7. During ten years the number of males employed in the manufacture of gloves in the United States increased from 2741 to 4503, and of females from 3675 to 7768. Find, to the nearest tenth, the per cent of increase of each.

8. The unwashed wool from a flock of sheep one year weighed 8840 lb. and was sold for $2210. The next year the yield, 9282 lb., was sold for $1949.22. Find the increase per cent in the yield and the decrease per cent in the price per pound.

STATE OF NEVADA

Year	Population
1860	6,857
1870	42,491
1880	62,266
1890	45,761
1900	42,335

9. Find, to the nearest tenth, the per cent of increase in population in Nevada from 1860 to 1870; from 1870 to 1880.

10. What was the per cent of decrease from 1880 to 1890? from 1890 to 1900?

11. Find, to the nearest hundredth, the per cent of decrease from 1870 to 1900.

355. Finding a number when the number increased by some per cent of itself is given.

1. If 3 times a number is 15, how do we find the number?

2. If $1\frac{1}{3}$ times a number is 8, what is the number? A number increased by $\frac{1}{3}$ of itself equals 8. What is the number?

3. If 1.25, or $1\frac{1}{4}$, times a number is 10, what is the number? A number increased by .25 of itself equals 10. What is the number?

4. If 100% of a number is 12, what is the number? If 120% of a number is 12, what is the number? A number increased by 20% of itself equals 12. What is the number?

Find the number of which

5. 11 is 110%
6. 24 is 120%
7. 25 is 125%
8. 32 is 200%
9. 45 is 150%
10. 26 is 130%

What number increased by

11. 25% of itself equals 250?
12. 20% of itself equals 360?
13. $33\frac{1}{3}$% of itself equals 120?
14. $12\frac{1}{2}$% of itself equals 108?

WRITTEN EXERCISES

356. 1. What number increased by 32% of itself equals 165?

Solution. — The number + 32% of it = 132% of it = 165; that is, 1.32 times the number = 165.

Hence, the number = 165 ÷ 1.32 = 125.

2. What number increased by 75% of itself equals 672?

Suggestion. — Since 75% of the number is $\frac{3}{4}$ of it, 672 is $1\frac{3}{4}$ times the number.

What sum of money increased by

3. 43% of itself = \$354.64?
4. 68% of itself = \$890.40?
5. $16\frac{2}{3}$% of itself = \$693.84?
6. $62\frac{1}{2}$% of itself = \$459.16?

7. A clerk's salary was increased 18%. He then received $1652 a year. What was his salary before the increase?

8. A typewriter can transcribe her notes at the rate of 50 words per minute, which is 25 % faster than she could do it six months ago. What was her rate then?

9. The number of children in an orphan asylum has increased 8% this year, and there are 486 children in it at present. How many children were there in the asylum last year?

10. I paid $1250 for my automobile, or 60% more than for my horse and carriage. How much did my horse and carriage cost?

11. The cost of living for a family one year was $442, which was an increase of 4% over the cost for the year before. Find the cost for the year before.

12. The circulation of a publication has increased 26% in three years, and is at present 177,030. What was the circulation three years ago?

13. During a period of about 40 years from the close of the Civil War, the railway system of the United States increased 500%. If it comprised 210,000 miles at the end of this period, what was the mileage at the close of the war?

14. The production of woolen cloth in the United States increased 323% in six years, and then amounted to 274,950 yards per week. How many yards per week were produced at the beginning of the time?

15. A gas meter that ran 2.5% too fast registered the passage of 65,600 cubic feet of gas in a year. What was the actual amount of gas that passed through the meter?

16. The yearly value of the meat packing and slaughtering products in Kansas City increased 84.8% during ten years, and at the end of that time was $33,859,980. What was the value of these products at the beginning of the decade?

357. Finding a number when the number decreased by some per cent of itself is given.

1. What part of a number is left when $\frac{1}{3}$ of it has been taken away?

If $\frac{2}{3}$ of a number is 6, what is the number?

A number decreased by $\frac{1}{3}$ of itself equals 6. What is the number?

2. What part of a number is left when .25 of it has been taken away?

If .75, or $\frac{3}{4}$, of a number is 9, what is the number?

A number decreased by .25 of itself equals 9. What is the number?

3. What per cent of a number is left after taking away 10% of it? $100\% - 10\% = ?$

If 90% of a number equals 18, what is the number?

A number decreased by 10% of itself equals 18. What is the number?

4. What per cent of anything is left after taking away $12\frac{1}{2}\%$ of it? $100\% - 12\frac{1}{2}\% = ?$

If $87\frac{1}{2}\%$, or $\frac{7}{8}$, of a number is 21, what is the number?

A number decreased by $12\frac{1}{2}\%$ of itself equals 21. What is the number?

What number decreased by

5. 15% of itself equals 85?
6. 20% of itself equals 24?
7. $66\frac{2}{3}\%$ of itself equals 15?
8. $87\frac{1}{2}\%$ of itself equals 11?

WRITTEN EXERCISES

358. 1. What number decreased by 36% of itself equals 208?

Solution. — The number − 36% of it = 64% of it = 208; that is, .64 of the number = 208. Hence, the number = 208 ÷ .64 = 325.

2. What number decreased by $16\frac{2}{3}\%$ of itself equals 485?

Suggestion. — Since $16\frac{2}{3}\%$ of the number is $\frac{1}{6}$ of it, 485 is $\frac{5}{6}$ of the number.

What number decreased by

3. 32 % of itself = \$166.60? 5. $37\frac{1}{2}$ % of itself = \$453.75?

4. 65 % of itself = \$332.50? 6. $83\frac{1}{3}$ % of itself = \$742.36?

7. Walter lost 5 % in weight one year and then weighed 114 pounds. How much did he weigh at the beginning of the year?

8. If beef loses 20 % of its weight by roasting, find the weight of 14 pounds of roast beef before cooking.

9. Mr. Ford had 76 % of his money invested in houses, and the rest, \$7680, he put in the bank. How much money had he?

10. A man who had property worth \$15,600 found that this was 60 % less than he owed. How much did he owe?

11. If cloth shrinks $5\frac{1}{2}$ % of its length in being sponged, find the original length of a sponged piece 37.8 yards long.

12. Coal sold at \$5.50 per ton one winter, or $8\frac{1}{3}$ % less than the price the winter before. Find the price the winter before.

13. If $6\frac{1}{2}$ % of those engaged in a battle were killed and 39,270 survived, how many were engaged in the battle?

14. Some flour was damaged 25 % in transportation. If 600 barrels arrived in good condition, how many were shipped?

15. A grocer bought a box of 72 cakes of soap for \$4.50, which was $37\frac{1}{2}$ % less than the amount he received when he sold it. At what price per cake did he sell the soap?

16. In a state in which 19.8 % of the population are negroes, there are 952,375 whites. Find the population of the state.

17. A merchant had a stock of hats that cost \$2.50 each. When he had sold 75 % of them, the cost value of the rest was \$360. How many of these hats did he have in stock at first?

18. If a city decreased in population 16 % each year for two successive years and then had 8820 inhabitants, what was the population before the decrease?

Profit and Loss

359. 1. A jeweler bought a watch for \$80 and sold it at a gain of 25 % of the cost. How much did he gain?

For how much did he sell the watch?

2. Mr. Rich paid \$1200 for an automobile and after using it a year he sold it at a loss of $33\frac{1}{3}$ % of the cost. How much did he lose? How much did he receive for it?

3. Sugar that cost 5 cents a pound was sold for $5\frac{1}{2}$ cents a pound. How much was gained per pound? What part of the cost was the gain? What per cent of the cost was the gain?

4. When the gain equals $\frac{1}{5}$ of the cost, what per cent is gained? What per cent of the cost is the selling price?

5. When $\frac{1}{4}$ of the cost is lost, how is the loss expressed in per cent? How is the selling price expressed in per cent?

6. A dealer sold a gasoline launch for \$500 and gained 25 % of the cost. How much did the launch cost?

7. A bicycle was sold for \$40 at a loss of 20 %. Find the cost.

360. The per cent of gain or of loss is reckoned on the *cost* or the *sum invested.*

EXERCISES

361. 1. A suit of clothes that cost \$20 was sold at a profit of 25 %. How much was gained?

2. Gloves that were bought for 60¢ per pair were retailed at a gain of $33\frac{1}{3}$ %. What was the selling price?

3. A grocer made 10¢ by selling a pound of tea for 40¢. What per cent of the cost did he gain?

4. A city lot that cost \$500 was sold at a loss of 10 %. How much was lost?

5. A merchant bought silk for \$1 per yard and sold it at a profit of 25 %. What was the selling price?

6. Coffee bought for 28¢ a pound was sold for 35¢ a pound. What was the gain per cent?

7. When a hat that cost \$1.50 was sold at an advance of 50 %, what was the selling price?

8. A merchant sold a rug for \$48, thus gaining \$8. What was the gain per cent?

9. Some goods were sold at a profit of \$50. If 10 % was gained, find the cost of the goods.

10. A horse bought for \$150 was sold for \$250. What was the gain per cent?

11. A carriage was bought for \$120 and sold for \$100. Find the loss per cent.

12. Find the selling price of cloth bought at 75¢ a yard and sold at a profit of 20 %.

13. By selling coal at \$6 a ton a gain of 20 % was made. How much did the coal cost?

14. A clothier sold 5 overcoats for what 8 cost him. What was the gain per cent?

15. A dry goods merchant sold flannel for 36¢ per yard at a gain of $12\frac{1}{2}$ %. How much did the flannel cost him?

16. Some goods that cost \$3000 were damaged by fire and sold at a loss of 12 %. How much was lost?

17. A man bought a house for \$4000 and later sold it at a gain of 15 %. What was the selling price?

18. A furniture dealer sold a parlor suit at a gain of $16\frac{2}{3}$ %. If he sold it for \$210, how much did he pay for it?

19. What was the loss per cent on a farm bought for \$3200 and sold for \$2400?

WRITTEN EXERCISES

362. **1.** Find the gain on a stove that cost \$36 and was sold for 15 % more than it cost.

2. I bought a phonograph for \$25 and sold it at a loss of 34 %. How much did I lose?

3. When raw silk costs \$3.84 per pound, for how much per pound must it be sold to make a profit of $16\frac{2}{3}$ %?

4. Grain that cost 66¢ per bushel was damaged so that it was sold for $41\frac{1}{4}$¢ per bushel. What was the per cent of loss?

5. A grocer paid \$14.40 for a tub of butter weighing 60 pounds. Find his selling price per pound, if he gained $8\frac{1}{3}$ %.

6. A quantity of leather that cost \$1648 was sold at a gain of $12\frac{1}{2}$ %. For how much was it sold?

7. A stationer bought blank note books at \$1.08 per dozen and sold them at 15¢ each. What per cent did he gain?

8. A haberdasher sold shirts at \$1.50 each, thereby gaining 20 %. How much did he pay for them per dozen?

9. Find the gain per cent on a gross of brooms bought for \$34.56 and sold at 28¢ each.

10. Find the cost per dozen pairs of infants' woolen hose that retail for 25¢ a pair at a gain of 60 %.

11. A man who paid \$8250 for a yacht sold it at a loss of 14 %. What was the selling price?

12. A lumber merchant sold 4800 feet of lumber for \$103.68, gaining 8 %. How much did it cost him per thousand feet?

13. Find the gain to New York fruit shippers, on a week's shipment of California oranges, consisting of 117 car loads of 362 boxes each, if they cost \$1.35 per box in New York and are sold at auction at an average profit of 20 %.

14. If apples are purchased for 60¢ per bushel and sold by the quart at a gain of $33\frac{1}{3}$ %, find the selling price per quart.

15. Codfish bought for $3.50 per hundredweight was sold in 5-pound boxes at 21¢ a box. Find the gain per cent.

16. A grocer bought a quantity of sugar at $4.50 per hundredweight and sold it at a profit of $11\frac{1}{9}$%. What was his selling price per pound?

17. What per cent is gained by buying pork at $22.50 per barrel (200 lb.) and retailing it at $13\frac{1}{2}$¢ a pound?

18. Find the gain per cent, to the nearest tenth, on neckties bought at $1.32 per dozen and sold at 25¢ each.

19. A man bought a ring for $35, which was $12\frac{1}{2}$% less than its value. He sold it for $12\frac{1}{2}$% more than its value. Find his gain.

20. When Georgia peaches are bought at $2.25 per bushel crate and are sold at 12¢ a quart, what is the gain per cent, supposing that each crate yields only 30 quarts of salable fruit?

21. Shoes that cost $18.48 per dozen pairs were sold at $2 per pair. Find the per cent of gain to the nearest tenth.

22. Mr. Jordan bought two houses, one for $2400 and the other for $5200. He sold the first at 120% of the cost, and on the second he lost 4%. Find his net gain on both transactions.

23. A dealer gained 20% on flour that he sold for $6 a barrel. The market advanced, and he sold the rest of his stock for $6.50 a barrel. What was his per cent of gain after the advance?

24. A merchant's complete stock of goods cost $50,000. He sold it at an average advance of 15% above cost and lost $2\frac{1}{4}$% of his sales in bad debts. Find his gain.

25. A wholesale fruit dealer paid $350 for oranges, $280 for bananas, and $320 for peaches. He sold the oranges at cost, the bananas at 130% of the cost, and the peaches at a loss of 5%. Did he gain or lose on all, and how much?

Commission and Brokerage

363. 1. I employed a man called a *commission merchant* to sell my potatoes in the city. He sold the crop for $800, and charged me 5% of this sum for his services. How much did he charge for his services; that is, what was his *commission?*

He took out the commission, $7 for carting the potatoes, and $3 for storing them, and then sent me the rest of the $800. How much did he send me; that is, what were the *net proceeds?*

2. An *agent* bought some cloth for $400, charging 3% of the cost for his services. How much was his commission?

3. A lawyer collected a debt of $500, receiving 10% of it for his services. What was his commission?

364. A person who buys or sells goods or transacts business for another is called an **agent**, a **commission merchant**, or a **broker**.

In general a commission merchant actually receives the goods, while a broker simply arranges for their sale or purchase, the goods being shipped directly from the seller to the purchaser.

365. The compensation of an agent is often reckoned as some per cent of the *value* involved, and is called **commission**, or **brokerage**.

Thus a seller's commission is some per cent of the amount of sales; a buyer's commission is some per cent of the cost; a collector's commission is some per cent of the money collected.

366. The sum left after the commission and other expenses have been paid is called the **net proceeds**.

EXERCISES

367. 1. An agent received 20% commission for selling pictures. How much did he earn by selling 50 at $2 each?

2. A book agent's sales one month amounted to $200. How much did he make that month, if his commission was 40%?

3. A commission merchant sold 100 barrels of flour at $6 a barrel. Find his commission at 3%.

4. The value of the maps sold by an agent one summer was $650. Find his commission at 50%.

5. Find the commission at 2% on 400 tons of coal sold for $5 a ton.

6. A lawyer received $15 for collecting a debt of $300. What was his rate of commission?

7. How much commission was received for selling 50 bbl. of apples at $2 per barrel, if the rate of commission was 7%?

8. How much was the builder's commission at 10% on a house that cost $4500? Find the architect's commission at 5%.

WRITTEN EXERCISES

368. 1. A commission merchant sold 2000 bushels of corn at 56¢ a bushel, commission 2%. He paid $123.30 freight and $27 cartage. Find the commission and the net proceeds.

SOLUTION

Amount of sales = 2000 × 56¢ =		$1120.00
Commission = 2% of $1120 =	$22.40	
Freight and cartage .	150.30	172.70
Net proceeds		$ 947.30

2. Find the commission and net proceeds of a sale of 150 dozen bunches of celery at 36¢ a dozen, if the rate of commission was 5%.

3. My agent sold goods for me to the amount of $4620. If he paid $85 for cartage and other expenses and charged 3% commission, what were the net proceeds?

4. An agent collected $745 for his employer and charged 7% commission. Find the commission and the net proceeds.

5. A commission merchant purchased rice for me to the amount of $2840.50. Find the commission at $3\frac{1}{2}$%.

6. Find the auctioneer's commission on a sale of rugs, the amount of the sale being \$97,546.50, and the rate of commission 2%.

7. Find the net proceeds from the sale of 250 bags of coffee, averaging 132 pounds each, at 14½¢ per pound, reckoning the commission at 2%.

8. What is the weekly income of a clerk who receives \$8 a week and a commission of 4% of his sales, if his sales average \$250 a week?

9. An agent sold 4 sewing machines at \$35 each, receiving a commission of 40%. The expense of delivering them was \$2.25 each. Find the commission and the net proceeds.

10. Find the commission and the net proceeds on a sale of 225 bales of cotton, averaging 500 pounds each, at 11¢ per pound. The rate of commission was 3%. Freight and other expenses amounted to \$125.

Find the commission and the net proceeds:

	Produce	Quantity	Price	Rate of Commission
11.	Cabbages	500 bbl.	75¢	5%
12.	Onions	250 bu.	85¢	6%
13.	Apples	360 bbl.	\$1.75	10%
14.	Peaches	120 baskets	\$1.25	8%
15.	Blackberries	960 qt.	12¢	7½%
16.	Butter	2880 lb.	18¾¢	5%
17.	Cheese	1280 lb.	8¼¢	5%
18.	Eggs	2250 doz.	22¢	5%
19.	Chickens (live)	1760 lb.	13¢	10%
20.	Chickens (dressed)	2450 lb.	16¢	5%

21. A lawyer collected 80% of a debt of \$2500 and charged 10% commission. How much did the creditor receive?

22. What was the income last year of a commercial traveler whose sales amounted to \$105,620, if his commission was $2\frac{3}{4}\%$?

23. How much business must a commission merchant do to earn \$125, if his average commission is $2\frac{1}{2}\%$?

24. Find the rate of commission, if the net proceeds from a sale of \$5000 are \$4875.

25. The net proceeds of a sale of produce were \$2451. If the commission was \$129, find the rate of commission.

26. If a lumber agent sold 80,000 feet of lumber at \$24 per thousand and received a commission of $4\frac{1}{4}\%$, how much was his commission?

27. An agent bought a block of 14 houses, each house worth \$3200, on a commission of $1\frac{1}{2}\%$. How much did he receive for his services?

28. The rent of a house was $\$37\frac{1}{2}$ per month, and the agent retained out of that sum a commission of $\$1\frac{1}{2}$ per month. What was the rate of commission?

29. Find the architect's commission on an office building that cost \$24,000, if he received $1\frac{3}{4}\%$ for drawing the plans, and $3\frac{1}{4}\%$ for superintending the construction.

30. A commercial traveler who sold goods on a commission had an income one year of \$3000 by selling goods to the amount of \$120,000. What was his rate of commission?

31. The commission at $3\frac{1}{2}\%$ for selling a quantity of oats was \$189. How many bushels were sold, if the price received was 30¢ per bushel?

32. A Cincinnati packer sent 150 barrels of mess pork to a commission merchant in Syracuse, who sold it at \$12.30 a barrel, paying \$77 freight charges, 10¢ a barrel cartage, and 4¢ a barrel storage. Find the commission at $2\frac{3}{4}\%$ and the net proceeds.

Commercial Discount

369. To meet the varying conditions of the market, discounts are often given from the prices published in catalogues and price lists, the list prices usually being higher than the market prices.

Sometimes, when goods are sold to be paid for at a future time, a discount is allowed for payment before that time.

It often happens that several successive discounts are allowed. The first is a discount from the list price, the second from the remainder, the third from the second remainder, and so on.

WRITTEN EXERCISES

370. 1. Find the net price of an article listed at $17.40, if the discounts are 25 % and 20 %.

List price	$17.40
Less 25 %	4.35
Remainder	13.05
Less 20 %	2.61
Net price	$10.44

The first discount is 25% of $17.40, or $4.35. Subtracting, we find the remainder, $13.05. The second discount is 20% of $13.05, or $2.61. Subtracting, the net price is found to be $10.44.

NOTE. — Find the discounts in the order (20 %, 25 %) and thus show that the result is the same in whatever order they are taken.

Find the net price of articles listed and discounted as follows:

2. $25; 20 %, 10 %
3. $40; 5 %, 15 %
4. $8.40; $33\frac{1}{3}$ %, $12\frac{1}{2}$ %
5. $5.70; 40 %, $16\frac{2}{3}$ %

Find the net price, net cost, and total discount on each:

	ARTICLES	PRICE	RATES OF DISCOUNT
6.	3 doz. food choppers	$24.00 per doz.	30 %, 10 %
7.	$4\frac{1}{2}$ doz. meat cleavers	9.00 per doz.	$33\frac{1}{3}$ %, 5 %
8.	6 doz. butcher knives	6.50 per doz.	40 %, 10 %
9.	$5\frac{1}{2}$ doz. coffee mills	8.00 per doz.	35 %, 20 %
10.	15 lawn mowers	10.00 each	60 %, 10 %, 5 %

11. A music dealer sold a piano listed at \$300 for 40% and 10% off. How much did he receive?

12. Find the net cost to a retailer of $2\frac{1}{2}$ dozen hammocks, the list price of which is \$16 per dozen; discounts 20%, $12\frac{1}{2}$%.

13. The list price of a set of George Eliot's works is \$30. If 40% discount is given, and 10% special, find the net cost.

14. Ford & Bond sold me $4\frac{1}{2}$ dozen thermometers at \$15 per dozen; discounts 20%, $7\frac{1}{2}$%. Find the total discount.

15. When the list price of sash cord is 30¢ per pound, with discounts of 15% and 5%, how much will 200 pounds cost?

16. George Snyder & Co. bought 60 bathrobes at discounts of 25% and 20%. If the bathrobes were listed at \$6 each, what was the net cost?

17. How much must be paid for 6 sets of harness, the list price being \$15.20 each, if there are discounts of 25% and 5%?

18. How much did I pay for 96,000 shipping tags listed at 25¢ per thousand; discounts 25%, 5%?

19. In paying for 2 dozen beaver traps, listed at \$16.50 per dozen, a sporting goods dealer received discounts of 60% and 10%. How much did he pay for the traps?

20. A dealer bought 12 six-pound sledges, listed at 30¢ per pound, at discounts of 75% and 10%. Find the net cost.

21. Ryder & Co. bought 720 pounds of galvanized sheet iron at discounts of 70% and $2\frac{1}{2}$%. If the list price was 15¢ per pound, how much was the net cost?

22. Mr. Fuller bought 150 feet of steel fence at \$1.10 per foot, and 2 gates at \$3.50 each. If he received a discount of 15%, and 5% for cash, what was the net cost?

23. I bought 2500 feet of fuse, the list price of which was \$3.20 per 1000 feet, and received a discount of 10% from the list price and $2\frac{1}{2}$% for paying cash. How much did it cost me?

371. Discounting bills.

When there are discounts that are the same for all items of a bill, it is customary to find the amount of the bill first and then the discount on the amount.

372. The amount of a bill less all discounts is called the **net amount.**

The amount of a bill before it is discounted is sometimes called the **gross amount.**

WRITTEN EXERCISES

373. In each of the following, find the net amount from the given gross amount and discount:

1. \$200; 20%, 5%	7. \$230; 5%, 2%
2. \$400; 25%, 10%	8. \$498; $16\frac{2}{3}$%, 4%
3. \$600; 10%, 10%	9. \$560; $37\frac{1}{2}$%, 15%
4. \$300; 15%, 20%	10. \$744; $12\frac{1}{2}$%, $33\frac{1}{3}$%
5. \$720; 50%, 10%	11. \$1000; 10%, 10%, 5%
6. \$850; 12%, 18%	12. \$2400; 25%, 20%, 8%

13. What is the difference between a discount of 35% on a bill of \$320, and two successive discounts of 20% and 15%?

14. Shafer & Son received discounts of 40% and 10% on a bill of \$350 for silver plate. Find the total discount.

15. A bill for school supplies amounted to \$475.50. What was the cash payment if there was a discount of $33\frac{1}{3}$%, and 5% for cash?

16. From a bill of \$840 for window glass, 90% and 10% discounts were allowed. What was the net amount of the bill?

17. Find the total discount on a bill of goods amounting to \$1280, with discounts of $12\frac{1}{2}$% and of 30%.

18. What is the net amount of a bill of \$496 for tinware, if the discounts are 75% and 10%, and 5% for cash?

19. The *terms* of the following bill are a trade discount of 5% and an additional discount of 2% for "cash."

Verify the gross amount and the net amount, if cash is paid.

BOSTON, MASS., *April 7*, 190*6*.

Mr. Ralph Boyd,
72 Tremont St.

Bought of A. G. CROOKS & CO., 284 FEDERAL ST.
WHOLESALE GROCERS

TERMS: *Trade 5%, 2% cash.*

10 pails ginger, 250 lb. 23¢	*$57*	*50*		
15 boxes cloves, 150 lb. 15¢	*22*	*50*		
20 " cinnamon, 200 lb. 27¢	*54*			
	134			
Discounts 5%, 2%	*9*	*25*	*124*	*75*
Received payment,				
A. G. Crooks & Co.				

Make out the bills and find the net amount of each:

20. T. W. Cook bought of Jas. Monroe: 8 grindstones @ \$5.50; 15 oilstones @ 60¢; $1\frac{3}{8}$ doz. scythes @ \$6. Discounts, $33\frac{1}{3}$%, 10%.

21. William Cline bought of H. N. Burt: 72 sets laundry irons @ \$1.50; 36 teakettles @ 65¢; 60 coffeepots @ 45¢; 3 gross egg beaters @ \$10.20. Discounts 40%, 15%.

22. Joseph Child bought of Walter White: 3 doz. hammers @ \$6; $2\frac{1}{2}$ doz. screwdrivers @ \$7; $1\frac{3}{4}$ doz. saws @ \$16; 15 bench planes @ \$1.90. Discounts 25%, 10%.

23. A. L. Dewitt bought of Ward, Craft & Co.: 30 linen tablecloths @ \$3.50; 40 doz. napkins @ \$3; 60 doz. handkerchiefs @ \$2.25. Discounts 15%, 5%.

Marking Goods

374. Merchants mark their goods to show the *cost* and the *selling price*, usually writing the first above a horizontal line and the second below.

To prevent customers from reading the mark and discovering the gain, some private mark is usually adopted.

The most usual device employed is some word or words of ten different letters to represent the ten Arabic numerals.

This is called a **key**.

Thus, if the key is "White Sambo," the corresponding letters and figures are

w	h	i	t	e	s	a	m	b	o
1	2	3	4	5	6	7	8	9	0

375. To avoid repeating a letter and thus giving a clew to the key, an extra letter, called a **repeater**, is used.

Thus, if the cost of an article is \$1.75 and the selling price \$2.99, using the key "White Sambo" with *k* for the repeater, the mark would be $\frac{\text{wae}}{\text{hbk}}$.

In the exercises, use this key and repeater unless otherwise specified.

WRITTEN EXERCISES

376. Interpret the following and find the per cent of gain for each:

1. $\frac{\text{whe}}{\text{weo}}$	**3.** $\frac{\text{tm}}{\text{et}}$	**5.** $\frac{\text{st}}{\text{mk}}$	**7.** $\frac{\text{hke}}{\text{hmk}}$	**9.** $\frac{\text{iaeo}}{\text{tihe}}$
2. $\frac{\text{hto}}{\text{iho}}$	**4.** $\frac{\text{ae}}{\text{who}}$	**6.** $\frac{\text{is}}{\text{th}}$	**8.** $\frac{\text{ito}}{\text{the}}$	**10.** $\frac{\text{etae}}{\text{samb}}$

In the following fill in the selling price at $33\frac{1}{3}\%$ above cost:

11. $\frac{\text{ht}}{\dots}$	**13.** $\frac{\text{tih}}{\dots}$	**15.** $\frac{\text{wtk}}{\dots}$	**17.** $\frac{\text{bs}}{\dots}$	**19.** $\frac{\text{hmks}}{\dots}$
12. $\frac{\text{sb[illegible]}}{\dots}$	**14.** $\frac{\text{sbo}}{\dots}$	**16.** $\frac{\text{amb}}{\dots}$	**18.** $\frac{\text{wih}}{\dots}$	**20.** $\frac{\text{tsie}}{\dots}$

Fill in the cost mark in each of the following, the given selling price being 20% above cost:

21. $\frac{\cdots}{\text{tm}}$	**23.** $\frac{\cdots}{\text{iok}}$	**25.** $\frac{\cdots}{\text{mt}}$	**27.** $\frac{\cdots}{\text{ith}}$	**29.** $\frac{\cdots}{\text{is}}$
22. $\frac{\cdots}{\text{sk}}$	**24.** $\frac{\cdots}{\text{aeo}}$	**26.** $\frac{\cdots}{\text{bs}}$	**28.** $\frac{\cdots}{\text{tbm}}$	**30.** $\frac{\cdots}{\text{smts}}$

Mark articles to sell at $37\frac{1}{2}$% above the following costs:

31. 24¢	**33.** 88¢	**35.** \$2.80	**37.** \$4.48	**39.** \$176
32. 40¢	**34.** 96¢	**36.** \$7.20	**38.** \$6.80	**40.** \$224

41. How shall a book dealer mark a book that cost \$1.20 so that he may give a discount of 16% from the marked price and still make a profit of 40%?

SOLUTION. 140% of \$1.20 = \$1.68, selling price.

Since the selling price is 16% less than the marked price, \$1.68 is 84% of the marked price.

Then the marked price = \$1.68 ÷ .84 = \$2.

Hence the complete mark should be $\frac{\text{who}}{\text{hok}}$.

Mark articles that cost as follows so that there will be a profit of 25% after giving the indicated discount:

42. 32¢, discount 20%	**45.** \$3.40, discount 15%
43. 72¢, discount 10%	**46.** \$6.72, discount $12\frac{1}{2}$%
44. \$48, discount 25%	**47.** \$8.40, discount $16\frac{2}{3}$%

Find the cost of one article when billed by the dozen as follows, and give complete marks (profit 40%), using the key "birthplace," repeater *n*, for the cost, and "White Sambo," repeater *k*, for the selling price:

48. Hats, \$21	**52.** Shirts, \$18
49. Caps, \$15.60	**53.** Gloves, \$16.80
50. Scarfs, \$24	**54.** Shoes, \$30
51. Neckties, \$7.20	**55.** Rubbers, \$8.40

REVIEW PROBLEMS IN INDUSTRIES

SUGGESTION. — When the problems are related to one another, the answer to each should be kept until the series is completed.

377. 1. A salesman bought a mileage book in New York, boarded the 8 : 30 A.M. Empire State Express, and rode to Buffalo. The next day he returned on the same train, leaving Buffalo at 1 : 00 P.M. Find his fare both ways at 2 ¢ per mile.

Miles	West, read downward.		East, read upward.		Miles
0	8:30	Lv. New York Ar.		10:00	440
143	11:10	Ar. Albany Lv.		7:00	297
143	11:13	Lv. Albany Ar.		6:57	297
238	12:55	Lv. Utica Lv.		5:02	202
291	1:58	Ar. Syracuse Lv.		3:57	149
291	2:01	Lv. Syracuse Ar.		3:54	149
371	3:25	Lv. Rochester Lv.		2:24	69
440	4:45	Ar. Buffalo Lv.		1:00	0

2. How long did it take him to go to Buffalo? to return?

3. Find the average speed per hour each way.

4. The train left New York with 320 passengers; at Albany 75 got off and 69 got on; at Utica 32 got off and 46 on; at Syracuse 40 got off and 48 on; and at Rochester 55 got off and 21 on. How many passengers went to Buffalo?

5. All passengers paid fare at the rate of 2 ¢ per mile, except 133 purchasers of regular tickets in New York, who paid \$.24 each in addition to the 2 ¢ per mile. Find the receipts from fares.

6. The parlor car company sold 21 seats to Albany @ \$1, 17 to Utica and Syracuse @ \$1.50, 38 to Rochester and Buffalo @ \$2, and the exclusive use of 2 drawing-rooms to Buffalo @ \$7. In addition they received 1 ¢ per mile from the railroad company for the rent of the car. Find the total parlor car receipts.

7. Find the pay of the engineers for a trip at 3.5 ¢ per mile.

8. It takes a steel steamship loaded with wheat 140 hours to go from Chicago to Montreal via the Welland Canal, and 160 hours to make the return trip. Sixty hours are spent in port. How many days does it take for the round trip?

9. The distance between the two ports is 1266 miles. How far does the vessel travel in a season of 13 round trips?

10. A vessel of this type carries, on an average, 75,000 bushels of wheat (60 pounds to the bushel) on each trip to Montreal and 1125 tons of other commodities on the return trip. How many more tons of freight does the vessel carry on a voyage to Montreal than on the return voyage?

11. Find the freight earnings for a season, at 5¢ per bushel "down" and $1 per ton "up."

12. How much more does the vessel earn going down than going up, each time? during the whole season?

13. Find the season's total expenses from the following:

Wages and board, $7350	Customs fees, etc., $500
Coal, 230 tons per round trip @ $3	Outfit and repairs, $2875
Engine expenses, oil, etc., $500	Insurance, $4600
Elevating 975,000 bu. wheat @ $\frac{1}{4}$¢	Management, $2000
Shoveling wheat, at $4 per 1000 bu.	General expenses, $2000
Wheat lost and damaged, $1950	Wear of vessel, $5750

14. Find the profit for the season; for 1 round trip.

15. Going up the St. Lawrence River the freight vessel must pass through canals. Their lengths are as follows: Lachine, 8.5 mi.; Soulanges, 14 mi.; Cornwall, 11 mi.; Farrans Point, 1 mi.; Rapide Plat, 3.5 mi.; Galops, 7.125 mi.; Murray, 5.167 mi. The Welland Canal is 26.75 mi. long. How many miles must the vessel pass through canals to reach Lake Erie?

16. The Welland Canal has 27 locks, giving a total lift from Lake Ontario to Lake Erie of 326.75 feet. Find the average lift.

17. A school rented 4 pianos, at \$4.50 per month each, for 10 months in a year. How much was paid for the use of pianos per year?

18. I pay my landlady \$10 per month for a room and \$4.20 per week for board. How much do room and board cost me per year (365 days)?

19. A man rented a typewriter at \$1.25 per week and kept it from the morning of June 2 to the morning of Oct. 27. Find the amount of rent he paid.

20. A man rented a house at \$492 per year. After living in it 8 months he sublet it for the rest of the year at \$35 per month. How much rent did he pay per month for the first 8 months? How much per month did he lose during the rest of the year?

21. A man rented a farm "on shares," agreeing to give the owner $\frac{2}{5}$ of each crop. He raised 1800 bushels of oats, 2350 bushels of wheat, and 4280 bushels of corn. How much of each crop had he left after paying the rent?

22. A farmer rented 220 acres of land at \$5.25 per acre, half to be paid Sept. 1 and half Feb. 1. Find the amount of each payment.

23. A sleeping and parlor car company charges the railroads 1¢ per car per mile for the use of its cars. Find the rent charged for twelve of these cars between Kansas City and Chicago, a distance of 458 miles.

24. A man secured a lease of a tenement for five years at \$2800 a year, agreeing to make all inside repairs. He sublet the premises to 12 families at the following rates per month: \$40; \$35; \$33; \$30; \$28; \$25; \$22; \$20; \$18; \$15; \$14; \$12. Repairs cost \$1425, coal \$494. He lost \$536 from unpaid rents and idle apartments. Find his gain.

25. A coffee planter had 85.8 acres of land on which were planted 500 coffee trees per acre. How many trees did his plantation contain?

26. The average yield of raw coffee was 1.6 pounds per tree. Find the yield from the 42,900 trees.

27. The planter sold his coffee on the plantation to a coffee buyer, who paid him \$4719 for the entire crop of 68,640 pounds. How much was received per pound?

28. How much did he gain, if the whole cost of production was \$2951.52?

29. How many cents per pound did it cost him to raise the coffee? How much was his gain on each pound?

30. How much did he gain on each acre?

31. The man who bought the coffee transported it on beasts of burden to the nearest shipping port at a cost of \$274.56, and sold it at $7\frac{1}{2}$¢ per pound. Find his gain.

32. The coffee was shipped to the New York market in bags containing an average of 132 pounds each. How many bags were required?

33. To send the coffee by ship to New York cost $37\frac{1}{2}$¢ per bag. How much were the freight charges on the 520 bags?

34. The coffee was imported by a coffee roaster. If coffee loses .15 of its weight in the process of roasting, how much did this lot of coffee weigh when roasted?

35. One year 11.75 pounds of coffee was the average amount used by each person in the United States, while in Great Britain the average per person was only .67 of a pound. At this rate how much more coffee would be used per year by a city of 75,000 inhabitants in the United States than by a city of the same population in Great Britain?

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36. On a tea plantation of 240 acres in India there were 512,640 tea plants. Find the number of plants per acre.

37. At the age of three years the plants were $4\frac{1}{2}$ feet high. If $2\frac{5}{6}$ feet were then pruned from each plant, how high was each?

38. A woman could pick only $7\frac{3}{4}$ pounds of leaves per day from plants of this age, but twice as much from plants eight years old. Find the earnings of a woman in 6 days, picking leaves from the 8-year-old plants at $\frac{3}{4}$¢ per pound.

39. If each plant furnished $1\frac{1}{8}$ pounds of green leaves during the year, what was the average yield of fresh leaves in pounds from an acre of 2136 plants?

40. It took $4\frac{1}{2}$ pounds of green leaves to make 1 pound of marketable tea. How much finished tea was produced per plant?

41. How many pounds of marketable tea were made from the green leaves (2403 lb.) of one acre? of the entire plantation?

42. In the first step of preparing the tea for market, the green leaves were spread to wither on trays. If 1890 pounds of fresh leaves were withered in one day and each pound required 10 square feet of space, how many trays of 6 square feet each were used?

43. In rolling the withered tea to break the oil cells, a machine rolled as much in 1 hour as a man did by hand in $4\frac{1}{3}$ days. If a good day's work by hand was 75 pounds, how much did the machine roll in a day of 9 hours?

44. When dried and graded, the tea was packed in chests holding 40 pounds each. How many chests were required for the 128,160 pounds of tea?

45. If the actual time consumed in packing one grade of the tea, or $\frac{1}{6}$ of the 3204 chests, was $22\frac{1}{4}$ hours, how many chests were packed per hour?

46. The sum received in Calcutta for the tea of this grade was $2990.40. Find the price received per pound.

47. Shipments of tea to the United States one year were as follows: from China, 53,157,000 pounds; from Japan, 42,700,000 pounds; from India, 7,679,000 pounds; from Great Britain, 6,647,000 pounds; and from other countries, 2,723,000 pounds. Find the total amount.

48. Of the tea shipped to Great Britain that year, 155,196,000 pounds were shipped from India and 11,048,000 pounds from China. How much more or less than the United States did Great Britain receive from each?

49. The average amount of tea consumed during a recent year by every 50 persons in each of the following countries was: Great Britain, 301.5 pounds; Russia, 47.5 pounds; Germany, 6 pounds; France, 3 pounds; United States, 65 pounds. Find the average consumption of tea per person in each country named.

India rubber is obtained by cutting or tapping the bark of certain tropical trees, which give out a milky fluid containing the rubber. The rubber is then hardened from the milk.

50. In the Amazon valley a native tapped 144 trees and secured $\frac{3}{8}$ gal. of milk per tree. How much milk did he secure?

51. Each of the 54 gallons of milk yielded $2\frac{1}{4}$ lb. of rubber. Find the yield from this tapping.

52. From June to December the trees were tapped 16 times. If each tapping yielded as much milk as the first, how much rubber did the season's work produce?

53. How many pounds of rubber were obtained per tree, the total yield being 1944 lb.?

54. The rubber was obtained from the liquid by dipping a paddle into it and then holding the paddle in smoke. If it took 27 hours to obtain the $121\frac{1}{2}$ lb. of rubber from the milk of one tapping, how much rubber was obtained in 1 hour?

55. If at the rate of $4\frac{1}{2}$ lb. per hour a loaf of rubber was made in $2\frac{2}{3}$ hr., how many loaves were made that season?

56. How many chests of 300 pounds capacity were used to pack them in, and how many loaves were left for a smaller package?

57. This rubber was sold in New York as *Fine Para* and brought $2410.56. What was the price per pound?

58. At this price find the value of the yield from one tree.

59. The Amazon district one year furnished $\frac{5}{12}$ of the world's supply of rubber, or 25,000 tons. Find the world's production.

60. The United States that year purchased 24,675 tons of rubber, of which $\frac{9}{25}$ was used for rubber boots and shoes. How many tons were thus used?

61. Of the rubber boots and shoes made, 24,686,643 pairs were men's, 18,847,865 women's, and 6,445,231 children's. Find the total number of pairs manufactured.

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62. A man engaged in silk culture bought 13 ounces of silkworm eggs at $1.75 per ounce. Find the cost of the eggs.

63. Each ounce contained 36,000 eggs. If $\frac{1}{2}$ oz. failed to hatch, how many silkworms were actually obtained?

64. After they began to hatch, the 450,000 silkworms hatched in five days. $\frac{1}{20}$ hatched the first day, $\frac{3}{8}$ the second, $\frac{2}{5}$ the third, $\frac{1}{8}$ the fourth, and $\frac{1}{20}$ the fifth. How many silkworms hatched each day?

65. The silkworms from each ounce of eggs consumed 1800 lb. of mulberry leaves during their feeding period. How many mulberry trees, bearing 125 lb. of leaves each, were needed to provide leaves for the silkworms from $12\frac{1}{2}$ ounces of eggs?

66. If $\frac{1}{4}$ of the worms died during this period, how many lived to spin cocoons?

67. When every silkworm had spun its cocoon, it was found that 250 fresh cocoons weighed one pound. What was the total weight of fresh cocoons spun by the 337,500 worms?

68. To prevent the insects emerging from the cocoons as moths, thus breaking the silk thread, they were killed or "choked" in ovens. If the 1350 lb. of cocoons lost .68 of their weight by this process, how much did the choked cocoons weigh?

69. At $\$\frac{7}{8}$ per pound, find the value of these cocoons, 432 lb.

70. Find the weight of fresh cocoons from the silkworms of 1 of the $12\frac{1}{2}$ ounces of eggs that hatched; the weight of these cocoons when choked; and the value of the choked cocoons.

71. The amount of raw silk obtained from the choked cocoons was .275 of their weight. How many pounds of silk did the whole number of cocoons furnish?

72. If it took 6 women 26.4 days to reel the 118.8 lb. of silk from the cocoons, how much silk did each woman reel per day?

73. Find the cost of the raw silk to a manufacturer who bought it at $4.15 per pound.

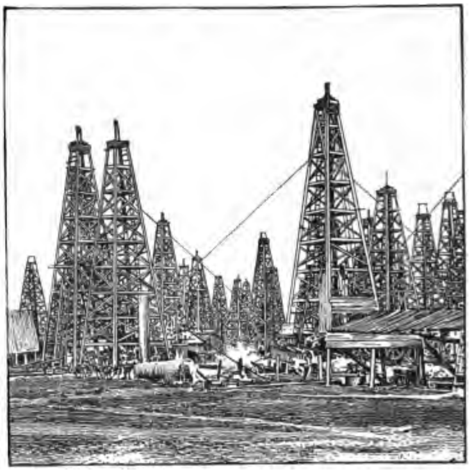
74. A man starting to raise silkworm eggs to sell, first bought 2 ounces, 40,000 eggs to the ounce. $\frac{3}{4}$ of the eggs hatched and the insects grew, spun cocoons, and emerged as moths. If $\frac{1}{2}$ of the moths laid eggs, 300 apiece, and $\frac{2}{3}$ of the eggs were marketable, how many ounces did the man have to sell?

75. Find the value of the 150 ounces of eggs @ $\$2\frac{1}{4}$.

76. The world's production of raw silk one year was: France, 1,256,625 pounds; Italy, 9,803,855 pounds; China, 12,288,565 pounds; Japan, 10,515,940 pounds; and other countries, 8,502,130 pounds. Find the total supply.

77. The price of raw silk per pound in New York was as follows, for the best grade from each place named: Italy, $4.40; Japan, $3.95; Shanghai, China, $4.70; Canton, $3.25. Find the cost of 240 pounds from each of these places.

78. In the construction of an oil derrick in Pennsylvania, 9500 feet of lumber were used. Find the cost of the lumber at $10.50 per M.



79. If $99.75, the cost of the lumber, was $37\frac{1}{2}\%$ of the total cost of the derrick, what was the expense of erecting the derrick?

80. The oil well underneath was drilled to a depth of 1280 feet. Find the cost of drilling it at 55¢ per foot.

81. If the rent of the drilling machinery used was 12¢ for each foot of the well drilled, for how much did the machinery rent?

82. The first 250 feet of the well was lined with a large size of pipe called casing, as a support to the sides of the hole. How much did the casing cost at 45¢ per foot?

83. Inside the casing and extending to the bottom of the well was 1280 feet of tubing worth $15\frac{1}{2}$¢ per foot. Find its cost.

84. Besides the tubing there was the same length of pumping rods to be used in pumping the oil. Find the cost of the pumping rods at $5\frac{1}{2}$¢ per foot.

85. To make the production of oil larger, a torpedo that weighed 85 pounds was exploded at the bottom of the well. At 96¢ per pound, how much did the torpedo cost?

86. Find the combined cost of derrick and well, if the only additional expense was for well fixtures, $24.25.

87. The well in 3 weeks produced 2352 barrels of oil. Find the yield per day; per hour.

88. A barrel of crude oil is 42 gallons. Find the yield per day in gallons.

89. The operator paid rent for the land in oil instead of in money. After paying $12\frac{1}{2}\%$ of the daily yield of 112 barrels for rent, how many barrels had he left?

90. If the price of crude oil in July was $1.35 per barrel, find the value, for that month, of the producer's portion, 98 barrels per day.

91. Find the value of the total yield during July.

92. This oil was pumped to a distant city through a pipe, interrupted at intervals by pumping stations. How far was the oil pumped, if the distances between stations were: 23.41 mi., 25.92 mi., 25.73 mi., 45.45 mi., 51.94 mi., 62.50 mi.?

93. If the yield of the well was 112 barrels per day, in how long a time did it produce enough oil to fill a tank at the end of the pipe line having a capacity of 30,800 barrels?

94. At the refinery the crude oil in a tank of this size yielded the following products, in 42-gallon barrels:

Gasoline, benzine, and naphtha,	3850 bbl.
Illuminating oil,	23,100 bbl.
Lubricating oil,	924 bbl.
Residuum and loss,	2926 bbl.

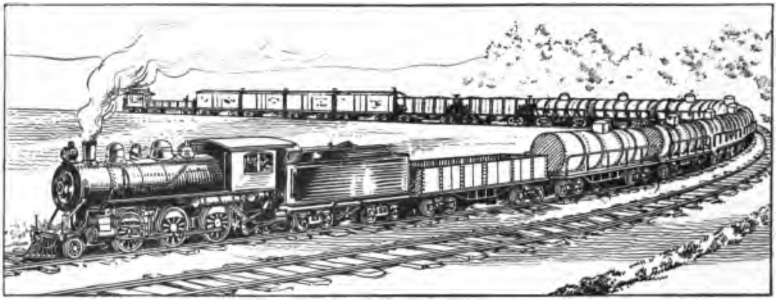
What per cent of the crude oil was each of the products?

95. Find the value of the naphtha, benzine, and gasoline at an average price of $7\frac{1}{2}$¢ per gallon.

96. The refined illuminating oil was put into 50-gallon barrels. How many such barrels were required?

97. Find the price per gallon of the illuminating oil, if it sold for $84,892.50.

98. Oil is often transported and sold in bulk instead of in barrels. When thus sold, $6\frac{1}{2}$ pounds are counted one gallon. How many gallons are there in a tank freight car that contains $24\frac{3}{8}$ tons of oil?



99. How many such tank cars, holding 7500 gallons each, would be required to load an ocean tank steamer whose capacity is 1,200,000 gallons of oil?

100. Find the weight in tons of this steamer's cargo of oil.

101. Refined oil is often shipped in 5-gallon cans, which are packed by twos in wooden cases $20\frac{3}{4}$ in. long and $10\frac{1}{2}$ in. wide. How many of these cases can be put in one layer in a compartment $41\frac{1}{2}$ ft. long and $31\frac{1}{2}$ ft. wide?

102 How many gallons will these 864 cases hold?

103 The cases are 15 in. high and there are 8640 gallons in one layer. How many gallons of oil will there be, if the cases are laid to a height of $6\frac{1}{4}$ ft.?

104. An oil delivery wagon weighed 2195 lb. empty, and 6420 lb. when filled with oil. Find its capacity in gallons.

105. The United States contributed 80,830,000 bbl. of oil to the world's production of 177,000,000 bbl. in a recent year. What per cent was produced by the United States?

106. In a cheese factory there were 6 vats for milk. Two of them held 6250 pounds of milk each, and the other 4 held 5400 pounds each. Find the capacity of all.

107. Milk brought to this factory was paid for according to the amount of butter fat it contained. How much was paid to a farmer who brought 640 pounds of milk that tested 3.75 % butter fat, if the factory price of butter fat was 27 ¢ per pound?

108. A farmer, by feeding his cows $2\frac{1}{2}$ dollars' worth of meal per day more than he had been feeding them, increased the percentage of butter fat from $3\frac{1}{4}$ % to 4 % and increased the daily yield of milk from 2000 pounds to 2200 pounds. How much did he gain per day by the experiment, if he was paid 26 ¢ per pound for butter fat?

109. A factory produced 320,000 pounds of cheese and 28,000 pounds of butter in a year. Find the value of the output at 15 ¢ per pound for cheese and 25 ¢ per pound for butter.

110. The Mohawk Valley Coöperative Cheese Company received 237,600 pounds of milk one month, and from it produced six hundred 40-pound cheeses. How many pounds of milk were required to make 1 pound of cheese? If the rest was whey, how many pounds of whey were produced?

111. The cheese was sold at $13\frac{1}{4}$ ¢ per pound; freight, commission, and other charges amounted to $580; the cost of manufacture was $238.85. Find the net proceeds, or the amount divided among the farmers at the end of the month.

112. The milk received that month contained 8910 pounds of butter fat, for which $2361.15 was paid. How much, then, was paid the farmers for each pound of butter fat?

113. Mr. Andrews took 8400 pounds of milk to the factory that month and was credited with 308 pounds of butter fat @ $26\frac{1}{2}$ ¢. How much did he receive as his share of the profits? What per cent of his milk was butter fat?

When crops are sold off a farm year after year, the soil becomes impoverished. As farm animals are good agents for restoring fertility to the soil, it is often profitable when a corn crop is large and low in price, to feed the surplus crop, producing "corn-fed" beef and pork.

Hogs are often fattened along with cattle, as they eat what the cattle waste.

114. An Illinois farmer bought 200 steers and 50 hogs on Nov. 1. He paid $7790 for the steers. If their total weight was 190,000 pounds, what was the price per 100 pounds?

115. Find the average weight per head; the average cost.

116. The steers weighed 190,000 lb., Nov. 1; 201,600 lb., Dec. 1; 213,600 lb., Jan. 1; 226,400 lb., Feb. 1; 240,800 lb., Mar. 1; and 256,400 lb., Apr. 1. What was the total gain in weight each month?

117. Find the average gain in weight per head for each month.

118. The cost of feed in November was $228; in December it was $408.30. Find the average cost of feeding 1 steer during each of these months.

119. How much did feed cost during each of the next two months, the record being as follows:

JANUARY	FEBRUARY
2200 bu. corn @ 27¢	36 loads elevator corn @ $3
20 tons hay @ $6.75	600 bu. corn @ 32¢
6 tons straw @ $1.90	18 tons hay @ $7.50
$\frac{3}{4}$ bbl. salt @ $1.20	Straw, etc., $17.20

120. What was the average cost of feeding 1 steer during January? during February?

121. The farmer paid in all $2850 for feed, of which $5.80 less than 36% was expended in the last month, March. How much did feed cost him in March?

122. Find the average cost of producing 100 pounds of beef, the increase in weight of the steers during the five months being from 190,000 pounds to 256,400 pounds.

123. How much less did it cost to produce 100 pounds of beef in November, when the gain in weight was 11,600 pounds and the cost $228, than in December, when the gain in weight was 12,000 pounds and the cost $408.30?

124. How much more did it cost to produce 100 pounds of beef in March, when the gain in weight was 15,600 pounds and the cost $1020.20, than in December?

125. If every 100 pounds of beef produced was worth $5.05, what was the total profit during November? during February, when the gain in weight was 14,400 pounds and the cost $452.20?

126. Find the loss during January, when the gain in weight was 12,800 pounds and the cost $741.30; during March.

127. The 36 loads of corn bought in February had been slightly damaged by water. How much was saved by buying this corn, if each load was worth, for feed, as much as 50 bushels of 32-cent corn?

128. The cattle cost $7790 in November; the total cost for feed was $2850. What was the average cost of the steers per head, April 1?

Apr. 1, the cattle were shipped to Chicago in 12 cars, freight 10¢ per 100 lb. on 256,400 lb., and sold through an agent at $5.05, commission $10 per car load. When sold they weighed 250,800 lb. Feed and care cost $45. Find:

129. Freight. **131.** Net proceeds of sale.

130. Commission. **132.** Shrinkage per head during shipment.

133. Find the total amount gained on the cattle.

134. The record for the hogs was as follows:

Nov. 1, bought 50 hogs at $5 each.

10 hogs died during the winter.

Extra feed cost $18.

Apr. 2, sold 40 hogs, 12,500 lb. at $4.35 per hundredweight.

Find the gain on fattening hogs.

135. Besides the gain on cattle and hogs, the value of fertility restored to the farm by fertilizing material, was:

10 % of value of grain fed, 8800 bu. worth 28¢ per bushel.

$2 for each ton of straw and of hay (74 tons).

How much did the stock-fattening experiment benefit the farm in fertility restored?

136. Find the total gain, deducting $130 for hired labor, $180 for the farmer's labor, and $85 for the use of money and tools.

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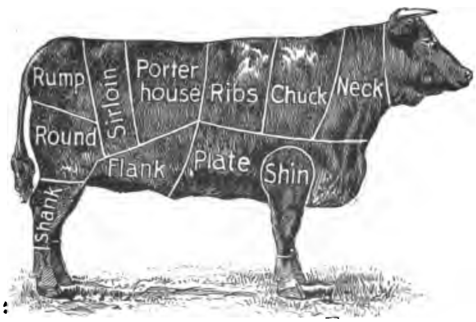
137. A steer weighing 1350 pounds yielded 783 pounds of dressed beef. What per cent of his weight was beef?



138. A packer buys this steer at $5 per 100 pounds. Expenses amount to $7.50. If he sells the beef at $9.50 per 100 pounds, the hide for $8.25, the fat for $5.50, and the offal for $2.25, how much will he gain?

139. Find the value of the beef at retail as follows:

Neck, 26 lb. @ 5¢; chuck, 137 lb. @ 8¢; ribs, 73 lb. @ 14¢; sirloin, 39 lb. @ 20¢; porterhouse, 94 lb. @ 22¢; rumps, 39 lb. @ 10¢; rounds, 122 lb. @ 14¢; shanks, 26 lb. @ 4¢; flanks, 47 lb. @ 6¢; plates, 120 lb. @ 5¢; shins, 60 lb. @ 4¢.

PART III

PRELIMINARY REVIEW

378. 1. Copy, point off, and read:

4710	32256	6445386	1234567890
2323	45045	7777412	406725304

2. Read:

.483	125.4	28.003	.000002
.574	62.875	5.0602	.00000075

In books of account, dollars and cents are separated by a line, as in the following exercises, instead of by decimal points.

Add and test in 9 minutes or less:

3.		4.		5.		6.		7.	
$78	50	$325	63	$2493	67	$3879		$105092	25
99	37	106	25	678	10	172	56	827219	40
29	17	204	16	3429	12	8	94	274416	29
4	25	75	24	117		5	62	98270	
	14		09	5276	30	476	28	7630	81
75	46	234	75	9417	37	2630	06	1192	11
7	44	308	42	148	04	5523		124	16
86	93	54	50	171	14	89649	44	796	96
17		4	19	9	18	24763	84	2643	10
63	22	2	43	481	92	5169	36	23344	66
23	68	192	05	25	42	18371	88	648872	18
50	29	416	23	8345	68	539	42	985020	60

Subtract, as indicated, and test:

8. 50,000 – 16,582
9. 84,164 – 27,629
10. \$193.80 – \$54.95
11. 4.81 – 1.3486
12. 6.0075 – 2.649
13. \$62,956 – \$438.88

How many years have elapsed since the following events?

14. Atlantic cable, 1858.
15. Morse telegraph, 1844.
16. Marconi telegraph, 1896.
17. First cotton gin, 1794.
18. First life-preserver, 1805.
19. First steamboat in U.S., 1786.
20. First railroad in U.S., 1826.
21. First newspaper in U.S., 1704.

Multiply:

22. 9.2 by 36.502
23. .47 by 100.64
24. 78.256 by 8.75
25. 4.6254 by 32.5
26. 738.49 by .496
27. 55.410 by .855
28. 42,720 by .00705
29. 81.625 by .00404
30. 3.6875 by .80608
31. 16 million by .15625
32. 7.5 million by 1.0844
33. 36.25 million by .0075

Divide as indicated:

34. $65 \div .64$
35. $15 \div .9375$
36. $64 \div .0625$
37. $17.4 \div 19.2$
38. $8.64 \div 3.84$
39. $39.6 \div 6.875$
40. $8.236 \div .145$
41. $319.5 \div .075$
42. $945 \div .448$
43. $918 \div .53125$
44. $33.176 \div .0464$
45. $10.2382 \div 49.7$

Find the prime factors of:

46. 270
47. 216
48. 225
49. 180
50. 324
51. 801
52. 495
53. 363
54. 567
55. 648
56. 780
57. 891
58. 1024
59. 1728
60. 13,068
61. 80,000

62. Reduce $\frac{2}{3}$ and $\frac{5}{12}$ each to a fraction whose denominator is 48; 96; 144; 240; 384; 528.

Reduce to lowest terms :

63. $\frac{42}{72}$ **65.** $\frac{144}{192}$ **67.** $\frac{125}{375}$ **69.** $\frac{450}{675}$ **71.** $\frac{375}{1000}$

64. $\frac{64}{80}$ **66.** $\frac{175}{210}$ **68.** $\frac{168}{192}$ **70.** $\frac{320}{768}$ **72.** $\frac{576}{1728}$

Reduce to a fraction :

73. $28\frac{1}{4}$ **75.** $216\frac{2}{3}$ **77.** $42\frac{7}{10}$ **79.** $29\frac{17}{18}$

74. $32\frac{5}{8}$ **76.** $324\frac{3}{4}$ **78.** $68\frac{5}{12}$ **80.** $56\frac{19}{24}$

Reduce to an integer or to a mixed number :

81. $\frac{87}{3}$ **83.** $\frac{196}{4}$ **85.** $\frac{531}{18}$ **87.** $\frac{515}{25}$ **89.** $\frac{1520}{20}$

82. $\frac{92}{4}$ **84.** $\frac{277}{8}$ **86.** $\frac{296}{24}$ **88.** $\frac{821}{12}$ **90.** $\frac{1360}{24}$

Reduce each decimal to a common fraction in its lowest terms :

91. .44 **94.** .0625 **97.** .15625 **100.** 53.125

92. .85 **95.** .3125 **98.** .21875 **101.** 65.625

93. .025 **96.** 20.75 **99.** .28125 **102.** 7.8125

Reduce to decimals, expressing each in the form best adapted, and carrying none farther than six places :

103. $\frac{5}{9}$ **106.** $\frac{84}{48}$ **109.** $\frac{5}{64}$ **112.** $3.4\frac{1}{2}$

104. $\frac{5}{12}$ **107.** $\frac{15}{16}$ **110.** $\frac{17}{40}$ **113.** $.85\frac{7}{11}$

105. $\frac{8}{15}$ **108.** $\frac{21}{25}$ **111.** $\frac{25}{36}$ **114.** $.00\frac{5}{16}$

Reduce to fractions having the least common denominator :

115. $\frac{5}{12}$ and $\frac{7}{15}$ **118.** $\frac{4}{9}, \frac{13}{24}, \frac{23}{36}$ **121.** $\frac{14}{15}, \frac{9}{20}, \frac{29}{30}$

116. $\frac{9}{10}$ and $\frac{9}{16}$ **119.** $\frac{13}{15}, \frac{18}{25}, \frac{31}{45}$ **122.** $\frac{7}{12}, \frac{11}{36}, \frac{23}{24}$

117. $\frac{13}{18}$ and $\frac{17}{24}$ **120.** $\frac{27}{48}, \frac{8}{15}, \frac{17}{10}$ **123.** $\frac{10}{12}, \frac{4}{28}, \frac{13}{42}$

Find the difference between :

124. $5\frac{3}{8}$ and $2\frac{3}{4}$ **126.** $\frac{15}{40}$ and $\frac{3}{16}$ **128.** $6\frac{4}{5}$ and $3\frac{2}{3}$

125. $4\frac{1}{2}$ and $1\frac{4}{5}$ **127.** $\frac{12}{15}$ and $\frac{18}{75}$ **129.** $\frac{14}{15}$ and $\frac{7}{12}$

Add:

130. $29\frac{2}{3}$	**131.** $42\frac{5}{8}$	**132.** $1.4\frac{3}{5}$	**133.** $3.33\frac{1}{3}$	**134.** $34\frac{5}{12}$
$18\frac{1}{4}$	$63\frac{1}{3}$	$7.6\frac{1}{6}$	$5.87\frac{1}{2}$	$69\frac{3}{10}$
$72\frac{5}{6}$	$24\frac{3}{4}$	$9.4\frac{1}{3}$	$7.83\frac{5}{6}$	$57\frac{9}{20}$
$46\frac{1}{2}$	$91\frac{1}{6}$	$2.3\frac{1}{2}$	$9.66\frac{2}{3}$	$40\frac{11}{15}$

135. Add $6.12\frac{1}{2}$, $8.4\frac{3}{4}$, and $9.06\frac{1}{4}$.

Suggestion. — Write $6.12\frac{1}{2}$, $8.47\frac{1}{2}$, and $9.06\frac{1}{4}$, or 6.125, 8.475, and 9.0625.

Multiply:

136. $\frac{10}{28}$ by $\frac{7}{40}$ **140.** 18.6 by $22\frac{1}{2}$ **144.** \$.$37\frac{1}{2}$ by 26

137. $\frac{25}{48}$ by $\frac{8}{10}$ **141.** 43.5 by $16\frac{1}{5}$ **145.** \$.$18\frac{3}{4}$ by 70

138. $\frac{10}{21}$ by $\frac{14}{15}$ **142.** 7.38 by $24\frac{1}{3}$ **146.** \$.$26\frac{3}{8}$ by 44

139. $\frac{33}{64}$ by $\frac{48}{55}$ **143.** 6.45 by $51\frac{2}{3}$ **147.** \$.$33\frac{1}{3}$ by 54

Divide:

148. $\frac{15}{24}$ by $\frac{2}{3}$ **152.** $899\frac{1}{2}$ by 21 **156.** 2730 by $24\frac{3}{8}$

149. $\frac{27}{36}$ by $\frac{3}{10}$ **153.** $616\frac{4}{5}$ by 36 **157.** 2216 by $69\frac{1}{4}$

150. $\frac{25}{48}$ by $\frac{5}{16}$ **154.** $657\frac{1}{2}$ by 25 **158.** 4112 by $42\frac{5}{6}$

151. $\frac{13}{40}$ by $\frac{39}{70}$ **155.** $635\frac{3}{5}$ by 35 **159.** 2800 by $33\frac{1}{3}$

160. Find the value of $\frac{2}{3}$ of $6 \div \frac{3}{4}$ of 8; of $\frac{2}{3} \times 6 \div \frac{3}{4} \times 8$.

Suggestion. — The word "of" indicates a closer connection than the signs $\times$ and $\div$; thus, $\frac{2}{3}$ of $6 \div \frac{3}{4}$ of $8 = (\frac{2}{3} \times 6) \div (\frac{3}{4} \times 8) = \frac{2}{3} \times 6 \times \frac{4}{3} \times \frac{1}{8}$, while $\frac{2}{3} \times 6 \div \frac{3}{4} \times 8 = \frac{2}{3} \times 6 \times \frac{4}{3} \times 8$.

Find the value of:

161. $\frac{4}{5}$ of $20 \div \frac{5}{6}$ of $30 \times \frac{7}{8}$ of 40 **164.** $\frac{5}{8} \times 18 + \frac{3}{4} \div 15$

162. $\frac{2}{3}$ of $14 \times \frac{3}{4}$ of $18 \div \frac{1}{6}$ of 42 **165.** $\frac{5}{6} \div 2\frac{2}{3} \times 14 \div \frac{5}{9}$

163. $\frac{5}{12}$ of $\frac{4}{5}$ of $\frac{7}{8} \div \frac{1}{2}$ of $\frac{14}{15}$ of $2\frac{1}{4}$ **166.** $\frac{4}{5} \div \frac{3}{10} + 1\frac{3}{5} \times 3\frac{1}{2}$

What part of

167. $\frac{3}{4}$ is $\frac{1}{2}$? **169.** $\frac{1}{3}$ is $\frac{1}{6}$? **171.** $4\frac{4}{5}$ is 4? **173.** 10 is $3\frac{1}{3}$?

168. 5 [illegible] is $\frac{1}{4}$? **170.** $\frac{4}{5}$ is $\frac{3}{4}$? **172.** $9\frac{1}{3}$ is 7? **174.** 12 is $6\frac{1}{2}$?

Find the number of which

175. 84 is $\frac{2}{3}$	**177.** 224 is $\frac{4}{5}$	**179.** $3\frac{3}{5}$ is $\frac{3}{10}$	**181.** $5\frac{1}{2}$ is $\frac{11}{4}$
176. 91 is $\frac{3}{4}$	**178.** 375 is $\frac{5}{6}$	**180.** $6\frac{5}{6}$ is $\frac{5}{12}$	**182.** $4\frac{1}{4}$ is $\frac{34}{5}$

379. 1. Add 37 thousand 250 and 8 hundredths, 64 thousand 208 and 9 tenths, 10 thousand and 70 thousandths, 856 thousand 926 and 25 ten-thousandths, 648 and 126 ten-thousandths, 70 and 2069 ten-thousandths.

2. The heaviest bell in the world weighs 432,000 pounds, and is in Moscow, Russia. How much heavier is it than the bell in the City Hall, New York, which weighs 22,500 pounds?

3. The United States cabinet officers receive a salary of $12,000 per year, each. What are the combined salaries of the 9 cabinet officers for a term of 4 years?

4. It takes a letter 45 days to go from New York to Shanghai, China, a distance of 14,445 miles. How many miles is it carried per day?

5. The United States 12-inch naval gun weighs $53\frac{3}{4}$ tons, the German gun of the same bore, $48\frac{7}{8}$ tons. How much less does the German gun weigh than the American?

6. The equipment of an American soldier in war time weighs $55\frac{1}{4}$ pounds. If the British soldier carries $3\frac{1}{8}$ pounds less and the German soldier $3\frac{1}{2}$ pounds more than the American, what is the weight of each one's equipment?

7. One year 74,510,064 pounds of paper were used in the United States for job printing. Find the value of this quantity of paper at 8.4¢ per pound.

8. The cable from San Francisco to Manila is laid in sections having the following lengths: San Francisco to Hawaii, 2089 miles; Hawaii to Wake Island, 2040 miles; Wake Island to Guam, 1290 miles; Guam to Manila, 1520 miles. Find the total length of the cable.

9. If Holland has 1500 miles of railroad, and only $\frac{2}{3}$ as many miles of railroad as of canals, what length of canals has she?

10. Recently .32 of the 260,000 seamen employed on British merchant vessels were foreigners. How many were foreigners?

11. If the flour mills near Liverpool are capable of producing each hour 625 sacks of flour of 280 pounds each, how many sacks can they produce in 7 hours? how many pounds?

To how many barrels of 196 pounds is this equal?

12. If the match factories in the Japanese city of Kobé one year manufactured matches to the value of 9,450,000 yen, what was this value in U. S. money, a yen being worth 49.8 ¢?

13. One season this country received German toys worth \$4,500,000. If all the toys exported by Germany were worth 3.4 times this amount, find their total value.

14. How much do girls who work in a German doll factory earn in 6 days of 10 hours each, if they receive 3.5 ¢ per hour?

15. India obtained in a season 222,200,000 pounds of tea from the 524,500 acres under tea cultivation. Find to the nearest tenth of a pound the average yield per acre.

16. On a South African ostrich farm of 48 acres there were 5 ostriches per acre. If each bird produced \$28.75 in feathers during the year, what was the income?

17. During a recent tourist season in Switzerland, 26,569 Germans stayed at Geneva hotels, 9618 Englishmen, 35,114 Swiss, 68,513 Frenchmen, 14,177 Americans, and 23,094 others. How many visitors were there?

18. The value of 15,000,000 pounds of chocolate exported from Switzerland in a year was \$5,100,000. The value of that exported to Great Britain was \$2,125,000; of that exported to the United States, \$578,000. How many pounds were exported to each of these countries?

19. If 220 trees in the Philippine Islands yield 180 pounds of hemp fiber for cordage, how many trees are necessary to provide fiber for a bale of 270 pounds?

20. If the average value of a load of diamond-bearing earth in a South African diamond mine is \$$6\frac{3}{4}$, find the value of the 424 loads put into one "washing pan" every twelve hours.

21. In a good year Bulgaria produced 13,770 pounds of attar of roses. How many acres of rose bushes were under cultivation, if the flowers from one acre produced $\frac{15}{16}$ of a pound of attar of roses?

22. How long did it take a woman to weave a Persian rug of 18 square feet, if she could weave 1 square foot in $3\frac{5}{6}$ weeks?

23. If $38\frac{1}{4}$ ounces of gold were extracted from 68 tons of ore, what was the amount of gold per ton of ore?

24. From 50,000 pounds of cod $68\frac{3}{4}$ gallons of cod-liver oil were obtained. Find the yield of oil per 1000 pounds of fish.

25. Find the value of cocoa at \$$\frac{1}{4}$ per pound obtained from $\frac{3}{8}$ of an acre of cacao trees, planted 408 trees to the acre, if $1\frac{1}{2}$ pounds of marketable cocoa are obtained per tree.

26. H. H. Moore bought of Lackawanna Coal Co. 1343 tons of coal @ \$3.40. Make out and receipt the bill, supplying necessary data.

27. A steamer on the enlarged Erie Canal will carry 900 tons of wheat and tow 3 barges, each carrying 100 tons, on the trip from Buffalo to New York. If the average load on the return trip is 1250 tons, how many tons of freight will the steamer and barges transport in a season of 10 round trips?

28. The cost per mile of running a suburban train by steam was: coal, $14\frac{1}{4}$¢; water, $\frac{1}{2}$¢; crew, $12\frac{1}{4}$¢; maintenance, $6\frac{1}{2}$¢; supplies, $\frac{5}{8}$¢. The cost with electrical equipment was: electric power, $10\frac{3}{8}$¢; crew, $6\frac{5}{8}$¢; maintenance, 4¢; supplies $\frac{1}{4}$¢. Find the saving per mile with electricity.

29. In $2\frac{1}{2}$ days a machine in a Maine paper mill turned out $43\frac{3}{4}$ tons of paper. How many tons did it make per day?

30. A factory is lighted by 50 electric incandescent lamps. If each lamp burns 720 hours per year at a cost of $\frac{13}{20}$¢ per hour, how much does it cost a year to light the factory?

31. A film for a moving picture consisted of a series of separate pictures, each $\frac{3}{4}$ of an inch long. If the film was 150 feet long, of how many individual pictures was it made up?

32. During a balloon contest, the balloon remained in the air $35\frac{3}{4}$ hours and traveled $33\frac{1}{3}$ miles per hour. How far did it travel?

33. A fan blower supplies a hall with 2760 cubic feet of air per minute. Find the weight of the air forced into the room in an hour, if 1 pound of it occupies $13\frac{1}{7}$ cubic feet.

34. Railroad statistics one year showed that there were 225,000 passenger coaches in the world, $\frac{2}{3}$ as many locomotives, and $13\frac{1}{3}$ times as many freight cars. Find the number of each.

35. The cost of a British third-class cruiser was $675,000. This was $5000 more than $\frac{4}{5}$ of the cost of a German warship of the same type. How much did the German cruiser cost?

36. Steel expands .00001093 ft. per foot of length for every degree of rise in temperature. If the length of a steel rail in freezing weather is 30 feet, how many inches longer will it be on a day when the temperature is 105° F.?

37. The average time of transmitting a 20-word cable message on the first cable was $6\frac{2}{3}$ minutes. Such a message can now be transmitted in $\frac{2}{5}$ of a minute. What part of the time originally required for transmission is the present time?

38. In the menagerie Harold saw a test in which a horse pulled a weight of 1875 pounds, or $\frac{3}{14}$ of the weight pulled by an elephant. A camel pulled $\frac{11}{15}$ as much as the horse. What was the pulling strength of each animal?

ALIQUOT PARTS

380. 1. How many times is \$.25 contained in \$1? .25 lb. in 1 lb.? .25 in 1? 25 in 100? 250 in 1000?

2. Mention several parts of \$1 that are exactly contained in \$1; parts of 1 that are exactly contained in 1, that is, **aliquot parts** of 1; aliquot parts of 100; aliquot parts of 1000.

3. What part of \$1 subtracted from \$1 will give \$.87$\frac{1}{2}$? \$.75? \$.90 \$.66$\frac{2}{3}$? \$.83$\frac{1}{3}$?

4. What part of \$1 added to \$1 will give \$1.10? \$1.12$\frac{1}{2}$? \$1.20? \$1.25? \$1.37$\frac{1}{2}$? \$1.50?

5. What is the cost of 44 articles at \$1 each? at \$.50, or \$$\frac{1}{2}$, each? at \$.25, or \$$\frac{1}{4}$, each? at \$1.25, or \$1+\$$\frac{1}{4}$, each?

6. From the cost of any number of articles at \$1 each, how may the cost at \$.50 each be found? at \$.25? at \$.12$\frac{1}{2}$?

381. From the parts of \$1 or any other unit, given on page 110, we may now construct the parts of 10, 100, 1000, thus:

Parts	Of 1	Of 10	Of 100	Of 1000
$\frac{1}{8}$	.125	1.25	12.5	125
$\frac{1}{4}$	.25	2.5	25	250
$\frac{3}{8}$	.375	3.75	37.5	375

and so on.

WRITTEN EXERCISES

382. 1. Find the cost of 27 buggies at \$75 each.

\$2700	At \$100 each, 27 buggies would cost \$2700.
675	At \$25 each, they would cost $\frac{1}{4}$ of \$2700, or \$675.
	At \$75 each, then, they cost \$2700 – \$675, or \$2025.
\$2025	The business man writes as few figures as possible.

In the following, count any fraction of a cent in results as an extra cent.

Find the cost of 85 articles:

2. At 20¢ each
3. At 90¢ each
4. At 16$\frac{2}{3}$¢ each
5. At 37$\frac{1}{2}$¢ each
6. At \$1.25 each
7. At \$1.75 each

Find the cost of 34 articles at:

8. \$25 each	**10.** \$250 each	**12.** \$12.50 each
9. \$75 each	**11.** \$225 each	**13.** \$37.50 each

Find products:

14. $27 \times .125$	**19.** 250×67	**24.** $25 \times 44 \times 19$
15. 48×12.5	**20.** 500×99	**25.** $50 \times 24 \times 77$
16. 71×750	**21.** 875×48	**26.** $32 \times 75 \times 22$
17. 62×37.5	**22.** $625 \times .81$	**27.** $75 \times 71 \times 72$
18. $94 \times .625$	**23.** 125×3.9	**28.** $66 \times 16 \times 1.25$

29. How many yards of cloth costing $66\frac{2}{3}$¢ per yard can be bought for \$144?

$$\begin{array}{r} 144 \text{ yd.} \\ \underline{72 \text{ yd.}} \\ 216 \text{ yd.} \end{array}$$

At \$1 per yard, 144 yards could be bought for \$144. But since the price is only \$$\frac{2}{3}$ per yard, $1\frac{1}{2}$ times as many yards can be bought; that is, $1\frac{1}{2} \times 144$ yards can be bought for \$144.

How many yards of cloth can be bought for \$120 at

30. $66\frac{2}{3}$¢ per yd.?	**33.** \$1.25 per yd.?
31. 75¢ per yd.?	**34.** \$1.50 per yd.?
32. $83\frac{1}{3}$¢ per yd.?	**35.** \$2.50 per yd.?

Divide:

36. 47 by .25	**41.** 316 by 500
37. 64 by 2.5	**42.** 428 by 250
38. 48 by 7.5	**43.** 715 by 625
39. 111 by 37.5	**44.** 240 by 12.5
40. 34.3 by 87.5	**45.** .425 by .125

46. Find the cost of 288 articles @ 50¢; @ 51¢; @ 49¢.

47. Find the cost of 432 articles @ $33\frac{1}{3}$¢; @ 33¢; @ $33\frac{1}{2}$¢.

48. A dealer bought 1024 pounds of tea at $47\frac{1}{2}$¢ per pound and sold it at 61¢ per pound. Find his gain.

ACCOUNTS

383. A record of business transactions is called an **account**.

384. The business of the day is usually recorded, as the transactions occur, in a book called a **daybook**. The records of the several transactions are afterward transferred, or **posted**, into another book called a **ledger**, where they are arranged in groups, each containing the transactions that belong to one class, as, for example, those relating to one person or firm.

These groups, or **ledger accounts**, headed by the name of the person, are arranged so that the amounts charged against him, called **debits**, appear on the *left* side and those in his favor, called **credits**, on the *right* side.

385. A person who owes a debt is called a **debtor**, and a person to whom a debt is owed is called a **creditor**.

386. The difference between the sum of the debits and the sum of the credits of an account is called the **balance**.

387. *Computing* the balance, *entering* it on the lesser side, and *ruling* up the account, is called **closing**, or **balancing**, the account.

Some business men do this at stated times, as once a month, the balance being *brought down* to continue the account. It is also done when an account is settled.

388. The following are a few abbreviations in common use :

Company,	Co.	Credit or creditor,	Cr.	Paid,	Pd.
Balance,	Bal.	Debit or debtor,	Dr.	Received,	Rec'd.
Account,	Acc't.	Merchandise,	Mdse.	Payment,	Pay't.

389. The symbol # placed *after* a number stands for *pounds*, but if placed *before*, it is an abbreviation for the word "number."

Thus, 24 # means "24 pounds," but # 24 means "number 24."

390. The following form illustrates a closed ledger account with John Graham, the balance being brought down to continue the account into the next month:

John Graham

Dr.											Cr.
190–			*			190–			*		
Jan.	*4*	*Flour*	*1*	*$15*	*40*	*Jan.*	*4*	*Cash*	*1*	*$10*	
"	*7*	*Oats*	*4*	*8*	*70*	"	*12*	*Carting*	*8*	*12*	*50*
"	"	*Cash*	"	*5*		"	*19*	"	*15*	*10*	
"	*15*	*Corn Meal*	*11*	*17*	*50*	"	*24*	"	*21*	*17*	
"	*24*	*Hay*	*21*	*24*		"	*31*	*Balance*		*32*	*47*
"	*29*	*Oats*	*34*	*11*	*37*						
				81	*97*					*81*	*97*
190–											
Feb.	*1*	*Balance*		*32*	*47*						

1. What are the debits in this account? Find their sum.

2. Name the credits. Find their sum.

3. How much is the balance? How is it found? See whether it has been correctly computed.

4. Is the balance in favor of or against John Graham?

On which side of the account would it have been entered, if it had been in his favor?

On which side would it have been brought down?

5. What is the *footing* of the debit side of the account? of the credit side? How do the footings compare?

Test. — When an account is properly closed, the footing of one side is equal to the footing of the other.

* These columns contain numbers of the daybook pages upon which the several transactions were entered.

WRITTEN EXERCISES

391. Prepare a ledger form for each of the following accounts, supply the year, enter the items, close and test the account:

1. George Griffin.

Debits. — Jan. 8, Groceries, $10.37; Jan. 22, Groceries, $18.91; Feb. 10, Groceries, $7.89; Feb. 24, Groceries, 48¢; Mar. 6, Groceries, $15.75; Mar. 20, Groceries, 66¢; Apr. 4, Groceries, $9.72; Apr. 23, Groceries, $13.96.

Credits. — Jan. 16, Cash, $5; Feb. 15, Cash, $14.50; Mar. 8, Labor, $6.30; Mar. 14, Cash, $17; Apr. 15, Cash, $20; May 3, Labor, $8.75; May 17, Cash, $5.

2. Thomas Hinds & Co.

Debits. — July 12, Mdse., $185.50; July 19, Cash, $210; July 25, Mdse., $119.63; July 30, Mdse., $223.29; Aug. 11, Mdse., $88.72; Aug. 18, Cash, $335; Sept. 10, Mdse., $47.54.

Credits. — July 3, Mdse., $237.45; July 19, Mdse., $149.80; July 28, Cash, $125; Aug. 6, Mdse., $92.24; Aug. 14, Mdse., $422.18; Aug. 29, Mdse., $116.40; Sept. 4, Cash, $25; Sept. 8, Mdse., $173.91; Sept. 24, Mdse., $275.16.

3. Johnson & Mason.

Debits. — May 6, 7 Carriages, $875; May 14, 5 Double Wagons, $450; May 20, 4 Runabouts, $210; May 25, 3 Surreys, $525; June 1, 3 Single Wagons, $225; June 8, 1 Phaeton, $200; and 6 Open Buggies, $480; June 15, 2 Buckboards, $100; and 2 Road Carts, $70.

Credits. — Jan. 4, White Ash Lumber, $449.25; Jan. 9, Hub Timber (elm), $375.40; Feb. 4, Spoke Timber (hickory), $274.65; Apr. 8, Basswood Lumber, $124.80; May 10, Cash, $400; May 18, Cash, $325; June 4, Cash, $560; June 12, Cash, $350.

METHODS OF SOLVING PROBLEMS

392. There is often more than one method of arriving at the answer of a problem, however simple it may be. Suppose that 5000 sheets of paper cost $8. How much will 10,000 sheets cost at the same price?

The answer may be found as follows:

1. **Indirectly**, by finding the cost of 1 sheet, then of 10,000 sheets. Find the answer by this method, called **unitary analysis**.

2. **Indirectly**, by finding the cost of 1 thousand sheets, then of 10 thousand sheets. Find the answer by this method.

3. **Directly**, by comparing the cost of 10,000 sheets with that of 5000 sheets. Find the answer by this method, called **analysis by comparison**. Which is the best method for this problem?

ORAL ANALYSIS

393. In the following problems try to discover the best method of solution. Give the answer first, then the steps in the solution, or the *analysis*, in few words.

One pupil may declare the answer and others give analyses. If more than one analysis is given, the best should be chosen after a comparison.

1. It requires 7 nails to fasten a shoe on a horse's foot. How many are required for 4 shoes?

Answer	Analysis
28 nails.	1 shoe requires 7 nails. 4 shoes require 4×7 nails.

2. Find the cost of 18 oranges at 3 for 10 cents.

Answer	Analysis
60 cents.	3 oranges cost 10¢.
	1 orange costs $\frac{1}{3}$ of 10¢.
	18 oranges cost $\frac{18}{3}$ of 10¢.

This is not the best analysis. Give a better one.

3. A boy had \$5. After buying a hat he had \$3 remaining. How much did he pay for the hat?

Analysis

All his money = \$5; one part = \$3; other part paid for hat = \$5 – \$3.

4. A man bought 60 pears at 3¢ each and sold them at 5¢ each. How much did he gain?

Analyze by finding the amounts paid and received; again by finding first the gain on 1 pear. Compare analyses.

5. Find the cost of 30 towels at 3 for \$1.

6. How far is it around a city block each of whose sides is 300 feet long?

7. How far will a motor car go in 8 hours at the rate of 25 miles an hour?

8. Mary has 40 stamps and Luther has 4 times as many. How many stamps have both?

9. A grocer bought 80 quarts of berries at 8¢ a quart and sold them at 10¢ a quart. Find his gain.

10. If a man smokes 2 10-cent cigars a day, how much will cigars cost him per month of 30 days?

11. Charles bought 8 bananas at 2¢ each and had 14¢ left. How much money had he at first?

12. Clarence had \$2 and bought 2 chickens at 75¢ each. How much money had he left?

13. Edwin bought 2 sailboats at 80¢ each and handed the dealer \$2. How much change was due him?

14. If 1 yard of ribbon costs 9¢, how many yards can be bought for $1.80?

ANALYSIS

1 yard costs 9¢; the number of yards that can be bought for $1.80 is the same as the number of times $1.80 contains $.09.

15. At a fire-engine house the time from 9 P.M. to 6 A.M. is divided into watches of 3 hours each. How many watches are there?

16. For this work 9 men are detailed every night. How many are there on each watch?

17. The hours of exercise for the horses are from 7 A.M. to 5 P.M. Only 2 of the 10 teams in a precinct can be exercised at a time. How many hours may each team be exercised?

18. The chemical wagon carries 750 feet of engine hose and 250 feet of chemical hose. What part is engine hose?

ANALYSES

1. 750 ft. = 3 × 250 ft.; 750 ft. + 250 ft. = 4 × 250 ft.; 3 of the 4 equal parts of 1000 ft., or $\frac{3}{4}$ of the whole, is engine hose.
2. 750 ft. + 250 ft. = 1000 ft.; 750 ft. = $\frac{750}{1000}$, or $\frac{3}{4}$ of 1000 ft.

19. The chemical wagon carries also a 60-gallon extinguisher and 2 3-gallon extinguishers. What part of the extinguishing liquid is carried in the large extinguisher?

20. The city paid $16,800 for 8 chemical wagons. Find the cost of each.

21. The hook-and-ladder men can erect their ladder in 52 seconds, or in —— of a minute.

22. At a zoölogical garden the animals eat 250 pounds of beef per day. Find the cost at $6 per 100 pounds.

23. The daily ration of bread is 100 pounds and of fish 30 pounds. Find the amount of each consumed in 30 days.

24. The weekly ration of vegetables is 8 barrels. How many barrels is that per year of 52 weeks?

25. The animals eat 280 bananas per week. How many dozen bananas do they eat per day?

26. The hippopotamus Caliph weighs about 6000 pounds. How many tons (2000 pounds) does he weigh?

27. Caliph eats 50 pounds of hay every day. How long does a ton of hay last him?

28. A baby elephant weighs about 200 pounds, an adult elephant about 10,000 pounds. Compare in two ways the weight at birth with the weight when full grown.

29. The boundary line between Massachusetts and New York is marked by 121 monuments, 83 of which are of granite and the rest of iron. How many more granite monuments are there than iron monuments?

30. I can buy ready-made screens for my 12 windows at 50¢ each, but better ones made to order will cost me $1.50 each. How much more will the better ones cost?

31. How many times as much will the better screens cost?

32. If the better screens will last 5 times as long as the poorer ones, how many times as expensive are the latter as the former?

33. If $12\frac{1}{2}$ yards of silk cost $15, how much will 25 yards cost? How much silk can be bought for $60?

34. If 1200 oysters cost $6, how much will 1000 cost?

35. If 100 pounds of meat cost $8, how much will 225 pounds cost? $50 will buy —— pounds.

36. If 1000 sheets of paper cost $3.20, how much will 750 sheets cost? $16 will buy —— sheets.

37. If $37\frac{1}{2}$ pounds of butter cost $7.50, how much will 100 pounds cost? $15 will buy —— pounds.

38. If 1000 bricks cost $10, how much will 2500 cost? How many can be bought for $125?

39. If $12\frac{1}{2}$ yards of matting cost \$5, how much will $62\frac{1}{2}$ yards cost? \$20 will buy —— yards.

40. If $2\frac{1}{2}$ pounds of peanut candy cost 50¢, how much will $6\frac{1}{2}$ pounds cost?

Analyses

1. $\frac{5}{2}$ lb. cost 50¢; $\frac{1}{2}$ lb. costs $\frac{1}{5}$ of 50¢; $\frac{13}{2}$ lb. cost 13 times $\frac{1}{5}$ of 50¢.

2. $\frac{5}{2}$ lb. cost 50¢; $\frac{2}{5}$ of $\frac{5}{2}$ lb., or 1 lb., costs $\frac{2}{5}$ of 50¢; $\frac{13}{2}$ lb. cost $\frac{13}{2}$ of $\frac{2}{5}$ of 50¢.

41. If $1\frac{1}{2}$ pounds of sirloin steak cost 30¢, how much will 5 pounds cost?

42. Mr. Jones bought $12\frac{1}{2}$ pounds of lamb for \$1.50 and sold me $3\frac{1}{2}$ pounds at cost. How much did I pay?

43. If $\frac{4}{5}$ of the value of a store is \$4000, how much is the store worth?

44. If 2 men can do a piece of work in 12 days, how long will it take 4 men to do it?

Analyses

1. The amount of work to be done is 2×12 days' work, or 24 days' work. In 1 day 4 men do 4 days' work. To do 24 days' work 4 men will require $\frac{24}{4}$ days, or 6 days.

2. 4 men can do twice as much work as 2 men in the same time, or the same work in half the time. It will take 4 men $\frac{1}{2}$ of 12 days.

45. If 5 men can paint a bridge in 10 days, how long will it take 10 men?

46. How long would it take 2 men to paint the bridge?

47. If 40 men can dig a ditch in 5 days, how long will it take 20 men?

48. A contractor has 30 days in which to do a piece of work. His present force of 60 men would require 50 days to do the work. How many more men must he employ?

49. A contractor who has 80 laborers in his employ can complete a piece of work in 30 days. If he discharges 20 laborers, how much longer will it take to complete the work?

WRITTEN ANALYSIS

394. Under modern business conditions problems must be solved with the utmost rapidity and with absolute accuracy. An inaccurate result is worthless — it may be disastrous.

No answer may be regarded as accurate unless it has been *checked*, preferably by solving the problem or parts of it in two different ways. Knowing this, yet keeping in mind the value of time, the business man sets down his work, briefly but in a neat form, so that he or any other person may review it at a glance with complete understanding.

395. 1. At a manufacturing plant, 9600 men are employed 55 hours a week. The weekly pay roll is $138,500. Find the average wage per hour to the nearest tenth of a cent.

Mental Estimate

10 thousand men working 50 hours each would do 500 thousand hours' work. 138 thousand dollars for 500 thousand hours' work is about $\$\frac{1}{4}$, or 25¢ per hour.

Written Analysis

1 man does 55 hours' work.
9600 men do 9600 × 55 hours' work.
9600 × 55 hours' work earns $138,500 in wages.

1 hour's work earns $\frac{\$138{,}500}{9600 \times 55}$ in wages.

Business Process

12	$\$13850\cancel{0}\cancel{0}$
8	115.417
11	14.427
5	1.312
	.262

After arriving mentally at the result given in the last line of the written analysis, the business man would employ this process instead of the cancellation process, because it is neater, more easily reviewed, and less liable to error.

Checks. — 1. The result agrees well with the estimate, consequently no large errors (such as pointing off wrong) have been made.

2. Reviewing the process rapidly, the work is found to be correct.

In solving the following problems, estimate the result in advance when you can; write brief analyses; arrange computations in a businesslike manner; check results.

2. A telephone company owned 32,425 telephones worth \$12.50 each. How much were all worth?

SUGGESTION. \$12.50 is $\frac{1}{8}$ of \$100.

3. A merchant bought 96 suits of clothes at \$12 each and sold them at \$19.50 each. How much did he gain?

4. He bought 125 overcoats at \$35 each, and sold them at \$60 each. Of this money \$425 was lost in bad debts. Find the average gain on each overcoat.

5. Of the pears picked from an orchard, 65 bushels were sold at the home canning factory at 95¢ per bushel. How much did they bring?

6. Also 284 bushels were shipped to Boston, kept in cold storage for a month, and sold at \$2.25 per bushel. Freight charges were \$32, storage 15¢ per bushel, and charges for selling 5¢ per bushel. Find the proceeds of this sale.

7. The medium-sized late pears were packed in boxes and sold at Christmas time at \$1.10 per box. There were 432 boxes. Find the proceeds, deducting \$77.50 for freight, etc.

8. The largest late pears were shipped in boxes to Liverpool for the holiday trade. There were 360 boxes, and they brought $6\frac{1}{2}$ shillings (1 shilling = $24\frac{1}{3}$¢) per box. Transportation and other charges amounted to $\frac{3}{4}$ shillings per box. Find the proceeds.

9. Find the net income from the pear orchard, deducting 45¢ per bushel and 25¢ per box, as the cost of production.

10. It required 1,836,000 bricks to build an office building. Find the cost of the bricks at \$9.25 per M.

11. A stove manufacturer bought $22\frac{1}{2}$ long tons (1 long ton = 2240 lb.) of pig iron at \$16.80 per ton. How much did it cost him? How much per pound did he pay?

Find the cost of:

12. 1650 pounds of lard at \$9.75 per 100 pounds.

13. 7425 bushels of corn at $49\frac{7}{8}$¢ per bushel.

14. 1460 yards of flannel at \$.75 per yard.

15. 2025 baskets of peaches at \$1.25 per basket.

16. 6847 pounds of sugar at \$4.25 per 100 pounds.

17. A car load of wheat, 1075 bushels @ $77\frac{3}{8}$¢.

18. A dealer sold a lot of rugs for \$9360, which was $\frac{1}{8}$ more than they cost him. How much did they cost him?

19. The Maynard Co. bought 12 pianos for \$3300, and sold each at \$25 more than $\frac{7}{6}$ of the cost. Find the selling price of each piano and the total gain.

20. In the settlement of an estate worth \$62,500 there were lawsuits that cost the estate \$43,750. What part of the estate was left to divide among the heirs?

21. The senior partner of a firm owned $\frac{4}{5}$ of the business, and sold $\frac{1}{3}$ of his share for \$15,120. At that rate how much was the whole business worth?

22. What is the cost of 10 pieces of cloth containing $42\frac{1}{2}$, $45\frac{3}{4}$, 46, 44, $47\frac{3}{4}$, $48\frac{1}{2}$, $48\frac{1}{2}$, $45\frac{3}{4}$, 47, and $43\frac{3}{4}$ yards, respectively, at $4\frac{3}{4}$¢ per yard?

23. A speculator bought 6000 bales of cotton averaging 500 pounds per bale, at 9.12¢ per pound. The market declined 4 points, or .04 of a cent, and he sold $\frac{1}{3}$ of the cotton; it then advanced 6 points, and he sold the rest. Did he gain or lose, and how much?

24. A man bought 4 car loads of corn, net weights 47,600 lb., 50,400 lb., 49,000 lb., and 53,200 lb., at $42\frac{3}{4}$¢ per bushel of 56 pounds. He sold the first car load at $42\frac{7}{8}$¢ per bushel, the second at $43\frac{5}{8}$¢, the third at $43\frac{3}{8}$¢, and the fourth at $44\frac{1}{4}$¢. How much did he gain?

ANALYSIS BY EQUATIONS

396. How many pounds added to 25 pounds will give 30 pounds?

The statement of the problem may be condensed to

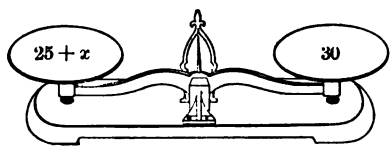
$$\begin{array}{r} 25 \text{ pounds} \\ \underline{+\ ?\ \text{ pounds}} \\ 30 \text{ pounds} \end{array} \quad \text{or} \quad \begin{array}{r} 25 \text{ pounds} \\ \underline{+\ x\ \text{ pounds}} \\ 30 \text{ pounds} \end{array} \quad \text{or} \quad 25 + x = 30$$

The letter x is only a convenient symbol for the **unknown number** (of pounds), or the number (of pounds) to be found. 25 and 30, on the other hand, are **known numbers.**

$25 + x = 30$, the briefest possible statement of the relation between the known and unknown numbers in the problem, is an **equation.** Finding the value of x is called **solving** the equation, $25 + x$ is the **first member** of the equation, and 30 is the **second member.**

Solution of Equations in x

397. 1. If 25 lb. is subtracted from the weight in each scale pan, the balance will be preserved.



In the same way, if 25 is subtracted from each member of the equation $25 + x = 30$, the equality will be preserved.

$$\begin{array}{rcr} 25 + x & = & 30 \\ \underline{25} & & \underline{25} \\ x & = & 5 \end{array}$$

2. What number subtracted from $x + 10$ will give x?

If the first member of $x + 10 = 12$ is decreased to x by subtracting 10, what must be done to the second member to preserve the equality?

Tell how the equation $x + 10 = 12$ may be solved.

3. Suppose that $x - 4 = 3$ and we wish to find the value of x. How much greater is x than $x - 4$?

If the first member of $x - 4 = 3$ is increased to x by adding 4, what must be done to the second member to preserve the equality? Tell how the equation may be solved.

The same number may be added to both members of an equation, or subtracted from both, without destroying the equality.

EXERCISES

398. First state what must be done to both members to change one member to x without destroying the equality; then solve:

1. $x + 6 = 8$	9. $x + 2 = 10$	17. $12 = 10 + x$
2. $x - 3 = 2$	10. $x - 5 = 11$	18. $15 = 11 + x$
3. $x - 4 = 5$	11. $x + 1 = 12$	19. $30 = 20 + x$
4. $x + 7 = 9$	12. $x - 7 = 10$	20. $14 = x + 10$
5. $x - 3 = 8$	13. $x + 9 = 12$	21. $22 = x + 20$
6. $x - 5 = 1$	14. $9 + x = 12$	22. $16 + x = 25$
7. $x + 5 = 7$	15. $5 + x = 15$	23. $20 + x = 24$
8. $x + 3 = 9$	16. $3 + x = 17$	24. $30 + x = 50$

399. 1. Just as $3 + 3 + 3 + 3 = 4 \times 3$, so $x + x + x + x = 4 \times x$. It is written $4x$, however, *without a multiplication sign between the figure and the letter.* What does $5x$ mean? $6x$?

2. If $x = 8$, what is the value of $2x$? of $3x$? of $\frac{5}{8}x$?

3. If $6x = 12$, what is the value of $1x$, or of x?

4. If $\frac{1}{3}x = 10$, what is the value of 3 times $\frac{1}{3}x$, or of x?

5. What must be done to both members of each of the following equations to give an equation whose first member is x?

$$\tfrac{1}{2}x = 3 \qquad \tfrac{1}{3}x = 5 \qquad 4x = 12 \qquad 5x = 35$$

Both members of an equation may be multiplied or divided by the same number without destroying the equality.

EXERCISES

400. First state what must be done to both members to change one member to x without destroying the equality; then solve:

1. $2x = 6$	7. $\frac{1}{2}x = 5$	13. $\frac{1}{2}x = 1.5$
2. $5x = 5$	8. $\frac{1}{3}x = 2$	14. $\frac{1}{2}x = 4.2$
3. $4x = 8$	9. $\frac{1}{4}x = 3$	15. $\frac{1}{4}x = .25$
4. $3x = 15$	10. $\frac{1}{5}x = 7$	16. $5x = 12$
5. $8x = 24$	11. $\frac{1}{6}x = 5$	17. $2x = 17$
6. $9x = 18$	12. $\frac{1}{8}x = 4$	18. $4x = 4.4$

WRITTEN EXERCISES

401. 1. What number increased by 6 is equal to 44?

SOLUTION

Let x = the number.

Then, $x + 6 = 44$

Subtracting 6 from both members, $x = 38$

Test. $38 + 6 = 44$.

2. What number increased by 15 is equal to 51?

3. What number decreased by 32 is equal to 60?

4. What number multiplied by 3 is equal to 78?

5. What number divided by 8 is equal to 62?

6. A certain number added to 87.5 gives 100. Find it.

7. Twelve times a certain number is 15. Find the number.

8. If $\frac{1}{6}$ of the number of books I have is 18, how many books have I?

9. If 20 is added to a certain number and 14 is subtracted from the sum, the result is 19. Find the number.

10. One half of a number, and 11 more, is equal to 37. Find $\frac{1}{2}$ of the number, then find the number.

WRITTEN EXERCISES

402. **1.** Solve the equation $\frac{3}{2}x = 15.$

FIRST SOLUTION

$$\tfrac{3}{2}x = 15$$

Dividing both members by 3, $\frac{1}{2}x = 5$
Multiplying both members by 2, $x = 10$

SECOND SOLUTION

By multiplying by 2 *before* dividing by 3, fractions may be avoided.

$$\tfrac{3}{2}x = 15$$

Multiplying both members by 2, $3x = 30$
Dividing both members by 3, $x = 10$
Test. $\frac{3}{2}$ of $10 = 15$.

Solve:

2. $\frac{3}{2}x = 9$ **6.** $\frac{3}{5}x = 21$ **10.** $\frac{5}{8}x = 15$

3. $\frac{4}{3}x = 8$ **7.** $\frac{2}{3}x = 30$ **11.** $\frac{7}{3}x = 21$

4. $\frac{5}{2}x = 10$ **8.** $\frac{4}{5}x = 28$ **12.** $\frac{7}{8}x = 63$

5. $\frac{2}{3}x = 14$ **9.** $\frac{5}{6}x = 20$ **13.** $\frac{6}{7}x = 48$

14. If $\frac{3}{4}$ of a certain number is 18, what is the number?

15. If $\frac{5}{8}$ of the number of pupils in a school is 250, what is the whole number of pupils?

16. A basket-ball team won 16 games, or $\frac{2}{3}$ of the games it played. Find the number of games it played.

17. If $\frac{3}{8}$ of the number of persons who went on an excursion to Niagara Falls were teachers, and 240 teachers went, find the whole number of persons who went.

18. Find the number of feet in the width of a street, if $\frac{3}{5}$ of the width, or 48 feet, lies between the curbstones.

19. On an elevated belt-line railway, $9\frac{3}{5}$ minutes, or $\frac{4}{15}$ of the time required to make a round trip, was consumed in stops. Find the number of minutes required to make a round trip.

403. **1.** How does the value of $6+4-4$ compare with 6? with $(6-4)+4$? with $(4-4)+6$?

2. Compare $5+7-2-3$ with $12-5$; with $5-(2+3)+7$; with $7-2-3+5$; with $7+(5-2-3)$; with $(-2-3+5)+7$.

3. 5, or $+5$, means $1+1+1+1+1$, that is, 5 units taken *additively*, or 5 **positive units**; -2 means $-1-1$, that is, 2 units taken *subtractively*, or 2 **negative units**. What does $+7$ mean? -3?

4. The expression $5+7-2-3$ is composed of 12 positive units and 5 negative units, and in whatever order or in whatever groups we unite the units, the 5 negative units will finally cancel 5 of the positive units, leaving 7 positive units, or $+7$.

5. Unite **terms** as indicated by their signs:

20	2 tens	2×10	$2t$	$2x$	$4x$
$+40$	$+4$ tens	$+4\times 10$	$+4t$	$+4x$	$-3x$
-30	-3 tens	-3×10	$-3t$	$-3x$	$+2x$

Such terms as $+4x$, $-3x$, and $+2x$ are called **like terms** because they have the same unit, x.

The multipliers, $+4$, -3, $+2$, are called **coefficients** of x.

In adding like terms, any number of negative units cancel an equal number of positive units.

EXERCISES

404. Add downward, then upward:

1. $+5x$	**2.** $+7x$	**3.** $+2x$	**4.** $+8x$	**5.** $+5x$
$+3x$	$-6x$	$-2x$	$-4x$	$+2x$
$-4x$	$+5x$	$+\ x$	$+9x$	$-7x$

Unite the terms of the following in various orders and by grouping in different ways, to find their sum:

6. $2x+5x-2x$

7. $7x-6x+4x$

8. $3x+8x-9x$

9. $6x-4x-2x+3x$

10. $8x-5x+4x-3x$

11. $9x-7x-3x+4x$

405. 1. Adding 7 to both members of the equation

$$x - 7 = 3,$$

we obtain $$x = 3 + 7, \text{ or } 10.$$

-7 has been removed from the first member, but reappears in the second member with the opposite sign.

2. Subtracting 5 from both members of the equation

$$x + 5 = 9,$$

we obtain $$x = 9 - 5, \text{ or } 4.$$

When $+5$ is removed, or **transposed**, from the first member to the second, what must be done to its sign?

3. Explain transposition of terms in the following:

$2x - 1 = 5;$	$3x + 2 = 11;$	$4x = 14 - 3x;$
$2x = 5 + 1.$	$3x = 11 - 2.$	$4x + 3x = 14.$

Any term may be transposed from one member of an equation to the other, provided its sign is changed.

WRITTEN EXERCISES

406. 1. Solve the equation $2x + 20 = 80 - 4x$.

FIRST SOLUTION

$$\begin{array}{rcl} 2x + 20 & = & +80 - 4x \\ \underline{4x, -20,} & & \underline{-20, +4x} \\ 6x & = & +60 \\ x & = & +10 \end{array}$$

SECOND SOLUTION

$$\begin{aligned} 2x + 20 &= 80 - 4x \\ 2x + 4x &= 80 - 20 \\ 6x &= 60 \\ x &= 10 \end{aligned}$$

The first step in solving an equation is to get the unknown terms into one member (usually the first member) and the known terms into the other member.

Adding $4x$ to both members (as in the first solution), or transposing $-4x$ from the second member to the first, and changing its sign (as in the second solution), places all unknown terms in the first member.

Subtracting 20 from both members, or transposing $+20$ from the first member to the second and changing its sign, places all known terms in the second member.

2. Solve $2x + 2x - 4x + 3x + 8x = 3x + 40$.

SOLUTION. — Since $2x + 2x - 4x = 0$, these terms may be omitted, or *canceled*, from the first member.

If the term $3x$ in the second member were transposed to the first member, it would become $-3x$ and so cancel $+3x$ in the first member. Consequently $+3x$ may be canceled from both members without transposing.

The equation becomes $8x = 40$

Dividing both members by 8, $x = 5$

Test. — We should always test the answer by finding whether the value obtained for x is such as to make the members of the original equation equal. Thus, if $x = 5$, the first member becomes

$$10 + 10 - 20 + 15 + 40, \text{ or } 55,$$

and the second $15 + 40$, or 55.

Solve and test:

3. $7x + 12 = 5x + 16$

4. $5x - 20 = 2x + 13$

5. $9x - 17 = 23 + x$

6. $22 - 6x = 40 - 8x$

7. $2x + 3x - 2x = 21$

8. $9x - 4x + 2x = 14$

9. $8x + 5x - 5x = 48$

10. $6x - 2x - x = 45$

11. $5x + 8x + 9x = 44$

12. $7x + 6x - 7x = 42$

13. $4x - 11 + 2x = 2x - 5$

14. $3x + 14 + 7x = 78 + 2x$

15. $44 - 3x - 2x = 79 - 12x$

16. $62 - 2x - 12 = 75 - 27x$

17. $14 - 21 - 30 = x - 2x - 21$

18. $22 - 15 + 21 = 2x - 4 + 6x$

19. $46 + 3x - 60 = 5x - 10 - 4x$

20. $8x - 20 - 2x = 30 - 2x + 14$

21. $5x + 16 - 6x = 16 + 24 - 6x$

22. $6x + 5x - 70 = 5x + 54 - 70$

23. $10x - 39 - 4x - 9x + 42 + 12x = 30 + 12 - 4x$

24. $16x + 12 - 75 + 2x - 12 - 110 = 8x - 50 - 25$

25. $11x - 60 + 5x + 17 - 2x - 3x + 41 = 106 + 2x$

26. $18x + 16 = 8 + 12x + 8 - 13 + 25x - 9 + 100 - 25x$

27. $14x - 35 = 9 - 11x + 4 + 16 - 10x + x + 136 - 16x$

28. Solve $\frac{1}{2}x - \frac{1}{3}x = 10 - \frac{1}{4}x$.

First Solution

	$\frac{1}{2}x - \frac{1}{3}x = 10 - \frac{1}{4}x$
Transposing,	$\frac{1}{2}x - \frac{1}{3}x + \frac{1}{4}x = 10$
Uniting terms,	$\frac{5}{12}x = 10$
Multiplying by 12,	$5x = 120$
Dividing by 5,	$x = 24$

If the steps of the first solution are taken in a different order, by multiplying by the l. c. d. 12 *before* transposing and uniting terms, the equation will be freed, or *cleared, of fractions*, thus:

Second Solution

	$\frac{1}{2}x - \frac{1}{3}x = 10 - \frac{1}{4}x$
Multiplying by 12,	$6x - 4x = 120 - 3x$
Transposing,	$6x - 4x + 3x = 120$
Uniting terms,	$5x = 120$
Dividing by 5,	$x = 24$

Solve and test:

29. $\frac{1}{4}x + \frac{1}{6}x = 10$

30. $\frac{3}{4}x - \frac{1}{9}x = 23$

31. $\frac{9}{10}x - 10 = \frac{2}{5}x$

32. $\frac{1}{2}x + \frac{7}{8}x = 440$

33. $\frac{2}{3}x = \frac{1}{3}x - \frac{7}{6}x + 27$

34. $\frac{4}{5}x + 12 = \frac{3}{8}x + 29$

35. $\frac{9}{4}x - 50 = 15 - \frac{1}{14}x$

36. $\frac{1}{2}x + \frac{1}{3}x + \frac{3}{4}x + \frac{4}{5}x = 143$

WRITTEN EXERCISES

407. 1. The sum of two numbers is 55 and the larger is 4 times the smaller. What are the numbers?

Solution. — Let x = smaller number.
Then $4x$ = larger number, and $x + 4x$, or $5x$, = sum.

Therefore,	$5x = 55$
	$x = 11$, smaller number
	$4x = 44$, larger number

2. Separate 116 into two parts, one of which shall be 3 times the other.

3. Two boys dug 160 clams. If one dug 3 times as many as the other, how many did each dig?

4. The water and steam in a boiler occupied 120 cubic feet of space, and the water occupied twice as much space as the steam. How many cubic feet of space did each occupy?

5. What number added to 5 times itself equals 12?

6. Two boys bought a boat for $45. One furnished 4 times as much money as the other. How much did each furnish?

7. During the summer they earned $52.50 by renting the boat. Find the amount of rent due each boy.

8. Separate 72 into two parts one of which shall be $\frac{1}{8}$ of the other.

SUGGESTION. — Let $x =$ the smaller part.

9. Separate 78 into two parts one of which shall be $\frac{1}{5}$ of the other.

10. The sum of a number and .04 of itself is 46.8. What is the number?

11. What number decreased by .35 of itself equals 52?

12. Messrs. Jones, Hollis & Frye invested $225,000 in a line of steamboats. Mr. Hollis invested 3 times as much as Mr. Jones, and Mr. Frye 5 times as much as Mr. Jones. How much did each invest?

13. At the end of a season they divided $2340 in profits according to their respective investments. How much did each receive?

14. It cost a mine owner $1.90 per ton to mine soft coal and ship it to market. The cost of shipping the coal was $.10 more per ton than the cost of mining it. Find the cost of mining it.

15. A wagon loaded with coal weighed 4200 pounds. The coal weighed 1800 pounds more than the wagon. How much did the wagon weigh? the coal?

16. The total height of a certain brick chimney in St. Louis is 172 feet. The height above ground is 2 feet more than 16 times the depth below. How high is the part above ground?

17. At the waterworks 2 large pumps and 4 small ones delivered 4800 gallons of water per minute. Each of the large pumps delivered 4 times as much water as each small pump. How many gallons per minute did each small pump deliver? each large pump?

18. One year 1500 violins were made in the United States. Twice as many were made in New York as in Massachusetts, and these two states made half of all that were made. How many violins were made in Massachusetts? in New York?

19. In lighting a hall a certain number of 16-candle power electric lamps and twice as many 20-candle power lamps were used. The total illumination amounted to 224 candle power. Find the number of lamps of each kind used.

20. It cost Mary 60¢ to send a telegram, at "30–2," or 30¢ for the first 10 words and 2¢ for each additional word. How many words did her message contain?

SOLUTION. — Let x = number of words in message.

Then, $x - 10$ = number of words in excess of 10 words.

Therefore, $30 + 2(x - 10) = 60$

Since $x - 10$ is 10 less than x, $2(x-10)$ is 20 less than $2x$.

$$30 + 2x - 20 = 60$$

$$x = 25$$

Test. — She paid 30¢ for 10 words and 30¢ for the 15 additional words.

Find the number of words in each of the following:

	Rate	Cost		Rate	Cost		Rate	Cost
21.	20–1	30¢	**24.**	30–2	66¢	**27.**	50–3	83¢
22.	25–2	39¢	**25.**	25–1	52¢	**28.**	60–4	96¢
23.	25–2	61¢	**26.**	40–3	55¢	**29.**	75–5	95¢

30. If you had 50¢, how many minutes could you converse over the telephone with a friend in a distant city, at 25¢ for the first three minutes and 5¢ for each additional minute?

31. Recently there were 435 piano factories in Germany. This was 15 more than 3 times the number in Berlin. How many piano factories were there in Berlin?

32. The total amount of plastering in the Agricultural Building at St. Louis was 64,000 square yards. There was 4000 square yards more than 4 times as much outside plastering as inside plastering. Find the amount of each.

33. What number increased by $\frac{1}{2}$ of itself equals 54?

34. If $\frac{7}{8}$ of a number is 112, what is the number?

35. What number decreased by $\frac{1}{5}$ of itself equals 84?

36. At one time $\frac{8}{19}$ of the miners in South Africa were Chinese. How many miners were there, if 21,000 were Chinese?

37. How many pounds of green hides are required to make 240 lb. of leather, if the leather weighs $\frac{3}{4}$ as much as the hides?

38. During a mild February, coke declined $\frac{3}{14}$ in market price. The price at the end of the month was $2.20 per ton. What was the price at the beginning of the month?

39. The width of the St. Lawrence River at Quebec at a point where it is spanned by a bridge is 1800 feet. This is $\frac{6}{11}$ of the length of the bridge. How long is the bridge?

40. The distance between two cities is 35 miles by rail. This is $\frac{5}{6}$ of the distance by boat. Find the distance by boat.

41. Of the steam vessels built on the Great Lakes one year, 21, or 5 less than $\frac{1}{3}$ of all, were of steel. How many steam vessels were built on the Lakes that year?

42. The Canadian, or Horseshoe Falls, in the Niagara River are 158 feet high. This is 8 feet more than $\frac{10}{11}$ of the height of the American Falls. Find the height of the American Falls.

43. A girl spent $\frac{1}{3}$ of her money for ice cream and $\frac{1}{4}$ of it for cake. She then had left 5 cents less than $\frac{1}{2}$ of her money. How much money had she at first?

SOLUTION. — Let x = number of cents she had at first.

Then she spent $\frac{1}{3}x$ cents for ice cream and $\frac{1}{4}x$ cents for cake, and had left $\frac{1}{2}x - 5$ cents.

Amount spent for ice cream + amount spent for cake + amount left = all her money.

Therefore,
$$\frac{1}{3}x + \frac{1}{4}x + \frac{1}{2}x - 5 = x$$
$$4x + 3x + 6x - 60 = 12x$$
$$x = 60$$

Test. — She had 60¢ at first, spent 20¢ for ice cream, 15¢ for cake, and had left 25¢, or 5¢ less than $\frac{1}{2}$ of 60¢.

44. After spending $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{6}$ of my money, I had 80 cents left. How much had I at first?

45. A man spent $700 more than $\frac{3}{5}$ of his income and then had $2200 left. What was his income?

46. A man deposited $\frac{2}{3}$ of his month's salary in a bank and spent $4 less than $\frac{4}{5}$ of the remainder. He then had $14 left. How much salary did he receive per month?

47. What is the number whose double, half, and third added to 33 will give 152?

48. Of the passengers on board an ocean liner $\frac{1}{6}$ were first-cabin passengers, $\frac{1}{3}$ were second-cabin passengers, and the rest, 805 persons, were steerage passengers. Find the whole number of passengers.

49. At a recent date Germany had 1025 ships of over 1000 tons capacity. There were 25 more than $\frac{1}{4}$ as many sailing vessels as steamers. Find the number of each.

50. If $\frac{3}{8}$ of the members of a boys' club live in the fifth ward, $\frac{1}{4}$ in the sixth, and the rest, 20 boys, in the seventh, how many boys belong to the club?

51. During March the A class deposited \$6 more in the school savings bank than the B class. $\frac{2}{3}$ of the A class deposits were equal to $\frac{4}{5}$ of the B class deposits. How much did each class deposit?

SOLUTION. — Let x = number of dollars deposited by B class.

Then, $x + 6$ = number of dollars deposited by A class.

Therefore $\frac{2}{3}(x + 6) = \frac{4}{5}x$.

We find $\frac{2}{3}$ of $(x + 6)$ by finding $\frac{2}{3}$ of x and $\frac{2}{3}$ of 6, and adding the results. Therefore $\frac{2}{3}(x + 6) = \frac{2}{3}x + 4$.

Hence,

$$\frac{2}{3}x + 4 = \frac{4}{5}x$$
$$10x + 60 = 12x$$
$$60 = 2x$$
$$x = 30,\ x + 6 = 36.$$

Test. — The A class deposited \$36; the B class, \$30. $\frac{2}{3}$ of \$36 = $\frac{4}{5}$ of \$30.

52. Separate 84 into two parts such that $\frac{1}{2}$ of the greater is equal to $\frac{5}{6}$ of the less.

SUGGESTION. — Let x = greater part, $84 - x$ = less part.

53. Separate 100 into two parts such that $\frac{1}{2}$ of the greater is equal to $\frac{3}{4}$ of the less.

54. The Massachusetts school term recently was 189 days. If $\frac{1}{3}$ of this term was $\frac{3}{5}$ of the school term in Alabama, what was the length of the school term in Alabama?

55. The weight of a locomotive, baggage car, and 7 coaches was 502.4 tons. The baggage car weighed $\frac{1}{4}$ as much as the locomotive, and each coach .27 as much as the locomotive. Find the weight of the locomotive; of the baggage car; of each coach.

56. The width of a room is $\frac{3}{4}$ of its length, and the distance around the room is 70 feet. Find the length and the width.

57. For every car load of iron ore dumped into a furnace, $\frac{7}{8}$ of a car load of coke was used for fuel and $\frac{3}{8}$ of a car load of limestone was used for a flux. In all 450 car loads of ore, coke, and limestone were used per day. How much of each was used per day in the furnace?

Solution of Equations in x and y

408. If 4 bananas and 9 oranges cost 35¢, and 4 bananas and 6 oranges cost 26¢, and it is required to find the cost of 1 of each, we may simplify the problem thus:

	4 bananas and 9 oranges cost 35¢	(1)
	4 bananas and 6 oranges cost 26¢	(2)
Subtracting,	3 oranges cost 9¢	(3)

By thus *eliminating* the cost of the bananas, we have obtained a relation, (3), more simple than either of the two given relations, (1) and (2), for it involves only one unknown cost.

Or, let x be the number of cents 1 banana costs, and y the number of cents 1 orange costs.

Then 4 bananas will cost $4x$ cents, 9 oranges $9y$ cents, etc.

$$4x + 9y = 35 \qquad (1)$$

$$4x + 6y = 26 \qquad (2)$$

Eliminating the x's,

$$3y = 9 \qquad (3)$$

$$y = 3, \text{ or 1 orange costs } 3¢.$$

Since $y = 3$, $9y$ in the first equation is equal to 27.

Substituting 3 for y in the first equation,

$$4x + 27 = 35 \qquad (4)$$

$$x = 2, \text{ or 1 banana costs } 2¢.$$

To **test** the answers we substitute 2 for x and 3 for y in the given equations.

Equation (1) becomes $8 + 27 = 35$, or $35 = 35$;

Equation (2) becomes $8 + 18 = 26$, or $26 = 26$.

Therefore the values obtained for x and y *satisfy* both equations, and the answers are correct.

This method of elimination is called **elimination by subtraction.**

In eliminating the x's on page 309, equal numbers, $4x + 6y$ and 26, are subtracted from both members of (1). Therefore the results are equal, giving a true equation, $3y = 9$.

If equals are subtracted from equals, the results are equal.

409. How must the equations $2x + 3y = 16$ and $5x - 3y = 19$ be combined to eliminate the y's?

$$\begin{array}{l} 2x + 3y = 16 \\ \underline{5x - 3y = 19} \\ 7x \qquad = 35 \end{array}$$

This method of elimination is called **elimination by addition.**

If equals are added to equals, the results are equal.

WRITTEN EXERCISES

410. 1. If $2x + 3y = 18$ and $2x + y = 10$, what is the value of each unknown number?

SOLUTION

$$2x + 3y = 18 \qquad (1)$$

$$2x + y = 10 \qquad (2)$$

Subtracting, $2y = 8$; $\therefore y = 4$

Substituting 4 for y in (2), $2x + 4 = 10$; $\therefore x = 3$

Test. — Substituting 3 for x and 4 for y in (1) and (2),

(1) becomes $6 + 12 = 18$, or $18 = 18$;

(2) becomes $6 + 4 = 10$, or $10 = 10$.

NOTES. — 1. The sign $\therefore$ means "*therefore*."

2. The value of y may be substituted in *either* of the given equations.

Solve the following and test results:

2. $\begin{cases} 5x + 2y = 22 \\ 5x + y = 21 \end{cases}$

3. $\begin{cases} 3x + 4y = 23 \\ 3x + 2y = 19 \end{cases}$

4. $\begin{cases} 3x - 4y = 16 \\ 5x + 4y = 48 \end{cases}$

5. $\begin{cases} 6x + 5y = 70 \\ x - 5y = 0 \end{cases}$

Solve and test:

6. $\begin{cases} 4x+5y=32 \\ 2x+5y=26 \end{cases}$

10. $\begin{cases} 5x-4y=8 \\ 3x-4y=0 \end{cases}$

7. $\begin{cases} 7x-2y=22 \\ 3x+2y=18 \end{cases}$

11. $\begin{cases} 8x+5y=18 \\ 8x+3y=14 \end{cases}$

8. $\begin{cases} 6x+7y=13 \\ 6x+\ \ y=7 \end{cases}$

12. $\begin{cases} 7x-3y=39 \\ 5x-3y=27 \end{cases}$

9. $\begin{cases} 9x-2y=41 \\ 7x-2y=31 \end{cases}$

13. $\begin{cases} 5y-4x=9 \\ 6y+4x=46 \end{cases}$

14. If $2x+3y=16$ and $5x+4y=33$, find x and y.

SOLUTION

$$2x+3y=16 \qquad (1)$$
$$5x+4y=33 \qquad (2)$$

We may eliminate either x or y. If we choose to eliminate x, we must first prepare the equations, so that x may have the same coefficient in each. Multiplying both members of (1) by 5, and both members of (2) by 2,

$$10x+15y=80 \qquad (3)$$

and $$\underline{10x+\ \ 8y=66} \qquad (4)$$

Subtracting (4) from (3), $7y=14$; $\therefore y=2$

Substituting 2 for y in (1), $2x+6=16$; $\therefore x=5$

Test.—These values give $10+6=16$ and $25+8=33$ in (1) and (2).

NOTE.—To eliminate y instead of x, proceed as follows:

Multiplying (1) by 4, $8x+12y=64$

Multiplying (2) by 3, $15x+12y=99$

Subtracting the *upper equation from the lower*, thus avoiding negative coefficients,

$$7x=35; \ \therefore x=5$$

Substituting 5 for x in (1), $10+3y=16$; $\therefore y=2$

Solve and test:

15. $\begin{cases} 9x + 2y = 20 \\ 3x + y = 7 \end{cases}$

16. $\begin{cases} 6x + 5y = 28 \\ 2x + 3y = 12 \end{cases}$

17. $\begin{cases} 7x + 4y = 40 \\ 3x + 2y = 18 \end{cases}$

18. $\begin{cases} 10x + 3y = 62 \\ 6x + 4y = 46 \end{cases}$

19. $\begin{cases} 11x + 8y = 37 \\ 5x + 6y = 18 \end{cases}$

20. $\begin{cases} 18x + 2y = 9\frac{2}{3} \\ 4x + 3y = 3 \end{cases}$

Equations in this book are intended to give aid in solving the more difficult problems of arithmetic, and are not given as exercises for their own sake. To keep within the limits of arithmetic such arithmetical problems or equations as,

"Find two numbers whose sum is 10 and whose difference is 2," or $\begin{cases} x + y = 10 \\ x - y = 2 \end{cases}$

should be solved by eliminating by addition or subtraction the unknown number found in the *negative* term or terms, if there are any.

Solve and test:

21. $\begin{cases} x + y = 10 \\ x - y = 2 \end{cases}$

22. $\begin{cases} 5x + 2y = 49 \\ 3x - 2y = 23 \end{cases}$

23. $\begin{cases} 4x - y = 27 \\ x - y = 3 \end{cases}$

24. $\begin{cases} y + 2x = 18 \\ y - 2x = 2 \end{cases}$

25. $\begin{cases} 2y - 3x = 5 \\ 5y + 4x = 93 \end{cases}$

26. $\begin{cases} 4x - 7y = 12 \\ 3x + 5y = 50 \end{cases}$

27. Solve the equations $\frac{3x}{2} - \frac{y}{3} = 1$, and $\frac{x}{4} + \frac{y}{5} = 4$.

Suggestion. — Multiplying the members of each equation by the l. c. m. of the denominators in that equation will *clear the equation of fractions.*

Thus, 6 times $\frac{3x}{2} = 9x$; 6 times $\frac{y}{3} = 2y$; 6 times $1 = 6$. Multiplying the members of the first equation by 6, then, changes it to $9x - 2y = 6$.

Solve and test:

28. $\begin{cases} \frac{x}{2}+\frac{y}{8}=7 \\ \frac{x}{3}+\frac{y}{4}=6 \end{cases}$

30. $\begin{cases} \frac{x}{3}-\frac{y}{9}=2 \\ \frac{x}{6}+\frac{y}{6}=5 \end{cases}$

29. $\begin{cases} \frac{x}{4}+\frac{y}{2}=13 \\ \frac{x}{5}-\frac{y}{8}=2 \end{cases}$

31. $\begin{cases} \frac{2x}{3}-\frac{3y}{4}=6 \\ \frac{3x}{4}-\frac{5y}{8}=12 \end{cases}$

Find two numbers related to each other as follows:

32. Sum $=14$; difference $=8$.

33. Sum of 2 times the first and 3 times the second $=34$; sum of 2 times the first and 5 times the second $=50$.

34. Sum $=18$; sum of the first and 2 times the second $=20$.

35. A grocer sold 2 boxes of raspberries and 3 of cherries to one customer for 54¢, and 3 boxes of raspberries and 2 of cherries to another for 56¢. Find the price of each per box.

36. A druggist wishes to put 500 grains of quinine into 3-grain and 2-grain capsules. He has 220 capsules. How many capsules of each size can he fill?

37. On the Fourth of July, 850 glasses of soda water were sold at a fountain, some at 5¢ each, the others at 10¢ each. The receipts were \$55. How many were sold at each price?

38. A fruit dealer bought 36 pineapples for \$2.50. He sold some at 12¢ each and the rest at 10¢ each, thereby gaining \$1.50. How many did he sell at each price?

39. An errand boy went to the bank to deposit some bills for his employer. Some of them were 1-dollar bills, and the rest 2-dollar bills. The number of bills was 38 and their value was \$50. Find the number of each.

40. A man noticed that a 15-word day message by telegraph cost him 40¢ and a 22-word day message 54¢, between the same two cities. Find the charge for the first ten words and for each additional word.

41. When 2 baskets of Delaware peaches cost 15¢ more than 3 of Whitestone peaches, and 3 baskets of Delaware peaches cost 45¢ less than 6 of Whitestone peaches, what is the price of each per basket?

42. At a factory where 1000 men and women were employed, the average daily wage was $2.50 for a man and $1.50 for a woman. If labor cost $2340 per day, how many men were employed and how many women?

43. The receipts from 300 tickets for a musical recital were $100. Adults were charged 50¢ and children 25¢, each. How many tickets of each kind were sold?

44. It required 60 inches of tape to bind the four edges of a card on which a photograph was mounted. The length of the card was 6 inches greater than the width. How many inches long was the card? how many inches wide?

45. A lieutenant of the U. S. navy received $150 per month while on sea duty and $127.50 per month while on shore duty. A lieutenant's salary for a year amounted to $1620. How many months was he on sea duty? on shore duty?

46. The great columns of Bedford stone in the Indianapolis postoffice building weigh 94 tons each, including the shafts and the capitals resting on them. Each shaft weighs 74 tons more than its capital. Find the weight of a shaft; of a capital.

47. The receipts from a football game were $700. Admission tickets to the grounds cost 50¢, and to the grand stand, 25¢ in addition. If twice as many persons had purchased tickets for the grand stand, the receipts would have been $800. How many tickets of each kind were sold?

DENOMINATE NUMBERS

REVIEW

EXERCISES

411. 1. Give these tables: liquid measures; dry measures; avoirdupois weight; troy weight; time measures; circular measures.

2. How many feet are there in 220 yards? in 96 inches?

3. What part of a gallon is 3 quarts? 5 pints?

4. How many quarts are there in 2 bushels? in $2\frac{1}{2}$ pecks?

5. Reduce 4 T. 5 cwt. to tons; to pounds; to hundredweight.

6. What part of an ounce is 10 pennyweights? Find the weight of a dozen silver spoons, each weighing 10 pwt.

7. Reduce 12 minutes to seconds; to a decimal of an hour.

8. Reduce $3° 45'$ to degrees; to minutes; $150''$ to minutes.

9. What part of a right angle is $45°$? $22° 30'$? $30°$? $60°$?

WRITTEN EXERCISES

412. Reduce:

1. 220 yd. to inches
2. $184\frac{3}{4}$ tons to pounds
3. 12 bu. 3 pk. to pecks
4. 1 day to minutes
5. 5.8 ft. to inches
6. $2° 24' 18''$ to seconds
7. Reduce 14,420 pounds to tons and pounds.
8. Reduce 30,000 seconds to hours and minutes.
9. What part of $360°$ is $67° 30'$? $86° 24'$? $7° 12'$?
10. What part of a ton is 4 cwt. 80 lb.? 16 cwt. 25 lb.?

Reduce:

11. 150 qt. 1 pt. to pints.

12. 2000 min. to hours.

13. 15,136 pounds to tons.

14. 275 ft. 10 in. to inches.

15. $365\frac{1}{4}$ days to weeks, days, and hours.

16. $\frac{1}{8}$ of a right angle to degrees and minutes.

17. 11,225 feet (height of Mt. Hood) to miles and feet.

18. Add 2 T. 5 cwt.; 1 T. 12 cwt.; 4 T. 15 cwt.; 3 T. 8 cwt.

19. Subtract 3 T. 17 cwt. from 10 T.; 22 T. 14 cwt. from 25 T.; 17 lb. 11 oz. from 42 lb.

20. Multiply 12 ft. 8 in. by 6; 2 yd. 27 in. by 4.

21. Divide 85 ft. 8 in. by 4; 181 ft. $8\frac{1}{2}$ in. by 7.

22. How much less than a right angle is $77° 42'$?

23. How much less than 2 right angles is $154° 28' 33''$?

24. Find $\frac{1}{4}$ of $16° 32' 30''$; $\frac{3}{4}$ of $16° 32' 30''$; $\frac{5}{8}$ of 125 ft. 4 in.

25. Find 5 times 11 lb. 10 oz. (av.); $\frac{5}{6}$ of 11 lb. 10 oz. (av.).

Find the time from to-day to the following dates:

26. Jan. 1, 1920

27. Feb. 8, 1915

28. Sept. 21, 1925

29. Apr. 18, 1930

30. Oct. 28, 1918

31. Aug. 11, 1922

32. The leakage from a 63-gallon hogshead of rock candy sirup was 2 qt. 1 pt. How much sirup remained?

33. If a skillful operator stamps the name on 24 gross 4 dozen pens per hour, how many will he stamp in 9 hours?

34. A man picked 94 bu. 2 pk. of cranberries in 12 days. What quantity did he pick per day, on the average?

35. My watch runs 36 hours after being wound. How many times must I wind it during April?

36. The balance wheel of a watch should make 5 oscillations per second. How many is that per hour?

37. If the balance wheel makes 17,985 oscillations per hour, how many minutes and seconds will the watch lose in a week?

38. A man owned 84 A. 82.5 sq. rd. of land and bought 16 A. 147.5 sq. rd. Afterward he sold $\frac{1}{3}$ of his land. How much land did he then own?

39. If a passenger locomotive consumes 86 lb. 9 oz. of coal per mile, how much coal will be consumed on a trip of 95 miles?

40. If one pound of corn yields 4 ounces of starch, how much starch can be obtained from a bushel of corn weighing 56 pounds?

41. An Indiana factory, running continuously, made 11,250 dozen fruit jars in 5 da. 15 hr. How many single jars did it make per day?

42. Find the value of the 312,500 great gross of brass buttons made in this country one year, at 20¢ per gross.

43. How many bushels of charcoal weighing 15 pounds per bushel are required to make a ton of gunpowder that is $\frac{3}{4}$ saltpeter, $\frac{1}{10}$ sulphur, and the remaining part charcoal?

44. One year 8028 gross of fountain pens were manufactured in this country. If the average wholesale value of each fountain pen was 78.9¢, find the value of all.

45. A shipment of candles to Calcutta consisted of 124 cases, each containing 21 lb. 14 oz. of candles. How much did the shipment of candles weigh?

46. Each case held 25 packets of 6 candles each. Find the weight of a packet; of a candle.

47. A stamp-canceling machine, working 1 hr. 15 min. 12 sec., canceled 13,912 stamps. Find its capacity per minute.

48. A vessel under its own steam passed through the Suez Canal in 15 hr. 45 min. The canal is 90 miles long. What was its rate of passage per hour to the nearest tenth of a mile?

49. How long will it take a torpedo to reach its mark 1870 yards away, if it travels $\frac{1}{2}$ of a mile per minute?

50. In the West Indies 120 barrels of limes were obtained from an acre of land. Each barrel yielded 8 gallons of juice, and each gallon of juice yielded $\frac{3}{4}$ of a pint of concentrated juice. How many gallons of concentrated juice were obtained from an acre of land?

FOREIGN MONEY

413. **Canada** has a decimal currency, the table being the same as that for United States money.

414. The unit of **English** or **sterling money** is the **pound** or **sovereign.**

The system is not a decimal system.

4 farthings (far.)	=	**1 penny (*d.*)**
12 pence	=	**1 shilling (*s.*)**
20 shillings	=	**1 pound (£)**

Farthings (not coined) are usually written as fractions of a penny.

The sign £ precedes the number of pounds.

415. The unit of **French money** is the **franc.**

The system is a decimal system.

100 centimes (c.) = 1 franc (fr.)

Centimes is pronounced *sän'-tēms'*.

The monetary unit of Belgium and Switzerland is the same as that of France. The following units are identical in weight and fineness with the franc: the *peseta* (Spain), the *lira* (Italy), the *drachma* (Greece), and the *bolivar* (Venezuela).

416. The unit of **German money** is the **mark.**

100 pfennigs (pf.) = 1 mark (M.)

The German money system is a decimal system.

417. These official **equivalents** may be learned.

1 pound = $4.8665
1 franc = $0.193
1 mark = $0.238

In estimates we think of the pound as $5, the franc as 20¢, and the mark as 25¢.

EXERCISES

418. Reduce:

1. £10 to shillings.
2. £1 10*s*. to pence.
3. ½ sovereign to pence.
4. 18.40 M. to pfennigs.
5. 450 c. to francs.
6. 6.75 fr. to centimes.
7. How many articles costing 6*d*. each can be bought for £1?
8. A man changed a 5-pound note and 3 half sovereigns for shillings. How many shillings did he receive?
9. How many toys at 60 pf. each can be bought for 12 M.?
10. How many handkerchiefs costing 75c. each can be bought for 1.50 fr.? for 3 fr.? for 7.5 fr.?

Find the cost of the following in the foreign money mentioned; then estimate the cost in United States money:

11. 4 Swiss statuettes purchased in Geneva, at 2.50 fr. each.
12. 30 bouquets purchased in Florence, at 50 centesimi (½ lira) each.

WRITTEN EXERCISES

419. Find, to the nearest cent, the value of:

1. £420
2. £24.75
3. £56.125
4. 1800 fr.
5. 144.50 fr.
6. 850.75 M.
7. 36.25 lire
8. 49.80 pesetas
9. 2000 drachmas
10. Find the value in U.S. money of a Bank of England note for £500; of a Bank of Naples note for 1000 lire.

11. Reduce £24 8*s*. 4*d*. to United States money.

$$\begin{array}{r|l} & \text{£} \;\; s. \;\; d. \\ & 24 \;\; 8 \;\; 4 \\ 12 & \underline{\qquad\quad 4} \\ 20 & \underline{\qquad 8.33+} \\ & 24.417- \end{array}$$

Since 1*d*. $= \frac{1}{12}$*s*., 4*d*. $= \frac{4}{12}$*s*. = .33+*s*. Bringing down the 8*s*., we have 8.33+*s*. Since 1*s*. = £$\frac{1}{20}$, 8.33+*s*. = £$\frac{8.33+}{20}$ = £.417−. Bringing down the £24, we have £24.417−. Then we multiply $4.8665 by 24.417, obtaining $118.83−.

NOTE. — When pounds are expressed to the nearest third decimal place, the error is within ½ of £.001, equal to about ¼ cent. Hence *pounds to the nearest thousandth give United States money to the nearest cent.*

Reduce to United States money to the nearest cent:

12. £8 3*s*. 4*d*.	**15.** £12 7*s*. 7*d*.	**18.** £100 8*s*. 1½*d*.
13. £5 2*s*. 6*d*.	**16.** £25 4*s*. 3*d*.	**19.** £125 6*s*. 10*d*.
14. £7 6*s*. 9*d*.	**17.** £66 8*s*. 5*d*.	**20.** £440 18*s*. 9½*d*.

21. Reduce $225.50 to English money.

$225.50 ÷ $4.8665 = 46.337+, the number of pounds
.337 × 20*s*. = 6.74*s*., and .74 × 12*d*. = 8.88*d*.

Hence, to the nearest penny, $225.50 = £46 6*s*. 9*d*.

NOTE. — To obtain the result correct to the nearest penny, the division need be carried only to the nearest third decimal place.

Reduce to English money, to the nearest penny:

22. $1000	**25.** $48.25	**28.** $77.14	**31.** $122.43
23. $3200	**26.** $62.75	**29.** $84.40	**32.** $760.50
24. $5500	**27.** $90.10	**30.** $50.07	**33.** $325.85

Add:

34. £865 4*s*. 8*d*.; £82 9*s*. 10½*d*.; £48 18*s*. 5½*d*.

35. £642 3*s*. 2*d*.; £66 0*s*. 6½*d*.; £34 15*s*. 3*d*.

36. Subtract each amount of money in exercise 35 from the amount written above it in exercise 34.

37. In a London fruit store I purchased 15 pounds of Baldwin apples @ 3*d.* and 20 pounds of California Newtowns @ 5*d.* How much change did I receive out of a sovereign?

38. Find the cost of twelve tons of house coal in London at £ 1 6*s.* per ton. Find the cost per ton in our money.

39. The cost of bread in some English cities is a penny per pound. How much does a baker receive for 85 1-lb. loaves of bread, 42 2-lb. loaves, and 64 $1\frac{1}{2}$-lb. loaves?

40. A party of three traveled by train from Bristol to London, 194 miles, at 2*d.* each per mile. Find the fare for all in English money, and the fare for each in United States money.

41. A cloth weaver in Germany received 20 pfennigs per hour. He worked 10 hours a day and 6 days a week. How much did he earn per week in U. S. money?

42. By some German railways, tea and coffee are sold to employees at 2 pfennigs per cup. If an employee buys 8 cups per week, how much will he expend per year of 52 weeks?

43. The toll on loaded vessels passing through the Suez Canal is 8 francs 50 centimes per ton of the vessel's capacity. Find the toll on a loaded vessel whose capacity is 3964 tons.

44. The receipts of the Suez Canal Company for tolls, one year, were 103,120,268 francs. Find the receipts in U. S. money.

45. An American tobacco company was fined, in Germany, 5000 marks for violating the laws of competition. Compute the fine in U. S. money.

46. The London Metropolitan Railway cost £ 500 per yard. A New York street railway cost $214.13 per yard. Find the difference in cost per yard in U. S. money.

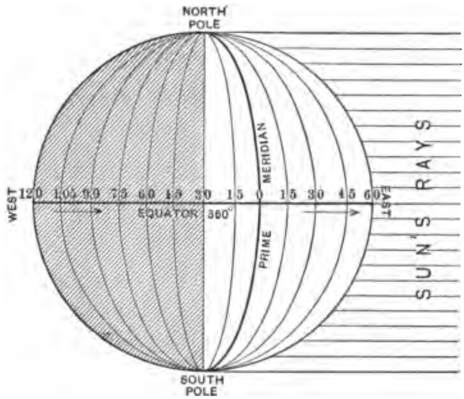
47. Great Britain built the New Zealand Government Railway at a cost of £ 7779 per mile. The length was 2212 miles. Find the cost in sterling money, and in U. S. money.

LONGITUDE AND TIME

420. An imaginary line passing north and south from one pole of the earth to the other is called a **meridian.**

421. The imaginary line around the earth running east and west halfway between the poles is called the **equator.**

Since the equator is the circumference of a circle, distances along it are measured in *degrees.*



422. Distances east and west are measured from some selected meridian, called the **prime meridian.**

The prime meridian commonly used is that passing through the Royal Observatory at Greenwich, England.

423. Distance east or west of the prime meridian, measured in degrees along the equator, is called **longitude.**

East longitude is the distance *east* of the prime meridian; *west longitude* is the distance *west* of it.

424. 1. Since the earth rotates on its axis once in 24 hours, any meridian passes through 360° in that time.

Then how many degrees of longitude pass under the sun's rays during 24 hours? during 1 hour?

2. Since 15° of longitude pass under the sun's rays in 1 hour, what part of a degree passes in 1 minute? How many minutes (′) of longitude pass in 1 minute of time?

3. Since 15′ of longitude pass under the sun's rays in 1 min., how many seconds (″) of longitude pass in 1 sec. of time?

425. As developed in the foregoing, the relation existing between *longitude* and *time* may be expressed briefly as follows:

360° of longitude correspond to 24 hours of time.
15° of longitude correspond to 1 hour of time.
15′ of longitude correspond to 1 minute of time.
15″ of longitude correspond to 1 second of time.

426. When the sun's rays are vertical at any point on a meridian, it is noon at all places on that meridian.

Since the earth turns from west to east, the sun *appears* to move from east to west. Therefore when it is noon at any place it is *before* noon, or **earlier**, at all places **west**, because the sun has not yet reached the meridians of those places. It is *after* noon, or **later**, at all places **east**, because the sun has already crossed the meridians of those places.

EXERCISES

427. 1. What is the difference in longitude between a place 15° west of Greenwich and a place 30° west?

Which place has the earlier time and how much earlier?

2. How far westward must one go to pass from meridian 15° E. to Greenwich? from Greenwich to 15° W.? How many degrees west of 15° E. is 15° W.? Compare their times.

Compare the times of places on the following meridians:

3. 15° W. and 45° W.
4. 15° W. and 60° W.
5. 45° E. and 30° E.
6. 35° E. and 20° E.
7. 42° E. and 12° E.
8. 30° E. and 30° W.
9. 10° E. and 5° W.
10. 20° E. and 10° W.
11. 40° E. and 20° W.
12. 30° E. and 45° W.

TABLE OF LONGITUDES

West Longitudes	°	′	″	East Longitudes	°	′	″	East Longitudes	°	′	″
Boston	71	03	50	Batavia	106	48	37	Bombay	72	48	56
New York	74	00	24	Melbourne	144	58	35	Cape Town	18	28	40
Washington	77	03	06	Tokyo	139	44	30	Berlin	13	23	44
Chicago	87	36	45	Shanghai	121	28	55	Hamburg	9	58	25
Denver	104	58	00	Manila	120	58	06	Amsterdam	4	53	04
San Francisco	122	24	32	Canton	113	16	30	Paris	2	20	14

In the following exercises refer to this table for longitudes and find answers to the *nearest* second.

WRITTEN EXERCISES

428. 1. When it is noon at San Francisco, what is the time at Washington?

$$\begin{array}{r} 122^\circ\ 24'\ 32'' \\ \underline{77^\circ\ \ 3'\ \ 6''} \\ 15\overline{)45^\circ\ 21'\ 26''} \\ 3^\circ\ \ 1'\ 26'' \end{array}$$

3 hr. 1 min. 26 sec. is the difference in time.

The difference in longitude is found to be 45° 21′ 26″.

Since 15° corresponds to 1 hr., 15′ to 1 min., and 15″ to 1 sec., the difference in time between San Francisco and Washington is as many hours, minutes, and seconds, respectively, as there are degrees, minutes, and seconds in $\frac{1}{15}$ of the difference in longitude.

Therefore the difference in time is 3 hr. 1 min. 26 sec., and since Washington is *east* of San Francisco, the time is *later* in Washington; that is, when it is noon at San Francisco it is 1 min. 26 sec. past 3 P.M. at Washington.

Find the true time in each of the following cities when the sun is on the meridian of Washington:

2. Boston
3. New York
4. Chicago
5. Berlin
6. Paris
7. London (Greenwich)
8. Hamburg
9. Amsterdam
10. Cape Town

11. What is the longitude of St. Petersburg, if its time is 6 hr. 57 min. 19 sec. later than that of New York?

hr.	min.	sec.					
6	57	19		104°	19′	45″	
		15		74°	00′	24″	W. (N.Y.)
104	19	45		30°	19′	21″	E. (St.P.)
104°	19′	45″, diff. in longitude					

Since 1 hr. corresponds to 15°, 1 min. to 15′, and 1 sec. to 15″, the difference in longitude between New York and St. Petersburg is as many degrees, minutes, and seconds, respectively, as there are hours, minutes, and seconds in 15 times the difference in time between the two places.

Therefore the difference in longitude is 104° 19′ 45″, and since the time of St. Petersburg is *later* than that of New York, St. Petersburg is *east* of New York; but New York is only 74° 00′ 24″ west of the prime meridian, and since the difference in longitude is greater than this, St. Petersburg must be east of the prime meridian. Subtracting, we find the longitude of St. Petersburg to be 30° 19′ 21″ E.

Find the longitude of places having the following times when the sun is on the meridian of Washington:

12. 2:21 P.M.	**15.** 10:18 A.M.	**18.** 7:32$\frac{1}{2}$ A.M.
13. 4:36 P.M.	**16.** 9:54 A.M.	**19.** 1:05$\frac{3}{4}$ P.M.
14. 3:42 P.M.	**17.** 6:09 A.M.	**20.** 15 sec. before 8 A.M.

21. The longitude of Norwich, Eng., is 1° 18′ W.; of Norwich, Conn., 72° 4′ W. How much later does the sun cross the meridian of the American city than that of the English city?

22. An astronomer in Boston observed the moon entering the shadow of an eclipse at 1:13 A.M. At what times did astronomers in Chicago, Denver, and San Francisco observe the same?

23. When the people of Boston were celebrating the passing of the 19th century at midnight Dec. 31, 1900, how long had Parisians been living in the 20th century? How much of the 19th century was left for the people of Galveston, 94° 47′ 26″ W.?

24. At noon a ship's chronometer carrying Greenwich time indicated 1:05 P.M. In what longitude was the ship?

25. A ship's chronometer carrying Greenwich time was 35 minutes slow Saturday noon and 13 minutes fast the following Tuesday noon. In what longitude was the ship at each observation? How far east or west did she sail?

26. When it is 6 P.M. Jan. 10 at San Francisco, what is the time and date at Tokyo?

First Solution

Since San Francisco is 122° 24′ 32″ W. and Tokyo is 139° 44′ 30″ E., Tokyo is 262° 9′ 2″ *east* of San Francisco, and consequently has 17 hr. 28 min. 36 sec. *later* time. Counting this time on from 6 P.M. Jan. 10, the time at Tokyo is found to be 28 min. 36 sec. after 11 A.M. Jan. 11.

Second Solution

Reckoning in the other direction, Tokyo is 360° – 262° 9′ 2″, or 97° 50′ 58″ *west* of San Francisco, and its time is therefore 6 hr. 31 min. 24 sec. *earlier*.

This apparent contradiction of the first solution is explained thus: the time of Tokyo is 17 hr. 28 min. 36 sec. later than that of San Francisco, or a day later *lacking* 6 hr. 31 min. 24 sec. A day later than 6 P.M. Jan. 10 is 6 P.M. Jan. 11, and a day later *less* 6 hr. 31 min. 24 sec. gives the Tokyo time, 28 min. 36 sec. after 11 A.M. Jan. 11..

Note. — When a ship sails *westward* over the 180th meridian the calendar is set *forward* one day; sailing *eastward* its calendar is set back one day. An irregular line corresponding in general with this meridian marks the place where the calendar changes. It is called the **international date line.**

When the sun is on the meridian of New York, on the first day of May, find the true time and date in:

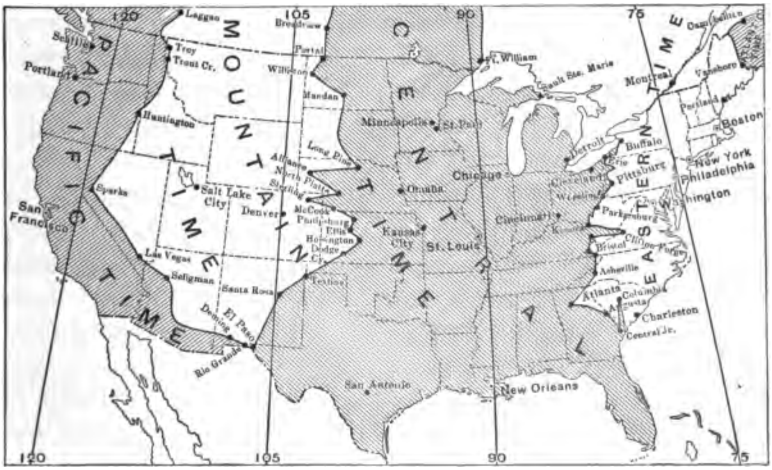
27. Manila	**29.** Canton	**31.** Bombay
28. Batavia	**30.** Melbourne	**32.** Cape Town

33. At noon, March 10, a ship weighed anchor at Hong-Kong, 114° 10′ 02″ E., and sailing eastward reached Honolulu, 157° 51′ 34″ W., at noon March 25. How long was the voyage?

Standard Time

429. In 1883, the railroads of the United States and Canada agreed upon a system of **standard time** that has come into general use because of its convenience. Under this system there are five **time belts**, each approximately 15° of longitude in width, and each having the time of its central meridian.

Each railway has selected the most convenient city on its own road at which to change from the standard time of one belt to that of the next. Since such towns on the various roads are seldom on the same meridian, the line connecting them forms an irregular boundary between the various belts; hence these time belts, shown on the following map, are neither equal in size nor uniform in shape.



430. The time belts are called **Atlantic**, with the time of the meridian of 60° W.; **Eastern**, with the time of the meridian of 75° W.; **Central**, with the time of the meridian of 90° W.; **Mountain**, with the time of the meridian of 105° W.; and **Pacific**, with the time of the meridian of 120° W.

It is evident that the time of the various belts differs by *hours*, the minute and second hands of all correct timepieces being the same at any instant. Central time is 1 hr. earlier than Eastern time; Mountain time is 1 hr. earlier than Central time; and Pacific time is 1 hr. earlier than Mountain time.

Correct time is telegraphed each day to all parts of the United States from the Naval Observatory at Washington.

431. Standard time has been adopted by most civilized governments of the world, the time meridian chosen being, with few exceptions, some multiple of 15° from the prime meridian through Greenwich.

In exercises refer to the map on page 327 for the time meridians of cities in the United States. The standard time meridian for each foreign city will be given in parentheses in the exercises where it is needed.

WRITTEN EXERCISES

432. 1. A certain business transaction was reported by telegraph from Chicago at 10:30 A.M., to New York, New Orleans, San Antonio, and Portland, Ore. Allowing no time for transmission, when did the message reach each city?

2. If the news of the opening of the St. Louis Exposition at 12:15 P.M. was immediately telegraphed all over the world, at what time did Salt Lake City receive it? Portland, Me.? Manila (120° E.)? Bombay (75° E.)? Berlin (15° E.)? London (0°)? Tokyo (135° E.)?

3. The news of the *Maine* disaster was cabled to Madrid (0°) from Havana (75° W.) at 2 A.M. Neglecting the time of transmission, find the time when the news reached Madrid.

4. Dewey's flagship, the *Olympia*, opened fire on the defenses of Manila (120° E.) at 5:41 A.M., May 1, 1898, and ceased firing at 7:40 A.M. to allow time for breakfast. Between what hours and on what day, Washington time, did the assault occur?

5. The first news of the assault reached Madrid (0°) at 6:20 P.M. the same day. How long was this after the *Olympia* opened fire?

6. Cable communication with Manila was severed at 10 A.M., London time, May 2, 1898. What was the clock time in Manila (120° E.) when the cable was cut?

7. A London paper received a dispatch from Cairo (30° E.) reporting an explosion in the British barracks. If the dispatch was received in London at 3:15 P.M., at what time was it sent?

8. If $1\frac{1}{4}$ hours later the news was cabled to New York (75° W.), find the time that it reached New York.

9. The first shock of the earthquake at San Francisco April 18, 1906, was recorded at 5:12 A.M. at the University of California, and at 20 seconds after 8:19 A.M. at Washington. How long did it take the shock to cross the continent?

10. The dispatch regarding the final surrender of Port Arthur (120° E.) was sent from that place at 10 P.M., Jan. 2, 1905. At what time was the news received in Tokyo (135° E.)? in St. Petersburg (30° E.)? in London? in New York?

11. The *Atlantic*, the winner of a yacht race from Sandy Hook (75° W.) to the Lizard, Eng. (0°), started at 12:15 P.M., May 17, and finished at 9:18 P.M., May 29. Find the *Atlantic's* time.

12. A ship in distress reports by wireless telegraph that she is in longitude 21° W. and has been pumping water 5 hours. A station on the English coast catches the message immediately at 2:27 P.M., Greenwich time. When did the pumps begin work?

13. Paris uses the time of her meridian, 2° 20′ 14″ E. The Paris stock exchange, or *Bourse*, closes at 3 P.M. At what time will closing quotations reach New York, if cabled immediately?

14. At 12 P.M., Saturday, Dec. 31, 1904, the chief of the U. S. Signal Service sent a message from Washington around the world *via* Chicago, Denver, San Francisco, Manila (120° E.), Tokyo (135° E.), Melbourne (150° E.), Bombay (75° E.), Berlin (15° E.), and London. What was the date and the clock time in each city at the instant of sending the message?

METRIC SYSTEM

433. The **metric system** is a *decimal* system of weights and measures, having for its **principal unit** the **meter**, to which all other units are related.

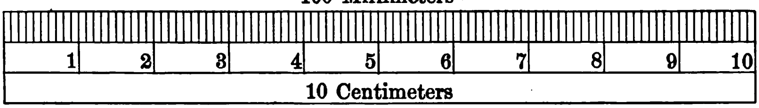
This system originated in France. Its use is required by law in many countries, and permitted in many others, including the United States. Congress has made it the official system in our Philippine possessions.

Its most general use is in the arts and sciences, where its convenience and accuracy have specially commended it.

MEASURES OF LENGTH

434. The primary unit of **length** is the **meter**, the unit of the system.

100 Millimeters



$\frac{1}{10}$ Meter, or 1 Decimeter

The length of the meter was intended to be a **ten-millionth part** of the distance from the equator to either pole, but subsequent calculations have shown it to differ slightly from that.

The length of the standard meter in the United States is 39.37 inches.

Other metric units of length are **decimal parts** of the meter and **multiples of 10** times the meter.

435. The primary units of surface, volume, capacity, weight, etc., are likewise subdivided and multiplied *decimally*, giving the other units of those measures. Consequently, in the metric

system names are simplified and made to show the size of each unit by giving to each *primary unit* the following *Latin prefixes* to indicate the *decimal parts* and *Greek prefixes* for the *multiples.*

LATIN		GREEK	
deci means	.1	**deka** means	10
centi means	.01	**hekto** means	100
milli means	.001	**kilo** means	1000

Thus, decimeter means $\frac{1}{10}$ of a meter; dekameter means 10 meters; etc
Another prefix sometimes used is **myria,** meaning 10,000.

436. Table of measures of length.

10 **millimeters (mm.)**	= 1 **centimeter (cm.)**
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 **meter (m.)**
10 meters	= 1 dekameter (Dm.)
10 dekameters	= 1 hektometer (Hm.)
10 hektometers	= 1 **kilometer (Km.)**

Abbreviations for *parts* of the primary unit begin with small letters, those for *multiples* of it with capitals.

The tables indicate important units by **heavy type.**

The **kilometer** (about .6 mi.) is used for long distances; the **meter** (about $3\frac{1}{4}$ ft., a little over 1 yd.), for shorter distances and to measure cloth, etc.; the **millimeter,** in the sciences and to express such small measurements as the thickness of wire.

437. The decimal relation of the units and their correspondence to the successive orders of units in the decimal notation is illustrated by the following:

Km.	Hm.	Dm.	m.		dm.	cm.	mm.
1	1	1	1	.	1	1	1

Since the meter occupies units' place, this number is read the same as 1111.111 m. Reduction to any other unit in the table is accomplished by simply moving the decimal point, thus:

1111.111 m. = 1.111111 Km. = 111111.1 cm. = etc.

EXERCISES

438. Express in meters or decimals of a meter:

1. 2 Km.
2. 4 mm.
3. 7 Hm.
4. 9 cm.
5. 5 Dm.
6. 8 dm.
7. 75 cm.
8. 37.5 cm.
9. 12.5 cm.
10. 250 mm.
11. 981 mm.
12. 3.275 Km.
13. 5 Km. 300 m.
14. 40 Km. 75 m.
15. 26 Km. 33.5 m.
16. 706 m. 82 cm.
17. 530 m. 75 cm.
18. 48 cm. 9.65 mm.

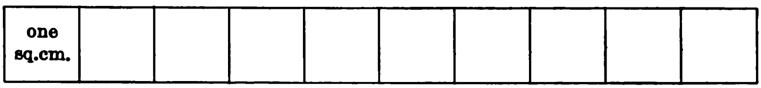
19. About how many feet are there in 40 meters? meters in 13 feet? miles in 15 kilometers? kilometers in 60 miles?

20. During practice marches the German soldier walks 30 Km. per day. About how many miles does he walk per day?

21. A foot is about 30 cm. About how many steps must a man take to walk 100 ft., if each step is $\frac{5}{6}$ of a meter long?

MEASURES OF SURFACE

439. 1. Draw a rectangle 10 cm. long and 1 cm. wide, and divide it into square centimeters. What is its area?



10 Square Centimeters

2. What is the area of a rectangle 10 cm. by 2 cm.? 10 cm. by 3 cm.? 10 cm. by 5 cm.? 10 cm. by 10 cm.?

3. Since 10 cm. = 1 dm., how many square centimeters are there in 1 square decimeter?

4. How many square decimeters equal 1 square meter? How many square meters equal 1 square dekameter?

440. *In metric square measure it requires* 100 *units of any denomination to make* 1 *unit of the next higher denomination.*

441. The primary unit of **surface** is the **square meter.**

The official equivalent of the square meter in the United States is 1.196 square yards.

442. Table of measures of surface.

100 square millimeters	=	1 square centimeter
100 square centimeters	=	1 square decimeter
100 square decimeters	=	1 **square meter**
100 square meters	=	1 square dekameter
100 square dekameters	=	1 square hektometer
100 square hektometers	=	1 **square kilometer**

The **square meter** (about 1.2 sq. yd.) is used for ordinary surfaces, such as floors, walls, etc.; the **square kilometer** (nearly .4 sq. mi.) for such large surfaces as the areas of countries.

443. From § 440 and the table, it is seen that the successive metric units of square measure occupy *two* orders of figures, thus:

sq. Km.		sq. Hm.		sq. Dm.		sq. m.			sq. dm.		sq. cm.		sq. mm.
1	0	1	0	1	0	1	.	0	1	0	1	0	1

Sometimes the abbreviation m^2 is used for sq. m.

444. The primary unit of **land measures** is the **are,** which is a *square dekameter.*

The official equivalent of the are is 119.6 square yards.

445. Table of land measures.

100 centares	=	1 **are**
100 ares	=	1 **hektare**

The **hektare** is nearly $2\frac{1}{2}$ acres.

446. The successive land units occupy two orders of figures:

Ha.		a.			ca.
1	0	1	.	0	1

EXERCISES

447. Reduce to square meters or decimals of a square meter:

1. 62.5 sq. dm.	3. 122.5 sq. Hm.	5. .125 sq. Dm.
2. 37.5 sq. dm.	4. 1.875 sq. Km.	6. 1000 ares.

7. Reduce to square millimeters: .5 sq. cm.; 1.25 sq. cm.

8. Reduce to square centimeters: 875 sq. mm.; 1500 sq. mm.

9. Find the area of a post card 14 cm. by 8 cm.

10. Find the area, in hektares, of a city block 1 Hm. square.

11. The large sulphur mines at Askhabad, Russia, cover an area of 6000 hektares. About how many acres do they cover?

MEASURES OF VOLUME

448. 1. How many centimeters are there in 1 decimeter?

2. Into how many cubic centimeters may a rectangular solid be divided, if it is 1 dm. long, 1 dm. wide, and 1 cm. thick? 2 cm. thick? 5 cm. thick? 10 cm., or 1 dm., thick?

3. Then how many cubic centimeters are there in a rectangular solid 1 dm. by 1 dm. by 1 dm., that is, in 1 cu. dm.?

4. How many cubic decimeters equal 1 cubic meter? How many cubic meters equal 1 cubic dekameter?

449. *In metric cubic measure it requires* 1000 *units of any denomination to make* 1 *unit of the next higher denomination.*

450. The primary unit of **volume** is the **cubic meter.**

The official equivalent of the cubic meter is 1.308 cubic yards.

451. Table of measures of volume.

1000 cubic millimeters = 1 **cubic centimeter**
1000 cubic centimeters = 1 **cubic decimeter**
1000 cubic decimeters = 1 **cubic meter**

and so on. The higher units, however, are little used.

452. From § 449 and the table, it is evident that the successive metric units of cubic measure occupy *three* orders:

cu. Km.			cu. Hm.			cu. Dm.			cu. m.			cu. dm.			cu. cm.			cu. mm.
1	0	0	1	0	0	1	0	0	1.	0	0	1	0	0	1	0	0	1

Sometimes the abbreviation m^3 is used for cu. m.

453. The primary unit of **wood measures** is the **stere**, which is a *cubic meter*.

454. Table of wood measures.

10 decisteres = 1 **stere**
10 steres = 1 dekastere

WRITTEN EXERCISES

455. 1. A book is 19 cm. long, $14\frac{1}{2}$ cm. wide, and 2 cm. thick. Express its volume in cubic centimeters.

2. How many steres of wood can be piled in a shed 6 m. long, 5 m. wide, and 3.2 m. high?

3. How many cubic centimeters of water will fill a cubical box, each inside dimension of which is 1 dm.?

4. A wall along the side of a park 1 Km. long is 1 m. thick and 1.6 m. high. Find the solid contents in cubic meters.

5. How thin must 1 cu. cm. of gold be beaten to cover a rectangle 40 cm. long and 15 cm. wide?

6. A block of white marble quarried at Paros, Greece, was 4 m. long and 1.25 m. square. Find its value at $28 per cu. m.

7. In a recent year 320 cu. Hm. of gas were produced in New York City. How many cubic meters were produced?

8. Timbers of imported Spanish mahogany are usually 61 cm. square and 3.05 m. long. Find the volume of such a timber.

MEASURES OF CAPACITY

456. The primary unit of **capacity** for both liquid and dry measures is the **liter** (lētẽr), which contains 1 *cubic decimeter*, or 1000 *cubic centimeters*.

The official equivalent of the liter is 1.0567 qt. (liquid). The liter is .908 qt. (dry).

457. Table of measures of capacity.

10 milliliters = 1 centiliter
10 centiliters = 1 deciliter
10 deciliters = 1 liter
10 liters = 1 dekaliter
10 dekaliters = 1 hektoliter

The **liter** (about 1 qt., liquid or dry) is used to measure moderate quantities; the **hektoliter** (nearly 2.84 bu.) is used to measure grain, fruit, vegetables, etc., in large quantities.

458. The successive units of capacity correspond to the orders of units in the decimal notation, thus:

Hl.	Dl.	l.		dl.	cl.	ml.
1	1	1	.	1	1	1

WRITTEN EXERCISES

459. 1. How many liters of rice will it take to fill a box the inside dimensions of which are 60 cm., 50 cm., and 40 cm.?

2. Find the cost, in Mexico, of 24 hektoliters of corn at $12 per 200 liters.

3. A Manila merchant bought 400 liters of olive oil at 25¢ per liter and retailed it at 30¢ per $\frac{3}{4}$ liter. Find his gain.

4. The French wheat harvest one year was $110\frac{1}{2}$ million hektoliters, grown on $6\frac{1}{2}$ million hektares. Find the yield per hektare.

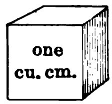
5. A Spanish olive orchard of 750 trees yielded 30 liters of olives per tree. How many hektoliters of olives did it yield?

6. An electric fire engine such as is used in France throws 350 liters of water per second. How many cubic meters of water do 4 such engines throw on a fire in an hour?

MEASURES OF WEIGHT

460. The primary unit of **weight** is the **gram**, which is the weight of 1 *cubic centimeter* of pure water at its greatest density.

The weight of 1000 cubic centimeters (1 cubic decimeter) of water, that is, of 1 *liter* of water, is called a **kilogram**, or a **kilo.**



The official equivalent of the kilogram is 2.2046 pounds avoirdupois.

461. Table of measures of weight.

10 milligrams = 1 centigram	10 grams = 1 dekagram
10 centigrams = 1 decigram	10 dekagrams = 1 hektogram
10 decigrams = 1 **gram**	10 hektograms = 1 **kilogram**

Higher units are:

10 kilograms = 1 **myriagram**
100 kilograms = 1 **metric quintal**
1000 kilograms = 1 **metric ton**

The **gram** (about $\frac{1}{5}$ the weight of a 5-cent piece) is used in weighing medicines, precious metals, etc.; the **kilogram** (about 2.2 lb.), in weighing groceries; and the **metric ton** (about 2205 lb.), in weighing bulky articles, such as hay, coal, etc.

A 5-cent piece weighs about 5 grams, and is about 2 cm. across and 2 mm. thick.

462. The successive units of weight and the orders of units in the decimal notation correspond, thus:

M.T.	Q.	Mg.	Kg.	Hg.	Dg.	g.		dg.	cg.	mg.
1	1	1	1	1	1	1	.	1	1	1

EXERCISES

463. Reduce to grams or decimals of a gram:

1. 3.5 Kg. 3. 2.5 dg. 5. 350 mg. 7. 49 Kg. 755 g.
2. 4.2 Hg. 4. 7.5 Dg. 6. 8400 mg. 8. 15 g. 200 mg.

9. Make a table of the units *milligram*, *gram*, *kilogram*, and *metric ton*.

10. If a 6-liter jar weighing $2\frac{1}{2}$ Kg. is filled with water, find the total weight of the jar and the water.

11. Find the weight of a cubic meter of water.

12. Ice is .92 as heavy as water. Find the weight of a cube of ice 30 cm. on each edge (about 1 cu. ft. of ice).

13. A boy who weighs 40 kilos weighs about —— pounds.

14. About how many 5-cent pieces weigh 1 kilo?

EQUIVALENTS

464. Hereafter, unless stated to the contrary, refer to the following equivalents when changing from one system to the other:

Common to Metric	Metric to Common
1 yd. = $\frac{3600}{3937}$ m. = .9144 m.	1 m. = 39.37 in.
1760 yd. = 1 mi. = 1.60935 Km.	1 Km. = .62137 mi.
1 sq. yd. = .836 sq. m.	1 sq. m. = 1.196 sq. yd.
1 A. = .4047 Ha.	1 Ha. = 2.471 A.
1 cu. yd. = .765 cu. m.	1 cu. m. = 1.308 cu. yd.
1 qt. (dry) = 1.1012 l.	1 l. = .908 qt. (dry)
1 qt. (liquid) = .94636 l.	1 l. = 1.0567 qt. (liquid)
1 bu. = .35239 Hl.	1 Hl. = 2.8377 bu.
1 oz. (troy) = 31.10348 g.	1 Kg. = 32.1507 oz. (troy)
1 lb. (av.) = .45359 Kg.	1 Kg. = 2.2046 lb. (av.)
1 T. = .90718 M.T.	1 M.T. = 1.1023 T.

WRITTEN EXERCISES

465. In changing from one system to the other, estimate results in advance; then use the equivalents on the preceding page, giving final inexact results to the nearest thousandth.

In exercises 1–9 reduce kilometers to miles, meters to feet, centimeters and millimeters to inches:

1. Height of spire of Strassburg Cathedral, 142 m.
2. Height of Pyramid of Cheops, 139.5 m.
3. Height of Leaning Tower of Pisa, 54.5 m.
4. Height of "203-meter Hill" near Port Arthur, 203 m.
5. Length of St. Gothard tunnel, Switzerland, 14.9 Km.
6. Length of Panama Railroad, Panama to Colon, 78 Km.
7. Length of French cable, Brest to New York, 5318.9 Km.
8. Length of hairspring of a watch, 23 cm.
9. Bore of Mauser rifle, 7 mm.

10. In Paris the height of a building to the eaves may not exceed the width of the street by more than 6 m., and may not exceed 20 m. in any case. How many feet high may the walls of a building be on a street 12 m. wide? on one 18 m. wide?

11. In a Parisian building 20 m. high to the eaves, the ceilings on the first floor must be at least 9 ft. 2 in. high. Express this height in meters.

12. There are 600,000 hektares of cork trees in Portugal, 300,000 in Spain, and 80,000 in Italy. How many square kilometers of cork tree forests are there in each country?

13. How many metric tons of lard are there in a shipment of 1000 tubs, each holding $12\frac{1}{2}$ Kg. of lard?

14. A South American sewer tunnel 1,278 m. long, 3.65 m. high, and 3 m. wide was cut through rock. How many cubic meters of rock were removed?

15. A large dredge removed 4500 cu. m. of mud per hour at a cost of 3 pfennigs per cubic meter. Find the cost in marks of running the dredge 12 hours.

16. Short cotton, grown in Algeria, averages 3 metric tons per hektare. Find the value of 5.5 Ha. of Algerian cotton at $9.25 per metric quintal.

17. A New York firm sent a letter to London weighing 135 g. How much postage was paid at 5¢ per 15 g.?

18. A coffee tree yielded 800 g. of berries. If $\frac{1}{5}$ of their weight was husk, how much marketable coffee was obtained?

19. In a recent year the output of gold in French Guiana amounted to 2541.35 Kg. How many troy ounces of gold were produced?

The **specific gravity** of a substance is its weight as compared with that of an equal volume of water.

20. If two liters of milk weigh 2.06 Kg., what is the specific gravity of milk, or how many times as heavy as water is it?

Solution

1 l. of water weighs 1 Kg.; 1 l. of milk weighs $\frac{1}{2}$ of 2.06 Kg., or 1.03 Kg. Therefore the specific gravity of milk is 1.03.

Find the specific gravity of the following substances:

21. Olive oil, if 40 cu. cm. weigh 36.6 g.

22. Mercury, if $\frac{1}{2}$ l. weighs 6.799 Kg.

23. Cork, if a piece 10 cm. by 8 cm. by 2 cm. weighs 384 dg.

24. Find the weight of 25 l. of naphtha, specific gravity .848.

25. A wholesale dealer carefully weighed a sample of cloth 40 mm. by 50 mm. and found its weight to be 62 cg. What was the weight of this cloth per meter, if it was 140 cm. wide?

26. Norway in a recent year exported 31,000,000 kilos of fresh codfish and 1,500,000 kilos more than half that amount of dried codfish. How many pounds of each did she export?

27. In a city in Greece, 800 private lights burning a certain kind of gas are in use. If each jet consumes 15 liters of gas per hour and burns on an average 4 hours per day, how much gas is consumed in the city per week?

28. The substance from which this gas is made gives out 300 liters of gas per kilo. How many boxes containing 50 kilos each are used per week?

29. A quart of water is less than a liter, but if frozen will make more than a liter of ice. Find how many cubic centimeters more, if water expands $\frac{2}{23}$ of its volume in freezing.

30. Find the cost of 17 metric quintals of chestnuts at 15 francs per sack of 100 kilos.

31. The length of a new French locomotive is 16.2 m. How much longer, to the nearest inch, is it than a large English locomotive whose length is 41 ft. 4 in.?

32. A shipment of coffee from Bahia, Brazil, to Bordeaux, France, consisted of 500 bags holding 60 Kg. each. Find the cost for freight in francs at 30 francs per 900 Kg.

33. In a quarry in Saxony an undercut resulted in the fall of 58,000 cu. m. of stone. Find the weight of the stone in metric tons, if its specific gravity was 2.7.

34. Greece sent to Great Britain one year 64,000 metric tons of currants, and to Holland and the United States $\frac{1}{2}$ and $\frac{5}{16}$ as much, respectively. Express these quantities in short tons.

35. The Bank of France pays 3437 francs for 1 kilogram of fine gold. Find, to the nearest cent, the equivalent price per troy ounce.

36. At a Berlin motor boat exhibit, an American exhibitor paid an entrance fee of 100 marks, and rented a floor space of $4\frac{1}{2}$ square dekameters at 10 marks per square meter. Find his expense in United States money.

PRACTICAL MEASUREMENTS

MISCELLANEOUS EXERCISES

466. 1. A skating rink 75 feet by 216 feet was flooded with 2 inches of water. How many gallons were used?

2. The cargo of grain from a vessel filled 12 elevator bins, each $8\frac{1}{3}$ feet square and 90 feet deep. How many bushels of wheat did the vessel carry? (1 bu. $= 1\frac{1}{4}$ cu. ft.)

3. The specific gravity of ice is .92. What must be the height of a room required to hold 184 tons of ice, making no allowance for packing, if the floor space is 20 feet square?

In the **centigrade** (C.) system of measuring temperature the freezing point is marked 0° and the boiling point 100°.

Since 100° C. = 180° F., 1° C. $= \frac{180}{100}$° F., or $\frac{9}{5}$° F.

Similarly, 1° F. $= \frac{100}{180}$° C., or $\frac{5}{9}$° C.

4. Express 15° C. in the Fahrenheit scale.

SOLUTION.—Since 1° C. $= \frac{9}{5}$° F., 15° C. $= 15 \times \frac{9}{5}$° F. above the freezing point, that is, above 32° F.

$$15^\circ \text{ C.} = \overset{3}{\cancel{15}} \times \frac{9^\circ}{\cancel{5}} \text{ F.} + 32^\circ \text{ F.} = 59^\circ \text{ F.}$$

5. Express 59° F. in the centigrade scale.

SOLUTION.—Since 59° F. means 59° − 32°, or 27 Fahrenheit degrees above the freezing point of water, and since each Fahrenheit degree above the freezing point is equivalent to $\frac{5}{9}$ of a centigrade degree above 0° C.,

$$59^\circ \text{ F.} = \tfrac{5}{9}(59 - 32) \text{ degrees centigrade} = 15^\circ \text{ C.}$$

6. Milk is pasteurized by heating it to $68\frac{1}{3}$° C. and keeping it at that temperature for a certain time. Express the temperature in the Fahrenheit scale.

7. Blood heat is about 98° F. At how many degrees would a centigrade thermometer register blood heat?

8. Water is heaviest at 4° C. Express this temperature in degrees Fahrenheit.

9. Olive oil freezes at 30° F., and mercury at $-37.9°$ F. Express these freezing points in the centigrade scale.

10. The roof of a house was 38′ long and it was 30′ from the ridge to the eaves on each side. How many pieces of slate 9″ wide were required to cover the roof, if they were laid 6″ to the weather?

11. Find the weight of slate on the roof, if a square foot weighed $4\frac{1}{2}$ pounds.

12. The roofer averaged 2 squares in 10 hours. If he received 35¢ per hour, how much did he receive for laying the roof?

13. Find the cost of the slate at $5.75 per square.

14. Adding $2.75 for hardware and breakage, what was the entire cost of the roof?

15. How many bunches of common shingles should be purchased to cover a roof $80' \times 32'$, estimating 4 bunches per square?

16. The pasting table used by a paper hanger consisted of four $\frac{3}{4}$-inch white pine boards 6 feet long and 10 inches wide. How many feet of lumber did it contain?

17. For a combination bookcase, 45 feet of plain oak were used at $30 per M, 10 feet of 3-ply oak at $95 per M, and 8 feet of quartered oak at $60 per M. This cost was $\frac{2}{11}$ of the cost of the materials used. Find the entire cost of materials.

18. A slab of ash 38 inches long, 11 inches wide, and $2\frac{3}{4}$ inches thick makes 4 baseball bats, which sell at 8¢ each at the mill. How many feet of ash are required to fill the order of a sporting goods company for baseball bats to the value of $46.08?

The second story of a cottage consisted of 4 bedrooms. Each was $18' \times 12'$ and $9'$ high, with 2 windows, each $3'6'' \times 6'$ and 1 door $3' \times 7'$. Find the following, deducting openings:

19. The number of laths required, if 1500 cover 100 sq. yd.

20. The cost of putting on the laths at 6¢ per sq. yd.

21. The number of days it will take a plasterer to lay on three coats, if he can plaster 144 sq. yd. of first coat per day, $\frac{2}{3}$ as much of second coat, and $\frac{1}{2}$ as much of third coat.

22. The cost of plastering the rooms at 33¢ per sq. yd.

23. How many days will be required to lay a mosaic tile floor in a room $48' \times 36'$, if the tiler can lay in a day 20 sheets of tile, each sheet $2' \times 1'$?

24. Find the cost of the floor at $1.55 per square foot.

25. In computing the bill for painting one side of a house 49 feet deep and 36 feet high, a painter, after deducting the area of 8 windows, each $3' \times 8'$, charged 27¢ per square yard, and 50¢ for each window. Find the cost to the owner.

26. A room $21' \times 18'$ has 3 windows and 2 doors. How many strips of wall paper 18 inches wide are necessary to paper the walls of the room, deducting 2 for each window and 2 for each door?

27. The room is $9'$ high. Making allowance for matching and for the width of baseboard and border, it is found that 3 strips can be cut from a roll. Find the cost of paper for the room at 50¢ per roll.

28. How much does the border cost at 7¢ per foot?

29. In papering the ceiling the long way, a roll makes but 1 strip. How much does this paper cost at 35¢ per roll?

30. Find the cost of carpeting the New York State senate chamber $60' \times 55'$ with Victoria Wilton carpet 27 in. wide at $2.35 per yard, making no allowance for matching the pattern.

31. A floor $16' \times 12'$ is to be covered with Axminster carpet 27 in. wide. Allowing 9 in. for matching the pattern, how much carpet will be required if the strips run the long way?

NOTE. — No allowance for matching is necessary for the first strip.

32. In getting sod for a park in an Illinois city, strips were cut 12″ wide and 9′ long, 80 strips making a wagon load. If the number of wagon loads was 250, how many *square yards* of sod were used?

33. The cost per square yard was: cutting 1.6¢, carting .9¢, laying 2.7¢, watering .6¢. Find the total cost of the sod.

34. Find the cost of excavating a cellar $28' \times 45'$ and 9′ deep, at 30¢ per *cubic yard.*

35. A short ton of Lehigh egg coal occupies about 35 cu. ft. What must be the depth of a car $30' \times 8'$ to hold 40 tons?

36. Find the cost of making a brick sidewalk $12' \times 318'$, including the following items. Find also the cost per *square yard.*

Excavating and leveling ground, $4.75.
6 loads of sand at 40¢; carting, $6; spreading, $3.13.
Bricks, 38 per square yard, at $7.50 per *thousand.*
Labor, 1 man laying 53 square yards per day, at $3 per day.

37. Find the cost of asphalt paving for 405 feet of street 60 feet wide in a Southern city, if the concrete foundation cost 65¢ per square yard and the asphalt $1.15 per square yard.

38. How much did it cost to construct a macadam road ½ mile long and 36 feet wide, at $.98½ per square yard?

39. Find the cost of paving the same road with asphalt at $2.36 per square yard.

40. One year 18,457 sq. yd. of brick pavement were laid in the streets of Helena, Mont. If 58 bricks were required for a square yard, how much did the bricks cost at $20 per M?

41. How many bricks are required for a wall 25′ long, 12′ high, and 3′ thick, if $22\frac{1}{2}$ bricks with mortar occupy 1 cubic foot?

42. Find the cost of the bricks at \$7.50 per M.

43. Find the cost of the wall at \$12 per M bricks, laid.

44. How many *cubic yards* of mortar are required, if 1000 bricks are laid in .7 of a cubic yard of mortar?

45. Find the weight of the wall, if 1 cu. ft. weighs 125 lb.

46. How much will it cost to build a wall 80′ long, 4′ high, and 3′ thick of hollow concrete blocks 32″ × 9″ × 12″, costing 25¢ each, laid?

47. The front wall of a house 22′ wide and 54′ high was of ashlar (cut stone) masonry 2′ thick. Deducting $\frac{1}{6}$ of the wall for openings, find the cost at \$25.20 per cubic yard.

48. Find the cost, at 50¢ per *square foot*, of dressing a stone doorstep 5′ long, 3′ wide, and 7″ thick, on the top, ends, and one long side.

49. How many cubic yards were there in a concrete wall 5′ high, 2′ thick, and 54′ long?

50. If 1 cubic yard of this concrete consisted of $1\frac{4}{5}$ *barrels* of cement of $3\frac{3}{4}$ cubic feet each, twice as much sand, and four times as much broken stone, how many barrels of cement and how many *cubic yards* of sand and of stone were used in all?

51. The cement cost the contractor \$2 per barrel; the sand 25¢ per cubic yard for loading, 20¢ for carting, and 15¢ for screening; and the broken stone 90¢ per cubic yard. If \$1 was the additional cost per cubic yard for mixing and laying the concrete, find the total cost to the contractor.

52. The specific gravity of clay is about 1.2. Find the weight of a cubic yard of clay.

53. The specific gravity of lead is 11.35. What is the value of a cubic foot of lead at $4\frac{1}{2}$¢ per pound?

54. A body immersed in water is buoyed up by a force equal to the weight of water displaced. If a man's body displaces $2\frac{1}{2}$ cubic feet of water, how much will he be buoyed up when swimming in fresh water? in salt water, specific gravity 1.03?

55. How much less force is required to lift a stone $5' \times 2' \times 1'$ to the surface of a stream than to lift it out?

56. A floating timber $20'$ long and $1'$ square is three fourths submerged. The water displaced weighs as much as the timber. Find the weight of the timber and its specific gravity.

The speed of vessels at sea is measured in **nautical miles** or **knots** per hour. The U. S. Coast Survey knot is **6080.27 feet**, or about the length of $1'$ of arc on the equator.

A sailor at the stern of a vessel throws overboard a log, or float, that remains stationary as the vessel proceeds. He then counts the number of knots of the line that are reeled out in a given time, usually 28 or 30 seconds. These knots are at such distances apart that the number reeled out in the given time is equal to the number of nautical miles an hour the ship is sailing. Thus, if the log falls behind 14 knots while the sand in the glass runs out, the ship is sailing 14 knots an hour. In the following use **1 knot = 1.15 statute miles.**

57. If the *Brooklyn* is making 22 knots an hour, how many statute miles an hour is she sailing? how many feet per second?

58. The new turbine Cunard steamships average $24\frac{1}{2}$ knots an hour. How many statute miles do they sail in an hour? in a day of 24 hours?

59. The cruiser *Washington* in a 4-hour trial made an average speed of 22.27 knots per hour. How many statute miles, to the nearest hundredth, did she travel per hour?

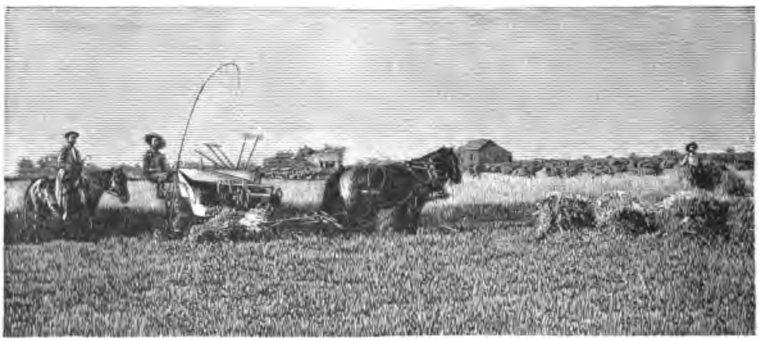
60. A ship sailed on the equator from 21° 50′ west longitude to 28° 14′ west longitude in 24 hours. Find her approximate speed in knots per hour.

REVIEW PROBLEMS IN INDUSTRIES

467. 1. A Kansas farmer had two fields sown to wheat, one $\frac{3}{4}$ of a mile long and $\frac{1}{2}$ of a mile wide, the other $\frac{3}{8}$ of a mile by $\frac{5}{8}$ of a mile. How many acres of wheat had he?

2. For plowing, 3 gang plows were used, each turning $12\frac{1}{2}$ acres per day of 10 hours. How many days and hours did it take to plow the 390 acres of both fields?

3. If $5\frac{1}{2}$ pecks of seed were sown per acre, find the cost, at 60 ¢ per bushel, of seed used on the 390 acres.



4. The wheat of the smaller field (150 acres) was reaped and bound into sheaves by a self-binding harvester, and afterward thrashed. If the yield was 3600 bushels, find the yield per acre. Find the cost of thrashing the wheat from this field at $4\frac{1}{2}$ ¢ per bushel.

5. If the larger field (240 acres) produced the same number of bushels per acre, find the whole yield of both fields.

6. On the larger field a combined harvester and thrasher was used, which reaped and thrashed the wheat and put it into 2-bushel sacks at the rate of 27 sacks per hour. How many days of 10 hours each did it take to reap the 5760 bushels?

7. The owner had his harvest of 9360 bushels carted to an elevator, where it was cleaned and loaded on freight cars at a cost of $1\frac{1}{2}$¢ per bushel. Find this item of expense.

8. Eighteen freight cars were used to carry this wheat. A bushel of wheat weighs 60 pounds. Find the load per car.

9. If the freight bill to Chicago amounted to $1010.88, what was the rate per 100 pounds?

10. At Chicago the charge for elevator storage was $\frac{3}{4}$¢ per bushel for the first 10 days, and $\frac{3}{8}$¢ for each succeeding 10 days. Find the cost of storing the 9360 bushels from August 8 to September 16 inclusive.

11. The farmer's entire expense was $2885.75. If the wheat was sold at 65¢ per bushel, what was his total gain?

12. It cost $4\frac{3}{4}$¢ per bushel to transport the 9360 bushels of wheat to New York by way of the Great Lakes, Erie canal, and Hudson River. The rate by rail was 16¢ per 100 pounds. How much more would the freight charges have been by rail?

13. If this crop of 9360 bushels made 4 pounds more than 2063 barrels of flour of 196 pounds each, what decimal part of the weight of the wheat was the weight of the flour?

14. How many bushels of wheat, to the nearest hundredth, did it take to make one barrel of flour?

15. If 100 pounds of flour make 135 1-lb. loaves of bread, how many loaves can a baker make from 5 barrels of flour?

16. Recently the average yield of wheat per acre in various countries was as follows: Great Britain, 31.8 bushels; France, 26; Germany, 19.4; Austria, 16.4; United States, 13.4; Russia, 9. Find the yield from 25 acres in each of these countries.

17. Our country's leading wheat states one year were Minnesota with 68,000,000 bu., Kansas with 65,000,000 bu., and North Dakota with 54,000,000 bu. If these states produced .34 of our entire crop, how much did we produce?

18. In the fall Harvey Smith accepted a position as timekeeper in a lumber camp. At $1.85 per day, how much did he earn in 156 working days?

19. With him in the camp were 34 men employed to chop down the trees and saw them into logs. If each received $2.25 per day, what were their combined wages for the 156 days?

20. The wages of 13 teamsters employed for this time amounted to $3853.20. How much did each receive per day?

21. The other occupants of the camp were: 1 cook at $45 per month, 1 chore boy at $30, and 1 scaler, who measured and marked the logs, at $45 per month. Find their combined wages for 6 months.

22. Another set of men was employed to make roads to the river. If each received $1.50 per day, how much did the 23 road makers receive in 12 weeks of 6 working days each?

23. The logs were drawn over these roads and rolled into the river. After the breaking up of the ice they were guided down to the mill by log drivers. Seventeen log drivers worked 35 days and received $3123.75 in wages. Find the daily wage of each.

24. What were the total wages paid to the men employed to procure the logs and get them to the mill?

25. At the mill the largest logs went to the band saw. The setter, who places the logs in position, received $2.55 per day;

the dogger, who fastens them, \$2; and the sawyer, \$5.50. Find the cost of this labor, if the mill was running 184 days.

26. The smaller logs went to the circular saw where they were squared. If the sawyer received \$3.75 per day and his assistant $\frac{8}{15}$ as much, find the earnings of each during 184 days.

27. These logs next went to the gang saw where they were sawed into boards of different thicknesses. How much was paid during the season for work at the gang saw, if the sawyer received \$2.75 per day and his 2 assistants \$2 each?

28. The final operation was edging and trimming, for which 2 edgers were employed at \$2 per day each, and 2 trimmers at \$1.85 each. Find their total wages for the season of 184 days.

29. The broken lumber and trimmings were then sorted by a boy and all suitable pieces cut into laths by a lathman. During the season, the boy and man together received \$690, of which $\frac{2}{5}$ was the boy's share. How much did each receive per day?

30. As the sawdust and waste were used for fuel, the only cost for power was the engineer's wages, \$3.50 per day, and the fireman's wages, \$2.25 per day. Find the cost for power.

31. The daily wages of the other employees were: saw filer, \$6.50; assistant, \$2; millwright, \$3.50; assistant, \$2.50; and 12 lumber handlers at \$1.85 each. Find their total earnings.

32. How much in all was paid for labor in the mill?

33. The average daily output of the mill was 95,000 feet of lumber. What was the output for the whole season?

34. Find the cost of transporting one day's output from the mill to the market at \$1.75 per M; the season's output.

35. If $\frac{3}{5}$ of the lumber was spruce worth \$22.25 per M, $\frac{1}{10}$ pine worth \$28.75 per M, and the rest hemlock worth \$19.75 per M, what was the value of one day's output of 95,000 feet?

36. If these were the average prices received, find the receipts from the lumber supplied by the mill during the season.

37. The entrance or shaft to an anthracite coal mine averaged 12 feet by 18 feet, and extended to a depth of 720 feet. How many cubic yards of material were removed for the shaft?

38. Find the cost of sinking and timbering the shaft at $3.75 per cubic yard.

39. If the average time required to sink and timber the shaft was 365 days per 292 feet, how many days did it take for this shaft?

40. For tunneling the underground passages, the cost per lineal yard was 90¢ for powder and $3.60 for labor. Find the cost of tunneling a passage 567 feet in length.

41. The cars in which the coal was carried from the miners to the shaft weighed 850 pounds and had a capacity of one long ton. What was the total weight of a train of 8 loaded cars?

42. The rails upon which these cars ran weighed 35 pounds per yard. At a cost of $13.20 per long ton, find the cost of rails for a track 180 yards long.

43. The cage in which the coal was raised from the mine ascended at the rate of 24 feet per second. How long did it take to reach the surface 720 feet above?

44. A miner and his laborer averaged $5\frac{1}{2}$ tons per day, for which they received 69¢ per ton. What were the earnings of each per day, if the miner received $\frac{2}{3}$ and the laborer $\frac{1}{3}$?

45. If they worked 24 days in November and mined $5\frac{1}{2}$ tons of coal per day, find the miner's share of the earnings.

46. It required, on the average, $10\frac{2}{3}$ ounces of powder to dislodge one ton of coal. Find the cost of the powder

used during the month, if a keg holding 25 pounds cost $1.50.

47. The miner's other expenses for November were: file, 18¢; sharpening and repairing tools, 96¢; fuse, 15¢; $1\frac{3}{8}$ gal. lard oil @ 48¢. What were his total expenses?

48. Deducting expenses from the $60.72 received for his labor, what were the miner's net earnings for the month?

49. The cost of running a mine locomotive included $4.25 per day for labor, 50¢ for coal and oil, and 60¢ for repairs. Find the cost for 24 days.

50. The mine locomotive did the same work formerly done by 15 mules and their drivers. Find the cost of mule power for 24 days, if feeding, shoeing, and tending the mules cost $9 per day and drivers' wages amounted to $7.25 per day.

51. How much was saved by using the locomotive instead of the mules?

52. After the coal was mined it was sent to the breaker where the slate was picked out by breaker boys, and the coal sorted into sizes. If the 43 breaker boys were paid $219.30 in 6 days, how much did each receive per day?

53. Of the coal mined, $\frac{3}{16}$ was lost by blasting, breaking, and adhering to the slate. Find the loss in one day when 1365 long tons of merchantable coal were obtained.

54. The mine operator's expenses on 850 long tons of chestnut coal that he sold at $3.40 per ton were $2.08 per ton for mining and 96¢ per ton for handling. Find his profits on this sale.

55. It cost the retailer who bought this coal 85¢ per ton for freight, and 75¢ per ton for handling. If he sold the coal at $6.25 per short ton, how much did he gain?

56. One year, when there was a miners' strike, Pennsylvania produced 36,940,710 tons of anthracite coal, and the next year increased the output by 23,301,750 tons. How many tons were produced the year after the strike?

PERCENTAGE

REVIEW

468. Find:

1. $46\frac{1}{2}\%$ of 5280 ft.

2. 125% of 5424 gal.

3. $87\frac{1}{2}\%$ of 15,992

4. $162\frac{1}{2}\%$ of 58,464

What per cent of

5. \$14 is \$16?

6. \$16 is \$14?

7. 900 yd. is 150 yd.?

8. 8 hr. is 2 hr. 40 min.?

9. 1 is .625?

10. .75 is $.12\frac{1}{2}$?

Find the number of which

11. \$864 is $2\frac{1}{4}\%$

12. 5500 is $62\frac{1}{2}\%$

13. \$2.25 is 5%

14. \$5.73 is $33\frac{1}{3}\%$

15. 5200 is $6\frac{1}{4}\%$

16. 4525 is $62\frac{1}{2}\%$

What number increased by

17. 15% of itself = 138?

18. 25% of itself = 365?

What number decreased by

19. $12\frac{1}{2}\%$ of itself = 2345?

20. $35\ \%$ of itself = 1612?

Find the value of x in each of the following exercises:

Base	Rate	Percentage	Base	Rate	Amount
21. \$375	16%	\$$x$	**29.** 7910	13%	x
22. \$$x$	$12\frac{1}{2}\%$	\$2.25	**30.** \$$x$	113%	\$10,650
23. .75	$x\%$	.125	**31.** x mi.	$133\frac{1}{3}\%$	3640 mi.
24. x	250%	$\frac{3}{4}$	Base	Rate	Difference
25. 2364	$\frac{1}{4}\%$	x	**32.** 1281 lb.	4%	x lb.
26. $\frac{5}{16}$	$x\%$	$\frac{1}{16}$	**33.** \$$x$	24%	\$1247.92
27. x	47%	$23\frac{1}{2}$	**34.** 6036	$83\frac{1}{3}\%$	x
28. 6 gal.	$x\%$	4 qt.	**35.** \$$x$	$66\frac{1}{3}\%$	\$4800.40

36. A commercial traveler receives a salary of \$25 a week and 2 % commission on his sales. If his sales amount to \$75,000 in a year, what is his income for the year?

37. Find the net cost to a dealer of 280 pounds of manila rope listed at $12\frac{1}{2}$¢ per pound, discounts 20 % and 5 %.

38. If 150 lb. of "lead," used in a medium grade of lead pencils, consisted of $88\frac{1}{2}$ lb. of graphite and the rest clay, find the per cent of each substance in the mixture.

39. In Oregon the average amount of standing timber per acre of forest land is 12,200 board feet, of which 66 % is red fir, 18 % pine, 5 % spruce, 5 % hemlock, 2 % cedar, and 4 % other kinds. Find the amount of each per acre.

40. One month an agent secured 346 subscriptions to a dollar magazine, earning \$86.50. Find his rate of commission.

41. An orange tree yielded 50 % more oranges the second time it bore than the first time. If the second crop amounted to 420 oranges, how many were there in the first crop?

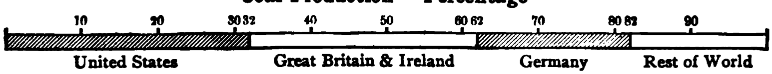
42. What per cent of the cocoa from 1000 trees, whose average yield is $1\frac{3}{4}$ pounds apiece, is cocoa butter, if the cocoa butter amounts to 910 pounds?

43. One year 151,590 skilled laborers arrived in the United States. This was $18\frac{3}{5}$ % of the total number of immigrants. How many immigrants arrived that year?

44. The rice crop of Japan one year was 204.6 million bushels, or 7 % less than normal. What is the normal crop?

45. In a year when the United States produced 352,000,000 short tons of coal, find from the following graph the production of the world; of Great Britain and Ireland; of Germany.

Coal Production — Percentage



46. Which gives the lower price for a piano listed at \$600, a direct discount of 45 % or successive discounts of 25 %, 15 %, and 5 % ? how much lower?

47. During five years the number of horses in this country increased from 13,500,000 to 17,010,000. What was the per cent of increase?

48. The tonnage of Russia's navy was 450,000 tons before the war with Japan and 225,000 tons after the war. Find the per cent of decrease.

49. What single discount is equivalent to two successive discounts of 15 % and 10 % ?

50. Lake Erie has an area of 9960 square miles. If it were larger by 24 square miles, its area would be 32 % of that of Lake Superior. How large is Lake Superior?

51. In the manufacture of 120,000 postal cards, 414 pounds of the material used was spruce pulp and 132 pounds was poplar pulp. If 1000 postal cards weigh 5 pounds, what per cent of the material was spruce? what per cent poplar?

52. The importation of cheese into Great Britain in 1880 was 125,000,000 lb. from the United States and 30,000,000 lb. from Canada. Twenty-five years later the amount from the United States was 80 % less, and that from Canada 600 % more. How much did Great Britain then import from each country?

53. The production of steel in Germany during a recent year was 920,000 tons, or 68 % less than the amount produced in this country, and $2\frac{2}{9}$ % more than the amount produced in England. What was the output of steel in each country?

54. A grain dealer in Mobile bought 8000 bushels of shelled corn through a commission merchant in St. Louis @ 22 ¢, commission 2 %. The freight cost \$806.40. Unloading cost \$30.40, cartage \$80, and elevator charges were $\frac{1}{2}$ ¢ per bushel. Find the total cost and the cost per bushel in the elevator at Mobile.

55. A dealer bought goods at 20 % less than their market value, and sold them at 20 % above the market value. What per cent did he gain?

SUGGESTION. 80 % of market value represents the cost, and 40 % of market value the gain. What part of 80 % is 40 %? what per cent?

56. What per cent is gained by buying clothing at 10 % below market value and selling it at $12\frac{1}{2}$ % above?

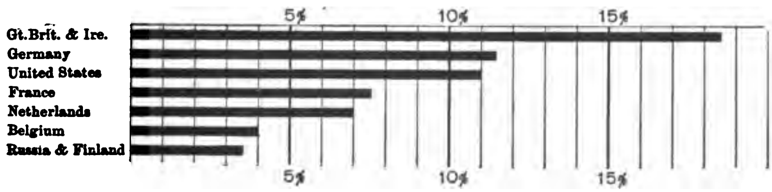
57. At what price must goods that cost \$1.80 per yard be marked so that the seller may deduct 10 % from the marked price and still gain 20 %?

SUGGESTION. $\frac{6}{5}$ of \$1.80 = 90 % of marked price.

58. A dry goods merchant sold tablecloths at 20 % less than the marked price, yet gained $16\frac{2}{3}$ %. At what price were they marked, if they cost him \$3.60 apiece?

59. A manufacturer sold 50 pieces of velveteen at auction at \$48 per piece. On one half of them he gained 20 % and on the rest he lost 20 %. Did he gain or lose on all and how much?

60. When the commerce of the world for a year amounted to \$21,900,000,000, the shares of the chief nations were approximately as represented by the following graphs. Find the value of each nation's share.



61. From these graphs find, to the nearest tenth, what per cent the commerce of each nation was of the commerce of Great Britain and Ireland; also what per cent the commerce of each nation was of the commerce of the United States.

GOVERNMENT REVENUE

469. The expenses of government are paid, for the most part, by **taxes**, which are sums of money levied on persons, on property, or on business; and by **duties** or **customs**, which are charges on goods imported from other countries.

The expenses of our national government consist of outlays for the army and the navy, for pensions, for the salaries of officers, etc. The state expends money for salaries of state officers, for educational purposes, and for the care of criminals and defective persons; while local governments need money for maintaining their various departments, such as Fire Department, Police Department, Department of Education, etc.

Taxes

470. Fixed property, such as land and structures built upon it, is called **real estate.**

471. Movable property, such as money, bonds, mortgages, cattle, lumber, etc., is called **personal property.**

472. State and local taxes are usually a certain *per cent* of the *estimated value* of real estate and of personal property.

In addition to the property tax, some states collect a *poll tax* (head tax), varying from $1 to $3, from each male citizen over 21 years of age.

473. The officers who estimate, or *assess*, the value of the property subject to taxation are called **assessors.**

Property is not generally assessed at its full value.

474. The officer who collects the taxes, when there is a special officer for the purpose, is called a **collector**; he receives either a salary or a per cent of the tax collected.

The collector's fee is sometimes paid by the person who pays the tax. In such cases it is not included in the amount of tax to be raised.

475. The rate of taxation is usually expressed as a *number of mills* on each *dollar*, or a *number of cents* on each *hundred dollars* of assessed valuation. A tax rate of 1 mill means a tax of 1 mill on each dollar, and is therefore a tax of $\frac{1}{10}\%$.

EXERCISES

476. 1. What is my tax on property assessed at $2000, if the tax rate is 10 mills on a dollar?

Find the tax on the following property:

2. $1500 @ 3 mills	5. $3600 @ $1\frac{1}{4}$ mills
3. $6000 @ 4 mills	6. $10,000 @ $5\frac{1}{2}$ mills
4. $7500 @ 2 mills	7. $20,000 @ $4\frac{3}{4}$ mills

Find the tax rate expressed as mills on a dollar:

	Valuation	Tax		Valuation	Tax		Valuation	Tax
8.	$1500	$4.50	11.	$10,000	$40	14.	$100,000	$100
9.	$2500	$5.00	12.	$15,000	$45	15.	$250,000	$625
10.	$4000	$6.00	13.	$75,000	$300	16.	$300,000	$900

Find the assessed valuation:

	Tax	Rate		Tax	Rate		Tax	Rate
17.	$10	1 mill	19.	$75	5 mills	21.	$5000	5 mills
18.	$40	2 mills	20.	$600	3 mills	22.	$8000	4 mills

WRITTEN EXERCISES

477. 1. A union school district levied a tax of $3420 on an assessed valuation of $456,000. Find the tax rate; also the tax paid by a citizen whose real property was assessed at $9000 and personal property at $3000.

Solution

$3420 ÷ $456,000 = .0075, or $7\frac{1}{2}$ mills on a dollar.
Tax on $9000 + $3000, or $12,000 = .0075 of $12,000 = $90.

2. If the assessed valuation of a town is $12,346,000, what will be the rate of tax to raise $82,965.12?

3. How much must a citizen of this town pay, including the collector's fee of 1%, if his property is assessed at $12,500?

4. The assessed valuation of a city is $15,900,000, and the estimated cost of maintaining the schools is $27,825. Find the rate of school tax in mills.

5. A town wishes to raise by taxation $206,200. It is estimated to contain 9115 persons subject to a poll tax of $1 each. Its assessed valuation is $23,462,500. What will be the rate of property tax?

SUGGESTION.—First deduct the poll tax from the amount to be raised.

6. How much must a citizen of this town pay, if he is subject to a poll tax, and has real property assessed at $40,000 and personal property at $25,000, the collector's fee being $\frac{1}{2}$%?

7. How much county tax must be raised to pay for a courthouse that cost $53,595, if the tax includes $\frac{3}{4}$% for collection?

SUGGESTION. $53,595 = 99$\frac{1}{4}$% of the tax to be raised.

8. The taxable property in a certain city consists of $93,100,000 real estate and $36,400,000 personal property. The tax rate is 18.6 mills per dollar on an assessed valuation of 80% of the actual value. Find the amount of tax raised.

9. Find, to the nearest cent, the tax for each borough of Greater New York in a year when the valuation and tax rates were:

BOROUGH	TAX RATE PER $100	REALTY	PERSONALTY
Manhattan	$1.413	$3,483,793,382	$549,843,253
Bronx	1.413	247,090,767	14,762,041
Brooklyn	1.489	853,742,357	100,052,348
Queens	1.475	123,781,723	10,176,900
Richmond	1.496	43,124,597	6,031,550

10. One year the tax rate of a city was $2.14 per $100; the next year an economical administration lowered it to $1.90. Find the reduction in taxes on property assessed at $64,500.

11. Find the total state tax raised in the State of New York in a year when the assessed valuation of the property in the state was $7,738,165,640, the tax rate being $\frac{154}{1000}$ of a mill per dollar.

To facilitate calculation, **tax tables** are often used.

TAX TABLE. RATE, 1.74 MILLS ON $1

PROP.	TAX	PROP.	TAX	PROP.	TAX	PROP.	TAX
$1	$0.002	$10	$0.017	$100	$0.174	$1000	$ 1.74
2	.003	20	.035	200	.348	2000	3.48
3	.005	30	.052	300	.522	3000	5.22
4	.007	40	.070	400	.696	4000	6.96
5	.009	50	.087	500	.870	5000	8.70
6	.010	60	.104	600	1.044	6000	10.44
7	.012	70	.122	700	1.218	7000	12.18
8	.014	80	.139	800	1.392	8000	13.92
9	.016	90	.157	900	1.566	9000	15.66

12. Find by the table Mr. Butler's tax on property assessed at $9594.

SOLUTION

Tax on $9000 = $15.66
Tax on 500 = .87
Tax on 90 = .157
Tax on 4 = .007
Tax on $9594 = $16.694 = $16.70

13. Mr. Hall's property is valued at $2504, and his poll tax is $1. Find his tax by the table.

Find by the table the tax on property valued at:

14. $3556 **16.** $70,345 **18.** $643,200 **20.** $750,000

15. $4012 **17.** $89,614 **19.** $521,000 **21.** $641,000

Duties or Customs

478. The income for the support of our national government is derived from **duties** or **customs** and from **internal revenue**, which consists chiefly of taxes on spirits, tobacco, etc.

479. The duties are collected at **customhouses**, which are maintained at *ports of entry*.

480. Imported goods may be divided into four classes: first, goods admitted free of duty; second, goods subject to an **ad valorem duty**, that is, a certain per cent of the cost of the goods; third, goods subject to a **specific duty**, a certain amount per yard, pound, etc., without regard to value; fourth, goods subject to both specific and ad valorem duties.

For example, coffee is admitted free; laces pay an ad valorem duty of 60%; the specific duty on wheat is 25¢ per bushel; and the duty on linen collars is 20% ad valorem and 40¢ per dozen.

Before computing specific duties allowance is often made for *tare*, or the weight of the box, bag, etc.

481. A schedule of duties on merchandise is called a **tariff**.

WRITTEN EXERCISES

482. Find the duty on:

	Imports	Value and Quantity	Tariff
1.	Blankets	$150, 450 lb.	30% + 22¢ per lb.
2.	Clocks	$4125	40%
3.	Eggs	3000 doz.	5¢ per doz.
4.	Quicksilver	100 flasks (76.5 lb. each)	7¢ per lb.
5.	Skins, tanned	$6430	20%
6.	Table knives	$600, 100 doz.	15% + 16¢ each
7.	Umbrellas	$7500	50%
8.	Wheat	10,000 bu.	25¢ per bu.

9. The specific duty on cheese is 6¢ per pound. Find the duty on 750 pounds of cheese.

10. The ad valorem duty on jewelry is 60%. Find the duty on a bracelet appraised at $550.

11. Find the duty on 400 boxes of raisins valued at $1.50 a box, each box containing 36 pounds, if the rate is 1¢ a pound and 35% ad valorem.

12. Find the duty on 25,000 cigars, invoiced at $62.50 per M, weighing on an average $10\frac{1}{2}$ ounces per box of 50 cigars. The tariff rate is $4.50 per pound and 25% ad valorem.

13. A merchant imported 80 dozen razors valued at $6 a dozen. He paid a duty of $1.75 a dozen and 20% ad valorem. Transportation cost $4.00. How much did each razor cost him?

14. Find the duty on 600 dozen linen handkerchiefs costing 7*s.* 6*d.* per dozen at 50% ad valorem.

15. Compute the duty; also the dutiable value:

MANIFEST NO. *751.* INVOICED AT *Bristol, Eng. June 21, 1906.*

INWARD FOREIGN ENTRY OF MERCHANDISE

IMPORTED BY *Bates & Holman* IN THE STEAMER *Holloway*

Henry Maxwell, Master. From *Liverpool, Eng.*

Arrived *July 5, '06.*

MARKS	NUMBERS	PACKAGES AND CONTENTS	QUANTITY	FREE LIST	VALUE	AD VALOREM DUTY		SPECIFIC DUTY		TOTAL DUTY
A.A.	*1624*	*8 cases woolen yarn*	*2400 lb.*		*£252.10.0*	*40%*	£ . . .	*38½¢ lb.*	$ . .	
⊕	*833*	*12 cases cotton hose*	*384 doz.*		*£149.12.0*	*15%*	£ . . .	*70¢ doz.*	$ . .	
									$ . .	
					£		£ . . .			
					$		$ . . .			$ . . .

16. A shipment of skein silk, costing 2754 francs in Lyons and weighing 324 pounds net, was taxed 40 cents per pound and 15% ad valorem. How much duty was paid on it?

17. A man imported, from Paris, paintings valued at 125,400 francs upon which there was an ad valorem duty of 20%. Freight and insurance charges were \$84.50. How much did his paintings cost him?

18. Find the entire cost in this country of 2000 pounds of macaroni bought in Italy for 500 lire, if the duty was $1\frac{1}{2}$¢ per pound and the freight charges were \$10.60.

19. A merchant imported cut and stained glass invoiced at 12,440 crowns (20 crowns = \$4.052). If the tariff rate was 60% ad valorem, how much duty did he pay?

20. Find the duty on 3250 yards of English wool suiting costing 7*s.* $2\frac{1}{4}$*d.* per yard and weighing 1 pound per yard, at \$.44 per pound and 55% ad valorem.

21. If it takes $3\frac{1}{4}$ yards of this material to make a suit of clothes, how much does the tariff add to the cost of a suit?

INSURANCE

483. Indemnity against loss or damage is called **insurance.**

Indemnity against loss of property by fire is called **fire insurance**; against loss of property at sea, **marine insurance**; against loss by personal injuries, **accident insurance**; against loss by sickness, **health insurance**; against loss by death, **life insurance**; against loss by injuries to employees, **liability insurance.** There are many other forms of insurance.

484. The contract or written agreement between the person protected and the insurance company is called the **policy**, and the amount of indemnity, or "protection," the **face of the policy.**

485. The price paid for insurance is called the **premium.**

Insurance rates are expressed in per cent, or in cents per \$100, or in dollars and cents per \$1000.

Property Insurance

486. The principal kinds of **property insurance** are fire insurance, marine insurance, and such special forms as plate glass insurance, tornado insurance, burglar insurance, credit insurance, etc.

WRITTEN EXERCISES

487. 1. A frame house with a shingle roof was insured for \$3500 at 70¢ per \$100 for 1 year. Find the premium.

2. If the roof had been tin, the rate would have been 60¢ per \$100. Find the difference in premiums for 10 years.

3. Find the annual premium on a brick dwelling insured for \$5000 at 25¢ per \$100.

4. If the rate for 3 years is only twice the annual rate, how much will be saved in 12 years by insuring a house for \$5000 in 3-year periods, the annual rate being 25¢ per \$100?

5. If the premium paid for a policy is \$9.60 and the rate is 60¢ per \$100, what is the face of the policy?

6. If the premium is \$19.50 and the rate is $\frac{1}{2}$%, what is the face of the policy?

7. If the premium on a policy for \$3800 is \$15.20, what is the rate in cents per \$100? in per cent?

8. A house worth \$6000, insured for \$4500, was damaged to the extent of \$5000. Find the owner's loss; also the company's loss.

9. A store insured for \$5000 in the Phœnix, \$3000 in the Firemen's, and \$2000 in the Protective, was damaged by fire to the extent of \$4000. Find the loss of each company.

10. A merchant insured his stock of silk for \$15,000 for 15 days, and paid 14% of the annual rate, which was 2%. Find the amount of premium that he paid.

11. A canal boat contains 8000 bushels of corn worth 42¢ a bushel. If the corn is insured for $\frac{7}{8}$ of its value at 85¢ per $100, what will be the owner's net loss in case the cargo is destroyed?

12. A jeweler paid $3.50 a year to insure each of his two plate glass windows, and he also insured his stock for $25,000 against burglary, at $15 per $1000. Find his yearly premium for both.

Personal Insurance

488. The principal kinds of insurance of persons, or **personal insurance**, are life insurance, health insurance, accident insurance, and liability insurance. Life insurance is the most important kind.

489. The most important kinds of life insurance policies are:

1. **Ordinary Life Policies.** — The holder of an ordinary life policy pays a certain premium at the beginning of each year from the time he secures the policy until his death; at his death the company pays the face of the policy to his estate or to the persons named in the policy as his beneficiaries.

2. **Limited-payment Life Policies.** — These are paid for in a limited number of years, after which they are said to be *paid up*. Thus, a "20-payment life" policy is fully paid up in 20 years.

3. **Endowment Policies.** — These run for a specified number of years. The face of the policy is paid to the insured if he lives to the end of the term, or to his estate or beneficiaries if he dies before that time. Premiums are usually payable annually, for the whole term.

4. **Term Policies.** — These give only temporary insurance. No part of the face of the policy is paid unless the insured dies during the term.

490. The average number of years a healthy person of a given age has yet to live is called his **expectation of life**. As shown in the table on page 367, a person's expectation of life decreases as his age increases.

491. Life insurance premiums are given in dollars and cents per $1000. The rate depends upon the kind of policy and upon the age of the insured or his expectation of life.

Brief Table of Annual Premiums for Insurance of $1000 with Participation in Profits Annually

Age	Ordinary Life	20-Payment Life	10-Payment Life	20-Year Endowment	25-Year Endowment	30-Year Endowment	20-Year Term	Expectation of Life
Years								Years
20	$18.50	$26.90	$42.70	$48.10	$37.60	$31.00	$13.00	43.07
25	20.70	29.40	46.40	48.70	38.30	31.70	14.00	39.49
30	23.50	32.30	50.90	49.60	39.30	32.90	15.40	35.85
35	27.30	36.00	56.30	50.80	40.80	34.70	17.70	32.17
40	32.20	40.60	62.80	52.80	43.20	37.60	21.50	28.48
45	38.80	46.60	70.80	56.00	47.10	42.30	27.70	24.82
50	47.90	54.70	80.80	61.30	53.40	49.70	37.70	21.24
55	60.40	65.70	93.20	69.80				17.80
60	77.70	81.20	108.90					14.56

The age of the insured is his age at his nearest birthday.

Premiums, if annual, are paid at the *beginning* of each year, and dividends (profits) are returned at the *end* of each year or credited as a part of the next premium, or applied to purchase additional insurance.

WRITTEN EXERCISES

492. Use the table in solving exercises 1–7 inclusive.

1. How much will it cost annually to carry an ordinary life policy for $2000 (age 25)?

2. How much will it cost annually to carry a life policy for $5000 (age 25), if it is agreed that the insurance is to be fully paid for by the first 20 annual premiums?

3. How much will it cost annually to carry a 20-year endowment policy for $2500 (age 20)?

4. Suppose that a man 30 years old takes out an ordinary life policy for $1000 and dies at the end of his expectation of life. How many premiums will he have paid? If the dividends (including a post mortem dividend for .85 of a year) amount to $207.36, what will be the net cost of his insurance?

5. A man 25 years old takes out a 20-payment life policy for \$3000, and lives the full term. Find the total premium.

6. Suppose that he dies at the end of his expectation of life. If the dividends average \$7 annually for 20 years, and then \$2 annually for 19.49 years, find the net cost of his insurance.

7. A man 35 years old took out a 20-year term policy for \$4000. He paid 10 premiums, receiving \$300 in dividends, and then gave up his policy. What was the net cost of his insurance?

8. A man took out a 15-year endowment policy for \$3000 at \$68 per thousand, and paid ten annual premiums, receiving the following dividends: \$8.12, \$9.22, \$9.75, \$9.50, \$10.10, \$9.40, \$8.93, \$9.90, \$10.75, \$11.00. At the end of the tenth year he surrendered the policy to the company, and received a cash value of \$580. Did he gain or lose, and how much?

9. What was the net cost per thousand of his "protection" for these ten years?

10. A man secured a health policy that would pay him \$30 per week indemnity in case of illness. For each \$5 of weekly indemnity he paid \$8.75 per year. Find the cost of carrying the policy for 12 years.

11. During this time he was confined by illness 6 weeks at home and 8 weeks in a hospital. While he was at the hospital he received double indemnity. Find the total indemnity.

12. A man obtained insurance for \$4000 against accident for 9 months, and was charged 85% of the annual premium of \$11.25 for each \$1000. Find the cost of his "protection."

13. A manufacturer paid a liability company \$172.50 per month to assume all expenses that might arise from claims for injuries sustained by employees in the performance of their duties. This was $\frac{3}{4}$% of the average monthly pay roll. Find the amount of the monthly pay roll.

REVIEW PROBLEMS IN INDUSTRIES

493. **1.** A Louisiana sugar-cane plantation contained 120 acres. If it was 726 feet wide, how long was it?

2. The sugar cane was planted in rows that ran the long way of the field and were $5\frac{1}{2}$ feet apart. How many rows of cane did the width of the field allow?

3. How much seed cane was used for the plantation, if $5\frac{3}{4}$ tons were planted on each of the 120 acres?

4. At harvest time in November, 2340 tons of cane were obtained. Find the yield per acre.

5. The cost of producing the cane was $37.05 per acre. What was the cost per ton?

6. At the sugar mill the planter received 80 pounds of sugar for each of the 2340 tons of cane. What were his receipts for cane, if he sold the sugar at $3\frac{1}{2}$¢ per pound?

7. What was his whole gain? his gain per acre?

8. In the sugar mill the cane was crushed between rollers to extract the juice. The juice extracted was 75 % of the weight of the cane. How much was extracted from the 2340 tons?

9. A test showed that the juice was $13\frac{1}{3}$ % sugar. How many tons of sugar did the 1755 tons of juice contain?

10. Not all of the sugar in the juice could be extracted. Only 210.6 tons were obtained. What part of the 234 tons of sugar in the juice was extracted?

11. What per cent of the weight of the raw cane, then, was actually manufactured into sugar?

12. Since 9% of the cane was converted into sugar, find the yield of sugar from 1 ton of cane; from an acre, $19\frac{1}{2}$ tons.

13. After paying 80 lb. of sugar for each of the 2340 tons of cane, how much of the 210.6 tons of sugar did the mill owner have? How much was it worth at $3.50 per 100 lb.?

14. That part of the raw cane which remains after the extraction of the juice is called *bagasse*. As 75% of the cane was juice, how many tons of bagasse were there?

15. The 585 tons of bagasse were used for fuel and were equivalent to $97\frac{1}{2}$ tons of coal. Find its value per ton, if coal was worth $3.90 per ton.

16. By the addition of water to dissolve the juice while the cane is being crushed, the yield of sugar may be increased 5 pounds per ton of cane. How many more pounds are thus obtained by a mill that crushes 500 tons of cane per day for 95 days?

17. How much more value is obtained from a ton of cane by this process when sugar is worth 3.5¢ per pound?

18. If the water added amounts to 10% of the weight of cane used, how many pounds of water does one ton of cane take?

19. How many pounds of coal are needed to evaporate the 200 lb. of water added to a ton of cane, if 1 lb. of coal evaporates 8 lb. of water? Find the cost of the coal at $4 per ton.

20. If the only expense of the 5 extra pounds of sugar is for coal, how much is gained per ton of cane by adding water?

21. What, then, is the added gain during the season, if the mill crushes 500 tons per day for 95 days?

22. The average yield of sugar cane per acre is 18 tons in Louisiana, 34 tons in Cuba, and 55 tons in Hawaii. If the sugar extracted averages 8% of the weight of the cane in Louisiana and 12% in Cuba and Hawaii, find the average yield of sugar per acre in each locality.

23. A farmer in Michigan sowed a 24-acre field with sugar beet seed, 20 pounds to the acre. Find the cost of the seed at 15¢ per pound.

24. It cost $5.20 per acre for thinning, $6.10 per acre for weeding, and $5.75 per acre for pulling the beets and cutting off the tops. Find the sum of these expenses for 1 acre and for the whole field.

25. If these three expenses, $409.20, were 55% of the whole cost of the beets to the farmer, find the cost of the crop; the cost per acre.

26. The crop of sugar beets, amounting to 300 tons, was drawn to the factory in wagons holding $2\frac{1}{2}$ tons each. How many wagon loads were there?

27. At the factory the beets were bought by the ton but not until all dirt, called *tare*, had been removed. If the tare in this case was 8% of the entire weight, how many tons of clean beets were there?

28. The factory paid $4 per ton for beets that contained 12% of their weight in sugar, and 25¢ more per ton for each additional per cent. These beets tested 14%. How much did the farmer receive for his crop of 276 tons?

29. Find his profit on the crop.

30. Though testing 14%, each ton of beets yielded only 224 pounds of sugar. What per cent of the sugar was extracted?

31. The sugar extracted from the 276 tons of beets was sold at $4\frac{5}{8}$ ¢ per pound. How much was received for it?

32. If the total cost of the manufactured sugar to the operators of the factory was $4 per hundredweight, how much did they gain on each ton of clean beets?

33. This factory used 45,339 tons of beets during its working season of 105 days. How many tons did it use per day?

34. If 431.8 tons was only 85% of its daily capacity, find its capacity.

Find the cost of each of the following items, which were among this factory's principal expenses:

35. 9660 tons of coal @ $2.50.

36. 3220 tons of lime rock @ $1.90.

37. 21,000 pounds of sulphur @ $2\frac{1}{4}$ ¢.

38. 8350 yards of filter cloth @ 15 ¢.

39. 19,300 sugar bags @ 9 ¢.

40. 24,135 sugar barrels @ $26\frac{1}{3}$ ¢.

41. In a recent year Louisiana produced 283,500 tons of cane sugar, which was 7 % of the world's production. How much cane sugar was produced that year?

42. That year 24% of the whole yield of 4,050,000 tons was produced in Cuba. How many tons did Cuba produce?

43. The world's production of beet sugar in the same year was 5,800,000 tons. How many more tons of beet sugar were produced than of cane sugar?

44. How much of the beet sugar was furnished by the United States, if we produced $3\frac{3}{8}$ % of the world's crop?

45. What per cent of the combined production of cane and beet sugar did Europe furnish, if 5,614,500 tons were produced there?

Cotton from the field consists of seed and fiber together. At the cotton gin the seed is separated from the fiber, or lint, which is pressed into bales, covered with cloth called bagging, and bound with heavy iron wire called ties. Before long-distance shipment the bales are compressed to fill a still smaller space. The weight of a bale is spoken of as "gross," when it includes bagging and ties, "net," when the lint alone is considered.

46. A bale of cotton before compression occupies about 45 cubic feet of space. When compressed for shipment it is about 54 in. by 27 in. by 16 in. How much less space does a compressed bale occupy?

47. How many compressed bales of $13\frac{1}{2}$ cu. ft. can be put into the space that would be occupied by 3 uncompressed bales?

48. The average gross weight of a bale of American cotton is 500 pounds. If the weight of bagging and ties is 5 % of the gross weight, what is the net weight of a bale?

49. The average gross weights of bales in some other cotton countries are:

India, 400 pounds	Peru, 182 pounds
Brazil, 230 pounds	Egypt, 735 pounds

What per cent of the American 500-pound bale is each of these bales?

50. If it takes 1500 pounds of seed cotton from the field to produce a bale in which the lint weighs 480 pounds, what per cent of the seed cotton is lint? What per cent is seed?

51. If 20 acres produce 12,960 pounds of seed cotton, and $33\frac{1}{3}\%$ of it is lint, how many bales of 480 pounds net does it make?

52. What per cent of a bale does one acre produce?

53. How long would it take a laborer to pick enough seed cotton to make a bale of lint of this weight, if he picks 120 pounds of seed cotton per day?

54. If it takes the cotton gin 1 hr. 2 min. 30 sec. to separate 500 pounds of lint from the seed, how long does it take to gin one pound?

55. At the rate of 2 pounds of lint in 15 seconds how long does it take to gin a bale of 480 pounds net?

56. On 16 acres a cotton farmer raised 720 pounds of seed cotton per acre. After giving 25% of his crop to the owner for rent, how much did he have left?

57. It cost $51.84 to have the entire crop of 11,520 lb. of cotton picked. How much did each picker receive per 100 pounds?

58. If $66\frac{2}{3}\%$ of the farmer's 8640 pounds of cotton consisted of seed, how many bushels of seed of 30 pounds each did he have? how many pounds of lint?

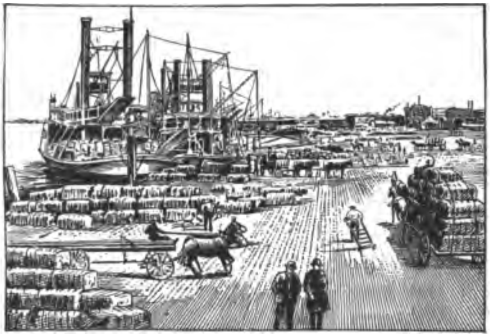
59. When the cotton was ginned and pressed, each bale of lint weighed 500 pounds, including 20 pounds of bagging and ties. How many such bales did the 2880 pounds of lint make?

60. How much did the farmer receive, if he sold the 5760 lb. of seed at 12¢ per bushel (30 lb.) and the 6 bales of lint (500 lb. each) at $6\frac{1}{2}$¢ per pound, gross weight?

61. Find his gain, if the total expense of raising and marketing the crop was 75% of the $218.04 received.

62. If the farmer had paid his rent in money, $\$3\frac{1}{2}$ an acre for 16 acres, and had sold the 4000 lb. gross of cotton @ $6\frac{1}{2}$¢ and the 256 bu. of seed @ 12¢, how much would he then have gained, the total expense aside from rent amounting to $167.53?

63. A New Orleans cotton buyer paid $1062.50 to ship 425 bales, averaging 500 pounds gross, to New York. Find the freight rate per hundredweight.



64. At that time the freight rate to Liverpool was only 35¢ per 100 pounds. How much less would it have cost to ship the same cotton to Liverpool?

65. A car load of cotton in the ordinary square bales consisted of 55 bales, averaging 500 pounds each in weight. How many pounds of cotton did the car carry?

66. Recently a large car load of cotton in the more compact round bales consisted of 160 bales, averaging $428\frac{1}{8}$ pounds each in weight. Find the weight of cotton carried by the car.

67. In a recent year the world's output of cotton was equal to 18,000,000 bales of 500 pounds net. Find the number of such bales contributed by each of the following countries according to the per cents given:

Egypt, $6\frac{2}{3}$%	East Indies, $16\frac{2}{3}$%
Brazil, $1\frac{2}{3}$%	United States, 75%

68. If Great Britain used 3,600,000 of these bales, the continent of Europe 5,100,000, and the United States 4,320,000, what per cent of the whole crop did each consume?

SIMPLE INTEREST

494. The sum paid for the use of money is called **interest.**

495. The money for the use of which interest is paid is called the **principal.**

496. The sum of the principal and the interest is called the **amount.**

497. Interest is reckoned as a certain *per cent* of the principal, and the **rate of interest** is the per cent paid for the use of the principal for *one year.*

498. In ordinary interest calculations, a month is regarded as 30 days, and a year as 12 months, or 360 days.

METHOD BY ALIQUOT PARTS

499. 1. If a man pays $6 for the use of $100 for 1 year, how much must he pay for the use of $100 for 2 yr.? for 3 yr.?

How much must he pay for the use of $100 for 6 months, or $\frac{1}{2}$ of a year? for 2 mo., or $\frac{1}{3}$ of 6 mo.? for 30 days, or $\frac{1}{2}$ of 2 mo.? for 15 days, or $\frac{1}{2}$ of 30 days?

2. When 6% is paid for the use of the principal for 1 year, what per cent of the principal is paid for its use for 2 yr.? for 6 mo.? for 2 mo.? for 6 da.?

3. Find the interest on $400 at 5% for 1 year; for 3 yr.; for 6 mo.; for 3 yr. 6 mo.

4. What is the interest at 4% on $600 for 1 year? for 2 months? for 15 days? for 1 yr. 2 mo. 15 da.?

WRITTEN EXERCISES

500. 1. Find the interest on \$325 for 3 years at 5%.

\$325	We may find 3 times 5% of \$325; or 3 times 5%, that is 15%, of \$325.	\$325
.05		.15
\$16.25		16 25
3		32 5
\$48.75		\$48.75

The interest equals the principal multiplied by the rate, multiplied by the number expressing the time in years.

Find the interest on:

2. \$250 for 2 yr. at 5%
3. \$500 for 5 yr. at 3%
4. \$275 for 3 yr. at 7%
5. \$5000 for 2 yr. at 4½%
6. \$6250 for 6 yr. at 4%
7. \$87.50 for 5 yr. at 3%
8. Find the interest on \$629 for 1 yr. 8 mo. at 7%.

SOLUTION

	\$629,	principal
	.07,	rate
	\$44.03,	int. for 1 yr.
½ of int. for 1 yr. =	22.015,	int. for 6 mo.
⅓ of int. for 6 mo. =	7.338,	int. for 2 mo.
	\$73.38,	int. for 1 yr. 8 mo.

NOTE.—It is sufficiently accurate to express final results to the nearest cent and intermediate results to the nearest mill.

Find, by aliquot parts, the interest on:

9. \$5000 for 1 yr. 6 mo. at 3%
10. \$5000 for 1 yr. 9 mo. at 4%
11. \$3225 from Aug. 1, 1906, to Nov. 1, 1907, at 4%
12. \$4175 from May 1, 1906, to Nov. 1, 1909, at 3½%

13. Find the interest and the amount of $500 for 3 yr. 5 mo. 22 da. at 5%.

Solution

Principal				$500	
Int. for 1 yr., 5% of $500	1 yr.	$25			
2 times int. for 1 yr.	2 yr.	50			
$\frac{1}{3}$ of int. for 1 yr.	4 mo.	8	333		
$\frac{1}{4}$ of int. for 4 mo.	1 mo.	2	083		
$\frac{1}{3}$ of int. for 1 mo.	10 da.		694		
$\frac{1}{3}$ " " " " "	10 da.		694		
$\frac{1}{5}$ of int. for 10 da.	2 da.		139		
Interest for 3 yr. 5 mo. 22 da.				86	94
Amount for 3 yr. 5 mo. 22 da.				$586	94

Find, by aliquot parts, the interest and the amount of:

14. $5000 for 1 yr. 3 mo. 3 da. at 6%

15. $5000 for 1 yr. 6 mo. 6 da. at 5%

16. $2500 for 1 yr. 7 mo. 15 da. at 7%

17. $3650 for 2 yr. 3 mo. 21 da. at 5%

18. $7500 for 3 yr. 2 mo. 18 da. at 4%

19. $875.25 for 2 yr. 5 mo. 16 da. at 6%

20. $169.75 for 3 yr. 11 mo. 10 da. at $3\frac{1}{2}$%

21. $15,000 for 60 days at $4\frac{1}{2}$%

22. $500,000 for 90 days at 2%

23. $3264.18 for 63 days at 5%

24. $4870.21 for 120 days at 6%

25. $3000 from Oct. 11, 1906, to Jan. 26, 1907, at 5%

26. $5000 from Aug. 16, 1907, to Dec. 28, 1909, at $3\frac{1}{2}$%

27. $466.30 from Nov. 12, 1906, to June 30, 1912, at 6%

28. $174.65 from Sept. 27, 1906, to Apr. 16, 1908, at 7%

29. $50,000 from Dec. 6, 1907, to June 4, 1908, at $2\frac{1}{2}$%

30. $125,000 from Oct. 22, 1906, to Sept. 11, 1907, at $4\frac{1}{2}$%

31. $320,000 from May 26, 1907, to July 15, 1910, at 5%

SIX PER CENT METHOD

501. The great bulk of money loaned in the money market is loaned for terms of 90 days, 60 days, or less. For such terms and a rate of 6% the most convenient unit of time is 2 months, or **60 days**, $\frac{1}{6}$ of a year. Hence,

The interest for 60 *days at* 6% *is* 1% *of the principal, found by moving the decimal point two places toward the left.*

By adding or subtracting parts or multiples of the interest for 60 days, the interest for any term may be found.

The following processes illustrate the method of finding the interest at 6% on $5000, for the terms mentioned, from the interest for 60 days:

30 days	10 days	6 days	5 days
2)$50.00	6)$50.00	10)$50.00	12)$50.00
$25.00	$ 8.33	$ 5.00	$ 4.17

90 days	11 days	125 days	53 days
$50.00	6)$50.00	$50.00	6)$50.00, 60 da.
$25.00	10) 8.333	12)50.00	−8.33, 10 da.
$75.00	.833	4.17	$41.67, 50 da.
	$9.17	$104.17	+2.50, 3 da.
			$44.17, 53 da.

This method is sometimes called the **banker's method.**

WRITTEN EXERCISES

502. Find the interest at 6% on:

1. $450 for 30 days
2. $750 for 90 days
3. $500 for 60 days
4. $825 for 63 days
5. $1200 for 45 days
6. $5000 for 60 days
7. $37,500 for 120 days
8. $75,000 for 125 days

503. For convenience, bankers often use the following:

TABLE OF DAYS INTERVENING BETWEEN DATES

	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
Mar.	306	337	365	31	61	92	122	153	184	214	245	275
Apr.	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	333	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

The number of days from any day of one month to the *same* day of another month is found by starting at the name of the first (in the left-hand column) and following across to the column headed with the name of the second. (Should Feb. 29th of a leap year intervene between dates, add 1 day.)

Suppose it is required to find the number of days from June 5 to Nov. 10. From the table we find that it is 153 days from June 5 to Nov. 5; adding 5 days, we find the required time to be 158 days.

WRITTEN EXERCISES

504. Find, by the six per cent method and the use of the table, the interest at 6 % on the following for the terms mentioned:

1. $5000, Mar. 6 to May 5
2. $2750, Apr. 10 to July 9
3. $3000, June 2 to July 5
4. $5000, July 5 to Nov. 2
5. $4500, Aug. 8 to Oct. 7
6. $7225, Sept. 1 to Dec. 3
7. $8400, Nov. 30 to Feb. 1
8. $1600, July 25 to Nov. 22
9. $22.50, June 30 to Aug. 2
10. $35.75, Sept. 26 to Nov. 15
11. $15,000, Aug. 21 to Sept. 4
12. $50,000, Dec. 22 to Jan. 7
13. $125,000, May 4 to July 1
14. $500,000, Nov. 6 to Feb. 2

505. Another form of the *six per cent method* often used makes the interest on $1 for the given time the basis of computation.

Thus, the interest on $1 for 1 year at 6% is $.06; for 1 month, $.005; for 6 days, $.001; and for 1 day, $.000$\frac{1}{6}$.

WRITTEN EXERCISES

506. 1. Find the interest on $240.50 for 2 yr. 4 mo. 14 da. at 6%.

Solution

Int. on $1 for 2 yr.	$.12
Int. on $1 for 4 mo.	.02
Int. on $1 for 14 da.	.002$\frac{1}{3}$
Int. on $1 for 2 yr. 4 mo. 14 da. =	.142$\frac{1}{3}$

The interest on $240.50 = 240.50 × $.142$\frac{1}{3}$, or $34.23.

Find the interest on the following at 6%:

2. $42.50 for 7 yr. 4 mo. 20 da.
3. $39.75 for 5 yr. 3 mo. 15 da.
4. $425.10 for 6 yr. 5 mo. 10 da.
5. $365.42 for 8 yr. 10 mo. 24 da.
6. $875.50 for 4 yr. 11 mo. 18 da.
7. $920.65 for 4 yr. 10 mo. 17 da.
8. $1500 from Mar. 14, 1906, to Apr. 3, 1907
9. $4800 from June 24, 1907, to Dec. 15, 1908
10. $3200 from Dec. 18, 1905, to Feb. 24, 1914
11. $64.45 from Feb. 28, 1907, to Oct. 31, 1912
12. $75,000 from Aug. 16, 1906, to Feb. 12, 1910
13. $56,000 from Feb. 24, 1907, to Dec. 17, 1909
14. $375.60 from May 22, 1903, to Mar. 16, 1908
15. $840.07 from Dec. 10, 1902, to Nov. 20, 1910
16. $225,000 from Oct. 4, 1906, to Feb. 11, 1908
17. $500,000 from Aug. 1, 1906, to Jan. 31, 1907

507. From the interest on any principal at 6% the interest at other rates may be found by adding or subtracting aliquot parts of the interest at 6%, as follows:

Adding	$\frac{1}{6}$ of itself	for the interest at **7%**
Subtracting	$\frac{1}{6}$ of itself	for the interest at **5%**
Adding	$\frac{1}{3}$ of itself	for the interest at **8%**
Subtracting	$\frac{1}{3}$ of itself	for the interest at **4%**
Adding	$\frac{1}{4}$ of itself	for the interest at **7½%**
Subtracting	$\frac{1}{4}$ of itself	for the interest at **4½%**
Dividing by	2	for the interest at **3%**

and so on.

WRITTEN EXERCISES

508. Find, by the six per cent method, the interest and the amount of:

1. $250 for 60 da. at 7%
2. $500 for 30 da. at 5%
3. $400 for 90 da. at 8%
4. $500 for 60 da. at 4%
5. $160 for 63 da. at 3%
6. $775 for 33 da. at 4%
7. $840 for 12 da. at 7½%
8. $450 for 15 da. at 4½%
9. $750 for 10 da. at 4%
10. $800 for 45 da. at 3%
11. $900 for 25 da. at 5%
12. $500 for 120 da. at 4½%
13. $5000 for 3 yr. 5 mo. 10 da. at 5%
14. $4000 for 2 yr. 7 mo. 24 da. at 8%
15. $5000 for 4 yr. 9 mo. 11 da. at 3%
16. $1400 for 2 yr. 1 mo. 22 da. at 4½%
17. $5000 for 4 mo. 2 da. at 3%; at 3½%
18. $7500 for 2 mo. 5 da. at 5%; at 5½%
19. $2440 from June 11, 1905, to Sept. 8, 1906, at 4%
20. $3685 from Feb. 17, 1906, to July 9, 1907, at 5%
21. $75.50 from Nov. 22, 1905, to May 10, 1906, at 7%
22. $825.25 from Sept. 8, 1904, to Apr. 3, 1910, at 7½%
23. $25,000 from May 16, 1907, to Aug. 14, 1908, at 3%

CANCELLATION METHOD

509. This method is a modification of the six per cent method.

When the rate is 6%, the interest for 60 days is 1% of the principal, and for 6 days, $\frac{1}{10}$% of the principal, or .001 of the principal. Hence,

The interest at 6% *for any number of days is* .001 *of the principal multiplied by the number of days divided by* 6.

The interest at any other rate is found by taking such a part of the interest at 6% as the given rate is of 6%.

WRITTEN EXERCISES

510. **1.** Find the interest on $6228 for 93 days at 6%; also at 4%.

SOLUTIONS

$$\overset{\$1.038}{\$\cancel{6.228}} \times \frac{93}{\cancel{6}} =$$

$$\$1.038 \times 93 = \$96.53$$

$$\overset{\$1.038}{\$\cancel{6.228}} \times \frac{\overset{31}{\cancel{93}}}{\cancel{6}} \times \frac{\overset{2}{\cancel{4}}}{\underset{\cancel{2}}{\cancel{6}}} =$$

$$\$1.038 \times 31 \times 2 = \$64.36$$

Find, by cancellation, the interest on the following for the given time and rate:

2. $3000, 12 days, 6%

3. $5000, 8 days, 4%

4. $7500, 15 days, 3%

5. $4365, 60 days, 5%

6. $8000, 10 days, 4%

7. $7200, 90 days, 4%

8. $3420, 42 days, 6%

9. $6250, 36 days, 4½%

10. $7200, 93 days, 3½%

11. $2400, July 6 to Oct. 4, 5%

12. $6300, June 4 to Aug. 3, 4%

13. $7800, Nov. 10 to Feb. 10, 3%

14. $8100, Dec. 12 to Feb. 1, 6%

15. $9600, May 16 to Aug. 14, 5%

16. $2700, Mar. 1 to June 2, 7%

17. $75.60, Apr. 1 to June 30, 7½%

18. $43.20, Nov. 4 to Jan. 3, 4½%

19. $85.50, Aug. 7 to Dec. 11, 5%

INTEREST TABLE METHOD

511. Bankers and others who have considerable interest to compute facilitate their work by the use of **interest tables.**

The following incomplete table illustrates the nature of such tables. This one is based on a year of 360 days and a rate of 6%.

Years	$ 1000	$ 2000	$ 3000	$ 4000	$ 5000	$ 6000	$ 7000	$ 8000	$ 9000
1	60	120	180	240	300	360	420	480	540
2	120	240	360	480	600	720	840	960	1080
3	180	360	540	720	900	1080	1260	1440	1620
4	240	480	720	960	1200	1440	1680	1920	2160
5	300	600	900	1200	1500	1800	2100	2400	2700
6	360	720	1080	1440	1800	2160	2520	2880	3240
...	...	...	...	...	...	...	...	...	...

Months	$ 1000	$ 2000	$ 3000	$ 4000	$ 5000	$ 6000	$ 7000	$ 8000	$ 9000
1	5	10	15	20	25	30	35	40	45
2	10	20	30	40	50	60	70	80	90
3	15	30	45	60	75	90	105	120	135
4	20	40	60	80	100	120	140	160	180
5	25	50	75	100	125	150	175	200	225
6	30	60	90	120	150	180	210	240	270
7	35	70	105	140	175	210	245	280	315
8	40	80	120	160	200	240	280	320	360
...	...	...	...	...	...	...	...	...	...

Days	$ 1000	$ 2000	$ 3000	$ 4000	$ 5000	$ 6000	$ 7000	$ 8000	$ 9000
1	.167	.333	.50	.667	.833	1.00	1.167	1.333	1.50
2	.333	.667	1.00	1.333	1.667	2.00	2.333	2.667	3.00
3	.500	1.000	1.50	2.000	2.500	3.00	3.500	4.000	4.50
4	.667	1.333	2.00	2.667	3.333	4.00	4.667	5.333	6.00
5	.833	1.667	2.50	3.333	4.167	5.00	5.833	6.667	7.50
6	1.000	2.000	3.00	4.000	5.000	6.00	7.000	8.000	9.00
7	1.167	2.333	3.50	4.667	5.833	7.00	8 167	9.333	10.50
8	1.333	2.667	4.00	5.333	6.667	8.00	9.333	10.667	12.00
9	1.500	3.000	4.50	6.000	7.500	9.00	10.500	12.000	13.50
10	1.667	3.333	5.00	6.667	8.333	10.00	11.667	13.333	15.00
11	1.833	3.667	5.50	7.333	9.167	11.00	12.833	14.667	16.50
12	2.000	4.000	6.00	8.000	10.000	12.00	14.000	16.000	18.00
13	2.167	4.333	6.50	8.667	10.833	13.00	15.167	17.333	19.50
14	2.333	4.667	7.00	9.333	11.667	14.00	16.333	18.667	21.00
15	2.500	5.000	7.50	10.000	12.500	15.00	17.500	20.000	22.50
16	2.667	5.333	8.00	10.667	13.333	16.00	18.667	21.333	24.00
17	2.833	5.667	8.50	11.333	14.167	17.00	19.833	22.667	25.50
18	3.000	6.000	9.00	12.000	15.000	18.00	21.000	24.000	27.00
19	3.167	6.333	9.50	12.667	15.833	19.00	22.167	25.333	28.50
20	3.333	6.667	10.00	13.333	16.667	20.00	23.333	26.667	30.00
...	...	...	...	...	...	...	...	...	...

WRITTEN EXERCISES

512. **1.** Find, by the interest table, the interest on $2348 for 3 yr. 2 mo. 7 da. at 6%.

SOLUTION

Since the interest on any number of *hundred* dollars is .1 of that on the *same* number of *thousand* dollars; on any number of *tens* of dollars .01 of that on the *same* number of *thousand* dollars, etc., we have,

	3 YR.		2 MO.		7 DA.	
Int. on $2000 =	$360.00	+	$20.00	+	$2.333	
Int. on 300 =	54.00	+	3.00	+	.350	
Int. on 40 =	7.20	+	.40	+	.047	
Int. on 8 =	1.44	+	.08	+	.009	
Int. on $2348 =	$422.64	+	$23.48	+	$2.739	= $448.86

NOTE. — Though the table is not complete, interest for any number of years, months, and days may be found by it. For example, the interest for 8 yr. is the same as for 6 yr. + 2 yr.; for 11 mo., the same as for 8 mo. + 3 mo.; for 25 da., the same as for 20 da. + 5 da.

The student is already familiar with methods of finding the interest at other rates, from the interest at 6%.

Find, by the interest table, the interest on the following for the given time and rate:

2. $2500, 12 days, 6%

3. $3200, 14 days, 6%

4. $4500, 11 days, 6%

5. $5000, 6 days, 6%

6. $4800, 29 days, 6%

7. $8700, 25 days, 6%

8. $7435, 5 mo., 3%

9. $2467, 3 mo., 5%

10. $5762, 11 mo., 4½%

11. $3840, 10 days, 5%

12. $4070, 15 days, 4%

13. $3175, 21 days, 3%

14. $5000, Jan. 30, 1906, to Mar. 11, 1907, 4%

15. $8040, Feb. 16, 1906, to May 12, 1908, 6%

16. $5340.25, May 8, 1907, to Mar. 1, 1910, 5%

17. $8257.75, Aug. 4, 1905, to June 22, 1909, 4½%

18. $3375.45, Jan. 16 to Mar. 10, in a leap year, 3%

WRITTEN EXERCISES

513. Using any convenient method, find the interest at 6%, 4%, $4\frac{1}{2}$%, 3%, and 5% on:

1. $6440 for 63 da.
2. $3200 for 75 da.
3. $5000 for 2 da.
4. $5000 for 5 da.
5. $5000 for 10 da.
6. $5000 for 30 da.
7. $5000 for 60 da.
8. $5000 for 90 da.
9. $5000 for 120 da.
10. $5000 for 125 da.
11. $133.20 for 21 da.
12. $786.32 for 27 da.
13. $432.60 for 11 da.
14. $635.80 for 57 da.
15. $752.19 for 93 da.
16. $563.22 for 3 mo. 18 da.
17. $287.10 for 5 mo. 12 da.
18. $1000 for 1 yr. 2 mo. 9 da.
19. $1200 for 2 yr. 7 mo. 3 da.
20. $16,000, Feb. 1 to Feb. 28
21. $45,000, Jan. 10 to Jan. 20
22. $500,000, June 7 to July 1
23. $2,500,000 for 60 da.
24. $13,200,000 for 30 da.

25. A man lived on the income from $50,000. If $\frac{1}{4}$ of it was invested at 7% and the rest at 5%, find his annual income.

26. A resident of Colorado Springs borrowed money in the East at 5% and loaned it on good security at home at 8%. Find his annual gain on an investment of $75,000.

27. A coal dealer purchased his coal at $3.20 per ton cash on the first of April, and sold it 6 months later. If money was worth 5%, what was the real cost of the coal to him at the time of sale?

28. A man invested $10,000 in a gold mine and for 12 years received an annual return of 15% on his investment. After 12 years, however, the ore was all mined and the man lost his principal. How much better or worse off was he than if he had invested his money in safe securities yielding 4% interest, if he did not reinvest any interest in either case?

ACCURATE INTEREST

514. Interest computed by taking the exact number of days between dates and reckoning 365 days for a year is called **accurate**, or **exact, interest.**

Accurate interest, then, does not differ from ordinary interest for whole years, but for terms less than a year it is $\frac{360}{365}$ of ordinary interest, or it is ordinary interest less $\frac{5}{365}$, or $\frac{1}{73}$, of itself, when the latter is reckoned for the actual number of days.

It may also be computed by accurate interest tables.

Accurate interest is used by the United States government, by some banks, and to some extent in other business.

WRITTEN EXERCISES

515. Find the accurate interest on:

1. \$200 for 90 da. at 6%
2. \$500 for 33 da. at 6%
3. \$750 for 60 da. at 5%
4. \$480 for 45 da. at 4%
5. \$5000 for 120 da. at 3%
6. \$6400 for 111 da. at 4%
7. \$7200 for 214 da. at 5%
8. \$8100 for 179 da. at $4\frac{1}{2}$%

Using the table for days on page 380, find the accurate interest at 6% on:

9. \$150, Mar. 2 to July 15
10. \$444, Aug. 1 to Dec. 1
11. \$765, May 1 to Nov. 1
12. \$480, June 1 to Dec. 1
13. \$5000, Apr. 1 to Oct. 1
14. \$5000, May 1 to Nov. 1
15. \$4640, June 1 to Nov. 4
16. \$3875, July 6 to Dec. 9

17. How much less is the accurate interest on \$10,000 from June 15 to Sept. 15 at 3% than the interest computed in the ordinary way?

18. The United States pays accurate interest. How much interest is saved thereby on \$2,500,000, borrowed from Mar. 1 to July 1 at $4\frac{1}{2}$%?

PROBLEMS IN INTEREST

516. To find the rate.

1. What is the interest on $200 for 2 years at 1%?

2. Since $200 yields $4 interest in 2 years at 1%, at what rate will it yield $8 interest in the same time? $12 interest? $24 interest?

3. At what rate will $500 yield $100 interest in 5 years?

4. At what rate will $1000 yield $120 interest in 2 years?

WRITTEN EXERCISES

517. 1. At what rate must $800 be invested to yield an interest of $176 in 5 yr. 6 mo.?

SOLUTIONS

1. The interest on $800 at 1% for $5\frac{1}{2}$ years is $5\frac{1}{2}$% of $800, or $44.

Since a rate of 1% yields $44, it will require a rate of as many per cent to yield $176 as $44 is contained times in $176, or 4%.

2. Let x = the rate. Then the interest on $800 for $5\frac{1}{2}$ years is $5\frac{1}{2} \times \frac{x}{100}$ of $800, or $44x$ dollars. Since this interest is 176 dollars,

$$44x = 176.$$
$$\therefore x = 4, \text{ and the rate is } 4\%.$$

Test. — Interest on $800 for $5\frac{1}{2}$ years at 4% = $176.

The rate equals the given interest divided by the interest for the given time at 1%.

At what rate of interest will

2. $300 yield an interest of $60 in 4 yr.?

3. $500 yield an interest of $90 in 3 yr.?

4. $700 yield an interest of $70 in 2 yr. 6. mo.?

Find the rate of interest when the interest on

5. $500 for 5 yr. 3 mo. is $105

6. $900 for 3 yr. 4 mo. is $180

7. $150 for 4 yr. 6 mo. is $33.75

8. $225 for 5 yr. 8 mo. is $76.50

9. $3200 for 3 yr. 1 mo. 15 da. is $400

10. $4500 for 6 yr. 4 mo. 24 da. is $1296

11. I deposited $3200 in a bank and 6 months later was credited with $56 interest. What rate of interest was allowed?

12. A man borrowed $3000 and gave a written promise to pay the lender $3050 in four months. How much of the $3050 was interest? What rate of interest was paid?

13. At what rate will $600 amount to $720 in 4 years?

14. A house that cost $5400 rented for $576 per year. Taxes and repairs cost the owner $157.50 a year. Find the net annual income and the rate of interest realized on the investment.

15. The profits earned by a power plant in a year, after deducting all expenses and allowing for depreciation of the property, were $8187.50. The plant cost $25,000. What rate of interest was earned on the investment?

518. To find the time.

1. What interest will $200 earn in 1 year at 6%?

2. In what time will $200 at 6% earn $12 interest? $18? $30?

WRITTEN EXERCISES

519. 1. In what time will $750 at 6% yield $250 interest?

SOLUTIONS

1. The interest for 1 yr. at 6% is 6% of $750, or $45.

Hence the time required is $\frac{250}{45}$ yr., or $5\frac{5}{9}$ yr., or 5 yr. 6 mo. 20 da.

2. Let x = the number of years. Then $\frac{6x}{100} \times 750 = 250$.

Hence, $45x = 250$

$\therefore x = 5\frac{5}{9}$, and the time is $5\frac{5}{9}$ yr.

Test. — Interest on $750 for $5\frac{5}{9}$ years at 6% = $250.

The number of years equals the given interest divided by the interest at the given rate for 1 year.

Find how long each of the following sums at the given rate must draw interest to amount to \$1000:

SUGGESTION. — First find the total interest by subtracting the principal from the amount.

2. \$600 at 8%	4. \$900 at 4%	6. \$625 at $4\frac{1}{2}$%
3. \$800 at 5%	5. \$840 at 6%	7. \$875 at $3\frac{1}{2}$%

8. In how many years will any principal double itself at 4%? at 5%? at 2%? at 6%? at $4\frac{1}{2}$%?

SUGGESTION. — The interest to be earned is equal to the principal. How many times does the principal contain 4% of itself?

9. In how many years will any principal treble itself at 5%?

520. To find the principal.

WRITTEN EXERCISES

1. What principal invested at 5% per annum will yield a yearly income of \$1000?

SOLUTIONS

1. Since \$1 at 5% yields \$.05 interest per year, as many dollars must be invested to yield \$1000 per year as \$.05 is contained times in \$1000. \$1000 ÷ \$.05 = 20,000. Hence \$20,000 must be invested.

2. Let x = number of dollars that must be invested.

Then,
$$.05x = 1000.$$
$$x = 1000 \div .05 = 20{,}000.$$

Hence the principal required is \$20,000.

Test. 5% of \$20,000 = \$1000.

The principal equals the given interest divided by the interest on \$1 *for the given time at the given rate.*

What principal will yield an annual income of

2. \$125 at 5%?	4. \$750 at 6%?	6. \$1000 at 2%?
3. \$500 at 4%?	5. \$900 at $4\frac{1}{2}$%?	7. \$3000 at $7\frac{1}{2}$%?

What principal will yield an interest of

8. \$375 in 4 yr. at 5%?
9. \$900 in 2 yr. at 6%?
10. \$840 in 7 yr. at 3%?
11. \$500 in 2 yr. 6 mo. at 4%?
12. \$480 in 3 yr. 4 mo. at 6%?
13. \$1000 in 60 da. at 5%?

14. A man wishes to insure his life for an amount that invested at 4% will provide an annual income of \$3000 after his death. For what amount shall he insure his life?

15. A man bequeathed to a college enough money so that the interest on it at $3\frac{1}{2}$% would provide for 10 annual scholarships of \$350 each. Find the amount of the gift or endowment.

Present Worth and True Discount

521. 1. What will be the amount of \$100 in 1 year, if loaned at 6% interest? in 2 years? in 3 years?

2. If money is loaned at 6%, what is the value *now* of \$106 due 1 year hence? What is the present value, or *present worth*, of \$112 due 2 years hence? of \$118 due in 3 years?

3. When money is worth 6%, what sum should be deducted from a debt of \$106 paid 1 year before it is due? What *discount* should be allowed on \$1060 paid 1 year before it is due?

522. The **present worth** of a sum due at a future time is its *cash* value; or it is the sum that loaned at the current market rate would amount to the given sum in the given time.

523. The difference between the face value of a sum due at a future time and its present worth, or the sum that should be deducted for immediate payment, is sometimes called the **true discount.**

The present worth may be regarded as the *principal*, the true discount as the *interest*, and the sum due at a future time as the *amount*.

WRITTEN EXERCISES

524. 1. A man owing me $500 due in 1 yr. 3 mo. wishes to pay me now. How much should I accept in payment, money being worth 5%? Find the true discount.

Solutions

1. Since every dollar put at interest at 5% would amount to $1.0625 in 1 yr. 3 mo., it will require as many dollars now to amount to $500 in 1 yr. 3 mo. as $1.0625 is contained times in $500. $500 ÷ $1.0625 = 470.59 (to the nearest .01). Hence I should accept a cash payment of $470.59.

The true discount is $500 – $470.59, or $29.41.

2. In 1 yr. 3 mo., x dollars at 5% will amount to $1.0625x$ dollars, or to the sum due at that time, $500.

Hence,
$$1.0625x = 500.$$
$$\therefore x = 500 \div 1.0625 = 470.59.$$

Hence present worth = $470.59; true discount = $500 – $470.59, or $29.41.

The present worth equals the given amount divided by the amount of $1 for the given time at the given rate.

2. A merchant bought a bill of goods amounting to $800 on 60 days' credit. Find the equivalent cash value of the goods, if money was worth 6%.

3. What is the present worth of $1000 due 2 yr. 6 mo. from date, if money can be borrowed at 6%?

4. Find the true discount on $5000 paid 6 months before it is due, if money is worth 5% per annum.

5. What sum of money put at interest for 9 months at $3\frac{1}{2}$% will amount to $1642?

6. A person just 20 years old is named in a will as heir to $5000 due when he is 21 years old. What is the present worth of his inheritance, money being worth 5%?

7. I have an endowment policy for $5000 due in 10 months. Find its present value, money being worth 6%.

ANNUAL INTEREST

525. In some states when a written agreement contains the expression "with interest payable *annually*," simple interest may be collected upon the principal and upon each year's interest that has not been paid when due. This is called **annual interest.**

WRITTEN EXERCISES

526. 1. No interest having been paid, find the amount due in 3 years 6 months 6 days on $1000, with interest payable annually at 6%.

SOLUTION

Principal		$1000	
Simple interest on $1000 at 6% for 3 yr. 6 mo. 6 da. . . .		211	
The interest for each year is $60			
The 1st annual int, $60, remains unpaid for	2 yr. 6 mo. 6 da.		
The 2d annual int., $60, remains unpaid for	1 yr. 6 mo. 6 da.		
The 3d annual int., $60, remains unpaid for	6 mo. 6 da.		
Interest on $60 at 6% for . . .	4 yr. 6 mo. 18 da.	16	38
Amount of $1000		$1227	38

The amount equals the principal, plus the simple interest for the entire time, plus the interest on each year's interest for the time it remains unpaid.

Find the amount, with annual interest, of:

2. $6200 for 2 yr. 6 mo. at 6%
3. $5850 for 4 yr. 3 mo. at 4%
4. $8760 for 3 yr. 8 mo. at 5%
5. $10,000 for 2 yr. 4 mo. 24 da. at 3%
6. $24,000 for 3 yr. 3 mo. 12 da. at 5%
7. $36,500 for 1 yr. 7 mo. 18 da. at 6%

COMPOUND INTEREST

527. Interest on the principal and its unpaid interest, combined at regular intervals, is called **compound interest.**

Interest may be compounded with the principal annually, or semiannually, or quarterly, etc.

Compound interest cannot usually be enforced by law, even though it is specified in the contract. Nevertheless the subject is of importance since it is at the basis of computations concerning investments, for there is nothing to hinder a man from drawing his interest and investing it.

WRITTEN EXERCISES

528. 1. Find the amount of $400 for 2 yr. 4 mo. at 6 %, interest compounded annually; also find the compound interest.

SOLUTION

Principal	$400
Interest for 1st yr. at 6 %	24
Principal beginning 2d yr.	$424
Interest for 2d yr. at 6 %	25.44
Principal beginning 3d yr.	$449.44
Interest for 4 mo. at 6 %	8.99
Amount for 2 yr. 4 mo. at 6 %	$458.43

Since the amount of $400 at compound interest is $458.43, the compound interest is $458.43 – $400, or $58.43.

NOTE. — Unless otherwise specified, interest is understood to be compounded *annually*. If compounded semiannually, the rate must be considered one half the annual rate mentioned; if quarterly, one fourth, etc.

When the time consists of years, months, and days, the amount is to be found for the greatest number of entire periods, as years, half years, quarter years, etc., and the simple interest upon this for the rest of the time.

Find the amount and the compound interest of:

2. \$400 for 2 yr. at 5 %
3. \$1000 for 3 yr. at 4 %
4. \$1500 for 4 yr. 6 mo. at 6 %
5. \$5000 for 2 yr. at 6 %, payable semiannually
6. \$5000 for 2 yr. at 6 %, payable quarterly
7. \$8000 for 1 yr. 8 mo. at 4 %, payable quarterly
8. Find the amount of \$1000 for 10 years at 4 % compound interest; at 4 %, compounded semiannually.

SOLUTIONS

1. By the table on the next page the amount of \$1 at 4% compound interest for 10 years is \$1.480244. Hence the amount of \$1000 for the same time at the same rate is 1000 × \$1.480244, or \$1480.24.

2. If the rate is 4 % compounded semiannually, there are 20 interest periods and the rate for each is 2 %. Therefore the amount of \$1000 for 10 years at 4 %, compounded semiannually, is the same as the amount of \$1000 for 20 years at 2 %, compounded annually. By use of the table this amount is found to be \$1485.95.

Find, by the table, the amount at compound interest of:

9. \$1000 for 10 yr. at 5 %
10. \$5000 for 20 yr. at 3½ %
11. \$3250 for 12 yr. at 4½ %
12. \$725.32 for 16 yr. at 6 %
13. \$4000 for 8 yr. 3 mo. at 4 %
14. \$6000 for 6 yr. 8 mo. at 2½ %
15. \$7000 for 14 yr. 9 mo. at 3 %
16. \$86.48 for 18 yr. 10 mo. 15 da. at 6 %
17. \$75.50 for 15 yr. 6 mo. 3 da. at 4 %
18. \$5000 for 8 yr. at 6 %, payable semiannually
19. \$6500 for 4 yr. 4 mo. 10 da. at 4 %, payable quarterly

COMPOUND INTEREST TABLE

Amount of $1, at various rates, compound interest, 1 to 20 years

YEARS	1%	1¼%	1½%	2%	2½%	3%
1	1.010000	1.012500	1.015000	1.020000	1.025000	1.030000
2	1.020100	1.025156	1.030225	1.040400	1.050625	1.060900
3	1.030301	1.037971	1.045678	1.061208	1.076891	1.092727
4	1.040604	1.050945	1.061364	1.082432	1.103813	1.125509
5	1.051010	1.064082	1.077284	1.104081	1.131408	1.159274
6	1.061520	1.077383	1.093443	1.126162	1.159693	1.194052
7	1.072135	1.090850	1.109845	1.148686	1.188686	1.229874
8	1.082857	1.104486	1.126493	1.171659	1.218403	1.266770
9	1.093685	1.118292	1.143390	1.195093	1.248863	1.304773
10	1.104622	1.132271	1.160541	1.218994	1.280085	1.343916
11	1.115668	1.146424	1.177949	1.243374	1.312087	1.384234
12	1.126825	1.160755	1.195618	1.268242	1.344889	1.425761
13	1.138093	1.175264	1.213552	1.293607	1.378511	1.468534
14	1.149474	1.189955	1.231756	1.319479	1.412974	1.512590
15	1.160969	1.204829	1.250232	1.345868	1.448298	1.557967
16	1.172579	1.219889	1.268986	1.372786	1.484506	1.604706
17	1.184304	1.235138	1.288020	1.400241	1.521618	1.652848
18	1.196148	1.250477	1.307341	1.428246	1.559659	1.702433
19	1.208109	1.266108	1.326951	1.456811	1.598650	1.753506
20	1.220190	1.281935	1.346855	1.485947	1.638616	1.806111

YEARS	3½%	4%	4½%	5%	6%	7%
1	1.035000	1.040000	1.045000	1.050000	1.060000	1.070000
2	1.071225	1.081600	1.092025	1.102500	1.123600	1.144900
3	1.108718	1.124864	1.141166	1.157625	1.191016	1.225043
4	1.147523	1.169859	1.192519	1.215506	1.262477	1.310796
5	1.187686	1.216653	1.246182	1.276282	1.338226	1.402552
6	1.229255	1.265319	1.302260	1.340096	1.418519	1.500730
7	1.272279	1.315932	1.360862	1.407100	1.503630	1.605782
8	1.316809	1.368569	1.422101	1.477455	1.593848	1.718186
9	1.362897	1.423312	1.486095	1.551328	1.689479	1.838459
10	1.410599	1.480244	1.552969	1.628895	1.790848	1.967151
11	1.459970	1.539454	1.622853	1.710339	1.898299	2.104852
12	1.511069	1.601032	1.695881	1.795856	2.012197	2.252192
13	1.563956	1.665074	1.772196	1.885649	2.132928	2.409845
14	1.618695	1.731676	1.851945	1.979932	2.260904	2.578534
15	1.675349	1.800944	1.935282	2.078928	2.396558	2.759032
16	1.733986	1.872981	2.022370	2.182875	2.540352	2.952164
17	1.794676	1.947901	2.113377	2.292018	2.692773	3.158815
18	1.857489	2.025817	2.208479	2.406619	2.854339	3.379932
19	1.922501	2.106849	2.307860	2.526950	3.025600	3.616528
20	1.989789	2.191123	2.411714	2.653298	3.207136	3.869684

PART IV

PRELIMINARY REVIEW

529. Reduce:

1. 3 bu. 2 pk. 6 qt. to pints.
2. 4 T. 8 cwt. 9 oz. to ounces.
3. 1 mi. 27 rd. 4 yd. 2 ft. 6 in. to inches.
4. 5 A. 100 sq. rd. 18 sq. yd. 3 sq. ft. 36 sq. in. to sq. in.

Reduce to higher denominations:

5. 73,286″.
6. 208,738 sec.
7. 36,894 cu. in.
8. 48,682 gr. (troy).

9. A drill for sowing wheat covered 24 acres of ground per day. How long did it take to sow a field 216 rd. by 160 rd.?

10. Find the weight of a box of dynamite whose contents measure 4 ft. by $2\frac{1}{2}$ ft. by 3 ft., if 1 cubic foot weighs $103\frac{1}{8}$ lb.

11. If 420 tons of drinking water are consumed on a vessel during an Atlantic voyage, how many gallons are used?

12. Find the capacity in bushels of a Duluth grain elevator having 2,250,000 cubic feet of storage space. (1 bu. $= 1\frac{1}{4}$ cu. ft.)

13. A body floating in water displaces its own weight of water. The steamship *Baltic* weighs 40,000 tons. How many cubic feet of salt water, specific gravity 1.03, does she displace?

14. *A diamond carat is* 3.168 *grains.* The weight of the largest diamond in the world is $3025\frac{3}{4}$ carats. How much more than $1\frac{1}{4}$ pounds avoirdupois does the diamond weigh?

15. The English mint is compelled to coin any gold offered it, returning 77*s.* $10\frac{1}{2}$*d.* per ounce $\frac{11}{12}$ fine. Show that the standard weight of a sovereign is 123.27447 grains.

16. When the temperature falls from 15° F. to −10° F., how many Fahrenheit degrees does it fall? how many Centigrade degrees?

17. How much cheaper is it to paper a room $14' 8'' \times 12' 9''$ with double rolls that give 7 strips each, than with single rolls that give 3 strips each, if 9 strips are allowed for openings, the price per roll being 25 ¢, per double roll, 50 ¢?

18. If 1 gallon of stain will cover 400 square feet of soft wood, how much stain will be required to give a coat of stain to a white pine wainscoting 5 feet high around a room $20' \times 17' 6''$, deducting 75 square feet for openings?

19. Good cypress shingles may be laid $4\frac{1}{4}$ inches to the weather. How many, to the nearest shingle, are required to cover a square? How many bunches must be purchased to cover a roof $38' \times 35'$, estimating 875 shingles per square?

20. In fireproof construction expanded metal or wire laths are used instead of wooden ones. In plastering a room $30' \times 21'$ and 11′ high, deducting $\frac{1}{2}$ for 3 windows, each $7' \times 3' 9''$, and for 1 door $10' \times 10'$, how much more will fireproof construction cost at 70 ¢ per square yard than ordinary construction at 40 ¢?

21. Of the 20,000,000 bushels of wheat exported by the United States one year, 9,000,000 bushels went to Great Britain. What per cent of the wheat went to Great Britain?

22. The Hudson River ice crop in a poor season was 1,440,000 tons, or 68 % less than the crop of the previous season. How many tons of ice were harvested the previous season?

23. A salesman for an importing house is offered his choice of a salary of $2500 a year, or $1500 a year with a commission of 2 % on his sales, or 6 % commission on all sales. He accepts the second offer and sells goods to the value of $60,000. How much better off is he than by the first offer? how much worse off than by the third?

24. A manufacturer marked his goods down $12\frac{1}{2}\%$ from the list price on account of a fall in their market value, and allowed an additional discount of 4 % for cash. What was the entire discount, expressed in per cent?

25. A retail druggist gets a discount of 40 % from the wholesale list price and sells at the list price. What per cent of profit does he make?

For exercises 26–28 use the rates given in the table on page 367.

26. By taking out a 20-payment life policy for \$2000 at the age of 25, how much more will a man have paid by the end of the term than by taking out a 10-payment life policy?

27. When 35 years old Mr. Adams took out a 20-year endowment policy for \$4000. How much less would it have cost him for the term, if he had taken it out at the age of 20?

28. Suppose that a man 40 years of age takes out an ordinary life policy for \$2000 and lives 2 years beyond his expectation of life. How much more or less than the face of the policy will he have paid?

29. What is the value of a merchant's stock, if he insures it for $\frac{5}{6}$ of its value and pays \$210 at an insurance rate of $1\frac{2}{5}\%$?

30. The premium on a building insured at \$8.75 per \$1000 is \$66.50. For what sum is the building insured? Express the rate in per cent.

31. Two companies insured a mill at $1\frac{1}{4}\%$, one for \$13,200, the other for \$22,000. Find the total annual premium.

Fire occasioned a loss of \$8400. What part of the loss had each company to pay? how much money?

32. A factory worth \$64,800 was insured for $\frac{3}{8}$ of its value at \$7.50 per \$1000. After 8 premiums had been paid, the factory was damaged by fire to the extent of \$50,000. What was the loss to the owners including the premiums paid?

33. A city's school tax one year, amounting to \$68,502, was raised by a tax rate of 5.25 mills per dollar. What was the total assessed valuation of taxable property in the city?

34. The assessed valuation of Mr. Roberts' store was \$19,800, or 90% of its true value. Besides paying a tax of 11.6 mills on the dollar, he insured the store for $\frac{3}{4}$ of its true value at \$1.35 per \$100. Find the cost of taxes and insurance.

35. The assessed valuation of a certain town is \$1,918,466. On this is to be raised a town tax of \$11,064.91, a county tax of \$3070.29, a state tax of \$228.99, and a stenographer's tax of \$97.10. Find the rate of tax on \$1000 to the nearest cent.

36. The duty on 6255 lb. of linseed oil was \$166.80. What was the schedule rate per gallon of $7\frac{1}{2}$ lb.?

37. Find the amount of specific duty, at 67¢ per long ton, on 25,200 bu. of bituminous coal, a bushel weighing 80 lb.

38. A vessel brought into the port of Boston 36 tons of barley upon which was levied a duty of 30¢ per bushel of 48 pounds. What was the amount of the duty?

39. A wholesale dealer paid 35% duty on 15 gross of doll heads and 750,000 marbles that cost him, respectively, \$1.60 per dozen and 12¢ per thousand. Including duty, what was the total cost?

40. At Philadelphia there was received a lumber shipment made up of 450 timbers, each $8'' \times 12'' \times 20'$; 360 timbers, each $9'' \times 16'' \times 24'$; and 1100 sawed boards, each $2'' \times 9'' \times 12'$. Find the duty at 1¢ per cubic foot for the timbers, and \$2 per thousand board feet for the boards.

41. An importation of drawing paper, size $24'' \times 20''$, consisted of 60,000 sheets weighing $15\frac{3}{4}$ lb. per ream and valued at 24¢ per pound. A ream at the customhouse being considered 180,000 sq. in., what was the total duty at $3\frac{1}{2}$¢ per pound and 15% ad valorem?

Give in U. S. money these costs in Norway (1 crown = 26.8 ¢):

42. 5 Kg. butter at $1\frac{1}{2}$ crowns per kilogram.

43. 14 M.T. hay at 65 crowns per metric ton.

44. 4 M.T. potatoes at $4\frac{1}{2}$ crowns per quintal.

45. 20 sacks flour, 100 Kg. each, at 19 crowns per quintal.

46. At the rate of 1 mm. per second, how many feet will a snail pass over in an hour?

47. If the earth revolves about the sun at the rate of 30 Km. per second, how many miles does it go per minute?

48. A motor touring car used only 9 liters of gasoline in going 100 kilometers. At this rate how many miles, to the nearest hundredth, could the car go on 1 gallon of gasoline?

49. When it is noon at Boston, whose longitude is 71° 3′ 50″ W., what time is it at Honolulu, 157° 51′ 34″ W.?

50. Hamburg, whose longitude is 9° 58′ 25″ E., has sunset 5 hr. 59 min. 25 sec. earlier than Charleston. What is the longitude of Charleston?

Using any convenient method, find the interest at 6 %, 7 %, $3\frac{1}{2}$ %, and 5 % on:

51. \$2000 for 30 da.

52. \$5000 for 63 da.

53. \$4500 for 15 da.

54. \$8000 for 90 da.

55. \$547.20 for 2 mo. 5 da.

56. \$4680 for 1 yr. 7 mo. 12 da.

57. \$5860 for 2 yr. 1 mo. 21 da.

58. \$30,000, May 19 to June 8.

59. Find the exact interest of \$21,900 from Dec. 17 to Jan. 31, at 5 %.

60. At 6 % find the amount of \$10,000 for 3 yr. 5 mo. 24 da. at simple interest; at annual interest; at compound interest.

61. A man withdraws the interest on his money at the bank as it falls due. He thus receives semiannually \$28.25, interest being at $2\frac{1}{2}$ % annually. What is the amount of his deposit and in how many years will it be equaled by the interest?

62. Mrs. Gordon deposited \$600 in a savings bank that paid interest quarterly. At the end of the first quarter she drew out the interest and $\frac{1}{3}$ of the principal, in all, \$205.25. What annual rate of interest did she receive?

63. Mr. Brown offers me \$8800 in cash for a house and lot, and Mr. White offers \$6000 in cash and \$3000 payable in 1 year without interest. Which is the better offer, and how much better, if money is worth 6 %? if money is worth 8 %?

530. Solve and test:

1. $2x - 36 = 60 - 6x$
2. $x + \frac{1}{7}x - \frac{1}{5}x = 33$
3. $7x - 30 = 10 + 16 - 7x$
4. $\frac{3}{8}x - \frac{1}{5}x + \frac{1}{4}x + \frac{1}{12}x = 8$
5. $9x + 13 - 6x - x = 6x + 12 - 4x + 13 - 3x$
6. $\frac{1}{2}x - 4 + \frac{3}{4}x + 3 - 14x = 19 + \frac{1}{4}x - 6x - 10 - 8x$
7. $\begin{cases} x + 2y = 7 \\ 2x - y = 4 \end{cases}$
8. $\begin{cases} 7x - 2y = 3 \\ 5x + 3y = 11 \end{cases}$
9. $\begin{cases} x + \frac{1}{2}y = 16 \\ \frac{1}{2}x + y = 20 \end{cases}$
10. $\begin{cases} \frac{1}{3}x - \frac{1}{5}y = 3 \\ \frac{1}{5}x + \frac{1}{2}y = 8 \end{cases}$

Solve the following by using letters to represent unknown numbers.

11. In target practice the battleship *Alabama* fired 32 shots. The successful shots outnumbered the unsuccessful ones by 12. How many shots hit the target?

12. The powder and the shell used in a twelve-inch gun together weigh 1265 lb. The powder weighs 15 lb. more than $\frac{1}{4}$ of the weight of the shell. Find the weight of each.

13. If a balloon that made a trip of 233 miles had gone at the rate of $\frac{3}{5}$ of a mile per minute, the distance traveled would have been 2 miles less. How long did the trip take?

14. A newspaper reporter saved $\frac{1}{6}$ of his weekly salary, or \$1 more than was saved by an artist on the same paper, whose salary was \$5 greater but who saved only $\frac{1}{7}$ of it. How much did the reporter earn per week?

15. Each topmast of a seven-masted schooner was 58 feet in length, or 4 feet more than $\frac{3}{5}$ as long as each lower mast. Find the length of each lower mast.

16. In making 5000 pounds of brass there were used $8\frac{1}{2}$ times as much copper as tin, and twice as much tin as zinc. How many pounds of each metal were used?

17. The steamship *Carmania* brought to New York a gold shipment of $15,278,500, of which the National City Bank received $2,778,500 more than all others. How much did this bank receive?

18. The duty paid on an importation of 40,000 shingles and 160,000 laths was $52, and that on 80,000 shingles and 70,000 laths was $41.50. Find the rate of duty per M on each.

19. A grocer paid $8.50 for a molasses pump and 5 feet of tubing. He paid 12 times as much for the pump as for each foot of tubing. How much did the pump cost? the tubing?

20. A shipment of 12,000 tons of coal arrived at Boston on 3 barges and 2 schooners. Each schooner held $3\frac{1}{2}$ times as much as each barge. Find the capacity of a barge; of a schooner.

21. A fruit dealer bought 100 baskets and 4 dozen boxes of figs for $21.68. If the price per box had been the same as the price per basket, he would have paid $.96 less. Find the price per basket and the price per box.

22. During a year of 365 days one locality had 6 days less of "clear" weather than of "cloudy" weather, and 4 days more of "clear" than of "partly cloudy" weather. Find the number of days of each kind of weather during the year.

23. At a certain time the circulation of money in the United States per person was $33.08. For a circulation of $36 per person with the same quantity of money the population would have had to be 6,895,580 less. What was the true population?

PROMISSORY NOTES

531. A written promise made by one person to pay to another a definite sum of money at a definite time is called a **promissory note**, or simply a **note**.

$4000.	Chicago, Ill., May 15, 1906.

....Three months.....after date...I...promise to pay to the order of Walter Winters................................

Four thousand and $\frac{00}{100}$ ~~~~~~~~~~~~~~~~ Dollars.

Value received, with interest at 5%.

John Simpson.

532. The *essentials* of a note are as follows:

1. It must be signed by the person who promises to pay. This person is called the **maker**, or **drawer**.

2. It must designate by name, or otherwise, the person to whom the money is to be paid. This person is called the **payee**.

3. The *sum* to be paid must be definite. It is called the **face** of the note.

The face of a note is usually written both in words and in figures.

4. The *time* of payment must be definite, in the sense that it is a time that can be determined and is not ambiguous.

It may be payable *on demand;* at a *specified time* after date, which must appear on the note; on or before a *specified date;* etc. When no time of payment is mentioned, the note is payable on demand.

A promise to pay "when trade will permit" is not binding.

533. A note should be definite in regard to the following particulars, according to the agreement between the maker and the payee:

1. Unless the *place* of payment is expressly stated, the residence or place of business of the maker is understood, and this must appear on the note.

2. When a note reads "with interest," but does not give the rate of interest, the legal rate in the state where the note was made may be collected.

3. If a note does not contain the words "with interest," no interest may be collected for the time it has to run; but if the note is not paid when due, it then begins to draw interest at the legal rate.

NOTE. — The words "value received" are usually included in a note, though they are not essential.

534. The following illustrates a **demand** note:

\$ 84.$\frac{65}{}$ *Baltimore, Md., Aug. 10, 1907.*

On demand ... I ... promise to pay to ....

Hiram Fairbanks .... or order,

Eighty-four and $\frac{65}{100}$ ~~~~~~~~~~~~~~~~ Dollars.

Silas Vincent.

The words "or order" mean that Silas Vincent will pay \$84.65 to any one to whom Hiram Fairbanks may order it paid.

A note payable "to bearer" is payable to any one who presents it.

EXERCISES

535. Examine the note in § 534 and answer these questions:

1. Who is the maker? the payee?
2. When is the note payable?
3. Where is the note payable?
4. What is the face of the note?
5. Answer the same questions in regard to the note shown on the previous page.

536. When a note becomes payable it is said to **mature.**

1. At common law, arising from the custom of merchants in the past, a note matures three days after the time specified therein. These three days are called **days of grace.** The majority of the states and territories, however, have abolished days of grace.

2. If a note falls due on Sunday, a Saturday half-holiday, or a legal holiday, it is usually payable on the next *succeeding* business day; but in some states it must be paid on the preceding business day.

NOTE. — In regard to the legal time of maturity of notes the teacher should be governed by the law of the state in which the school is situated.

537. If a note is drawn payable "to bearer," the payee may sell it, or **negotiate** it, by delivering it to the purchaser; and any purchaser may negotiate it in the same way.

If the note is drawn payable "to order," a simple delivery is not sufficient to negotiate it, for it is payable to the payee only, until he orders it paid to another person. To do this he writes his name on the back of the note together with any instructions or conditions he deems best. This is called **indorsing** the note.

If a note is drawn payable to the payee only, no words "to order" or "to bearer" being inserted, it is non-negotiable.

538. The person who indorses a note is called an **indorser**; the person who holds a note is called the **holder.**

The payee is the first holder; if he sells the note, the purchaser becomes the holder, etc.

539. Indorsements are of various forms:

1. Suppose that Hiram Fairbanks, the payee of the note in § 534, sells the note to Seth Bell and indorses it **in blank**, as shown here.

INDORSEMENT IN BLANK

This indorsement is really an agreement between Mr. Fairbanks and all subsequent holders of the note to this effect: "For value received, I, Hiram Fairbanks, transfer to the bearer all my

right and title in this note. I guarantee that it is genuine and that it will be paid when due, if not by the maker, then by myself."

2. If Mr. Fairbanks indorses in **full** when he sells the note to Mr. Bell, no one except Mr. Bell can negotiate the note, for it is payable to his order only.

INDORSEMENT IN FULL

Pay to the order of
Seth Bell.
Hiram Fairbanks.

This form of indorsement is useful in sending notes through the mails, for if lost or stolen, they are valueless to persons obtaining them.

3. Suppose that Mr. Fairbanks wishes to avoid responsibility for the payment of the note. He will write "without recourse" over his name.

QUALIFIED INDORSEMENT

Without recourse.
Hiram Fairbanks.

This is a **qualified** indorsement *in blank.*

4. If Mr. Fairbanks, in selling Mr. Bell the note, wishes to ***restrict*** its transfer so that Mr. Bell cannot sell it, he writes over his signature "Pay to Seth Bell only," or "Pay to Seth Bell."

RESTRICTIVE INDORSEMENT

Pay to Seth Bell.
Hiram Fairbanks.

This is a **restrictive** indorsement.

A restrictive indorsement makes the note non-negotiable.

540. If, when a note is unpaid at its maturity, the holder fails to **protest** it, that is, to notify the indorsers in a manner prescribed by law that it is unpaid, they are released from responsibility regarding its payment.

541. A higher rate of interest than that authorized by law is called **usury**.

The penalty for making usurious contracts varies in the different states, from the loss of the whole debt and interest to nothing.

WRITTEN EXERCISES

542. 1. Write a negotiable note for $122.50, yourself the maker and Reuben Hamilton the payee. Make it payable ninety days after date in the place where you live, with interest at the legal rate in your state.

2. Write a non-negotiable demand note for $200 payable by yourself to John H. Fassett, without interest.

3. Write a note with three indorsements, the first and second *in full*, the last *in blank*.

4. Write a note with two indorsements in full, the second *without recourse*.

5. Write two forms of negotiable notes for $795.36 payable in three months to John C. Fenton, with interest.

6. Indorse them properly to transfer one to the bearer and the other to Richard Gray or order.

7. Write a demand note with interest. Write indorsements to show that it has been transferred four times, each time by a different kind of an indorsement, the last being *restrictive*.

8. Suppose that you have sold to Sanford & Williams merchandise for $1000, terms half cash and half payable in sixty days. Draw the note, with their signature. Make the note negotiable.

9. Indorse the note for selling it to J. K. Avery in such a way that he can sell it. Do this in two ways.

10. Suppose that Mr. Avery sells the note to Henry Baldwin, and indorses it in such a way as to avoid responsibility for its payment. Write the indorsement.

11. Suppose that Mr. Baldwin is the final holder of the note. If Sanford & Williams do not pay the note at maturity, who must pay it? Explain each indorser's liability.

12. Who are responsible to Mr. Baldwin for the payment of the note? Who is responsible to you?

PARTIAL PAYMENTS

543. It frequently happens that the maker of a note cannot pay the whole amount at one time, but instead he makes **partial payments.**

A record of the partial payments is written, or **indorsed,** on the back of the note with the dates when they were made.

United States Rule

WRITTEN EXERCISES

544. 1. A note for $1000, dated Jan. 1, 1904, interest 6%, was indorsed as follows: Mar. 22, 1904, $125; June 1, 1905, $50; Aug. 4, 1905, $75; Nov. 1, 1905, $150. Find the amount due Jan. 1, 1906.

SOLUTION

Principal, Jan. 1, 1904		$1000.00
Int. to Mar. 22, 1904, — 2 mo. 21 da.		13.50
Amount		1013.50
First payment		125.00
New principal, Mar. 22, 1904		$888.50
Int. to June 1, 1905, — 1 yr. 2 mo. 9 da.		63.53
Second payment (which is less than int. due)	$50.00	
Int. from June 1 to Aug. 4 — 2 mo. 3 da. on $888.50		9.33
Amount		961.36
Third payment (to be added to second)	$75.00	125.00
New principal, Aug. 4, 1905		$836.36
Int. to Nov. 1, 1905, — 2 mo. 27 da.		12.13
Amount		848.49
Fourth payment		150.00
New principal, Nov. 1, 1905		$698.49
Int. to Jan. 1, 1906, — 2 mo.		6.98
Amount due Jan. 1, 1906		$705.47

United States Rule. — *Find the amount of the principal to a time when a payment, or the sum of two or more payments, equals or exceeds the interest due, and from the amount subtract such payment or payments.*

With the remainder as a new principal proceed as before.

In other words, partial payments are applied first to pay the interest due, and then, if anything is left, to reduce the principal.

Most of the states have adopted the United States Rule for computing the indebtedness when partial payments have been made. In states where other methods are prescribed the rules of those states should be followed.

2. A note for $1000, bearing 5% interest, and dated Jan. 2, 1906, had these payments indorsed on it: Apr. 2, 1906, $200; July 2, 1906, $300. How much was due Oct. 12, 1906?

3. A note for $400, dated Apr. 1, 1905, had indorsed on it the following payments: July 1, 1905, $25; Sept. 1, 1905, $30; Dec. 11, 1905, $100. How much was due Apr. 5, 1906, with interest at 4%?

4. A note for $1600, bearing 6% interest, dated Nov. 1, 1904, had indorsed on it the following partial payments: Feb. 1, 1905, $60; Oct. 20, 1905, $20; Nov. 1, 1905, $400. How much was due Apr. 10, 1906?

5. A note for $4000, bearing 5% interest, dated Aug. 15, 1904, had the following partial payments indorsed on it: Jan. 2, 1905, $40; May 11, 1905, $500. How much was due June 1, 1906?

6. A note for $1250, given Feb. 28, 1903, had indorsed on it the following payments: July 1, 1903, $100; Dec. 16, 1904, $200; Oct. 11, 1905, $600. Find the amount due Apr. 4, 1906, with interest at 6%.

7. A note for $80, dated Oct. 22, 1904, had written across the back of it, "Received on the within note, Feb. 1, 1905, $47." How much was due July 1, 1905, with interest at 4%?

Mercantile Rule

545. Business men often settle notes and accounts on which partial payments have been made, and which do not run longer than one year, by the following rule:

Mercantile Rule. — *Find the amount of the principal at the time of settlement.*

Find the amount of each payment from the time it was made until the time of settlement.

From the amount of the principal subtract the sum of the amounts of the payments.

The following exercises are to be solved by this rule.

WRITTEN EXERCISES

546. 1. A note for $1200 given May 18, 1904, had the following partial payments indorsed on it: Oct. 15, 1904, $300; Mar. 10, 1905, $50. How much was due May 18, 1905, with interest at 5%?

2. A note for $500, dated May 15, 1906, had these payments indorsed on it: July 10, 1906, $145; Oct. 16, 1906, $175. How much was due Jan. 1, 1907, interest at 6%?

3. A note for $850, dated June 1, 1905, had the following payments indorsed on it: Oct. 1, 1905, $100; Feb. 1, 1906, $150; May 1, 1906, $75. How much was due May 16, 1906, with interest at 6%?

4. A note for $175, dated June 20, 1906, had these indorsements: July 20, 1906, $75; Aug. 15, 1906, $25. How much was due Dec. 20, 1906, at 4% interest?

5. A note for $1600, dated Jan. 1, 1905, had the following payments indorsed on it: Mar. 2, 1905, $300; July 1, 1905, $25; Oct. 17, 1905, $80. With interest at 6%, how much was due Jan. 2, 1906?

Find also the amount due by the United States Rule.

BANKING

547. People do not usually carry much money about in their pockets or leave it in their houses or in their stores; they deposit it in banks for safe keeping.

If they intend to use the money soon, they place it in a **bank of deposit**, but if they do not expect to withdraw it for some time, it will be wiser to leave it in a **savings bank**, because the savings bank will pay interest upon small sums of money that are left for the *interest term*, which is commonly six months.

Banks of deposit do not usually pay interest on deposits unless the sums left with them are large.

548. The chief business of banks of deposit and discount, variously styled **commercial banks, national banks, state banks, private banks**, etc., is the receiving of deposits for safe keeping and the lending of money in various ways, as by buying notes at a discount. They also perform various financial services for business men and corporations.

549. National banks do business under the authority of national law. Besides the ordinary banking business, they issue demand notes, payable to bearer, called **bank notes**, or **bank bills**, which circulate as money.

550. A **trust company** is authorized under state law and is usually similar to a bank so far as loans and deposits are concerned, but it does other business peculiar to itself.

Trust companies, being restricted less than banks in regard to the kind of investments they may make, are usually able to pay interest on deposits. They do not issue bank notes.

551. Money that is deposited in a bank or trust company is usually payable to the depositor or his order on demand. The demand is made by a written order called a **check.**

Stub	Check
No. 460	No. 460 Omaha, Neb., Jan. 16, 1906.
Date Jan. 16, '06.	**Second National Bank** OF OMAHA
Payable to	
John Green	Pay to the order of John Green
For Mdse.	Forty-eight and $\frac{25}{100}$ Dollars.
Am't. \$48.^{25}	\$48.^{25} Samuel J. Slawson.

The terms **maker, payee, face, negotiable,** and the different kinds of **indorsement,** etc., apply to checks the same as to promissory notes.

To get this check cashed at the bank John Green, the payee, must indorse it, and then it will not be cashed unless he is known at the bank, or is identified by some one who is known there.

552. The **stub** that remains in the *check book* after the check has been torn out gives a complete record of the check.

After the check has been cashed at the bank, it will be canceled and later returned to Mr. Slawson. It then serves as a **receipt** from John Green since it has his name on the back of it.

553. When a depositor wishes to draw money for himself from his bank, he may write a check payable to the order of "Self," in which case he must indorse it before he can get it cashed; or, he may make it payable to the order of "Cash," in which case no indorsement is necessary.

554. A check payable to the payee "or bearer" is usually paid to any one presenting it at the bank on which it is drawn, if the bank is sure that the maker's signature is genuine.

555. Opening a Bank Account. — A person wishing to open an account at a bank takes his money there and leaves it with the "cashier" or "receiving teller" together with a **deposit slip** properly filled out.

He is then given a small book with the amount credited in it. He is usually furnished with a book of blank checks.

In order that the bank may have his true signature to refer to at any time, he is required to sign his name in a book of signatures.

When a stranger to the bank officials wishes to open an account, he usually has to be introduced by some reliable person or else he must furnish references.

DEPOSIT SLIP

DEPOSITED BY

Samuel J. Slawson

IN THE

Second National Bank

Omaha, Neb., *Jan. 1,* 1906.

Bills	$40	
Coin	14	75
Check on *1st Nat.* Bank	76	17
" " *Corn Exch.* "	22	38
Total	153	30

A deposit slip is handed in each time a deposit is made, and the amount is credited in the depositor's bank book, which he brings with him.

WRITTEN EXERCISES

556. 1. A man's balance at the First National Bank was $846.20. Find his balance after checking out $45, $3.75, $75.50, $13.62, $175.25, and $126.10.

2. On the first of April a man's balance at the Franklin National Bank was $142.34. Apr. 3 he deposited $112.50; Apr. 10 he deposited $60.40 and a check for $18.75; Apr. 17 he deposited a check for $65; Apr. 24 he deposited $92; Apr. 30 he drew out $125. Find his balance in the bank on the first of May.

3. Henry Brown's balance at the bank May 1 was $3472.38. During the month he deposited $84.60, $250, $36.22, $80.01, $72.35, $39.17, $24.64, $7.77, $4.65, and $96, and drew out $125, $36, $72.50, $18.10, $23.90, $500, and $61.37. What was his balance June 1?

4. A man's balance in the bank at the beginning of a week was $196.50. During the week he deposited $234.60, $490.75, $325.50, $416, $325, and $410.75, and paid by check a bill of goods for $1250 less 3 %, another for $98.50 less 2 %, and $375 for store rent. Find his balance at the end of the week.

5. Suppose that you sell Charles Raymond of your city an automobile for $1850 less 5 % for cash, receiving his check on the Second National Bank for the amount. Write the check, supplying the necessary details.

6. Indorse the check in blank and deposit it to your credit in the Bank of Commerce, together with $75 in bills and $16.63 in coin, filling out a deposit slip in due form.

7. Draw against your deposit in the Bank of Commerce, check No. 1 for $125, payable to "cash"; draw check No. 2 for $80, payable to the order of "self" and indorse it; draw check No. 3 for $83.65, payable to the order of Frank A. Culver.

8. Write the stub record of each check, the first two being for personal expenses and the third for automobile repairs.

9. Suppose that Frank A. Culver applies your check No. 3 in part payment of his bill at Carroll Brothers' grocery, and that Carroll Brothers deposit it in the Bank of Commerce. Write the indorsements that the check will show.

10. John Dixon pays George White $37.50 per month for rent by check on the Mercantile Bank. Supplying the date and place, write one of these checks in such a form that Mrs. White, who is not known at the bank, may cash it there.

BANK DISCOUNT

557. Lending Money. — Besides lending that part of its own capital that is not needed for other purposes, a bank of deposit and discount is able to lend a large part of the money that is deposited with it, for it is not likely that the depositors will all want the whole of their money at the same time.

558. Borrowing Money. — A great part of the capital employed in business is borrowed. This money is largely obtained from banks. Banks lend for the most part on notes or on some other form of negotiable paper. As a rule the names of at least two persons are required on the paper; one person will be the borrower, the other, a person who becomes responsible with the borrower for the payment of the loan, by *indorsing* the note.

559. If Mr. Ford wishes to borrow money at a bank and Albert Ross agrees to indorse his note, Mr. Ford may make out a promissory note as follows:

$1000.00		*Philadelphia, Pa., Aug. 8, 1906.*
Ninety days *after date, for value received,* I *promise*		
to pay to the order of Albert Ross		
One thousand $\frac{00}{100}$ *Dollars*		
at The Merchants' National Bank.		
No. 73.	*Due*	John Ford.

When Albert Ross has endorsed the note, John Ford may get money on it at the bank, if the officials consider both men reliable. He will not, however, get the whole sum of $1000. The bank will take out *interest in advance* for the time the note has yet to run and will give Mr. Ford the balance.

560. Simple interest, collected in advance, upon the sum due on a note at maturity is called **bank discount.**

When the note bears interest, the bank discount is reckoned on the *amount* of the note at maturity.

561. The sum due on a note at its maturity less the bank discount is called the **proceeds** of the note.

562. The *number of days* from the time when a note is discounted to the time when it legally matures is called the **term of discount.**

In states that allow days of grace care should be taken to include them.

WRITTEN EXERCISES

563. 1. Find the date of maturity, the term of discount, the bank discount, and the proceeds of the note in § 559, if it was discounted Sept. 16, 1906, at 6 %.

Solution

The **date of maturity** is 90 days after Aug. 8, 1906, which is found to be Nov. 6, 1906.

The **term of discount** is from Sept. 16, 1906, to Nov. 6, 1906, or 51 days (see table, page 380).

The **bank discount** is the interest on \$1000 for 51 days at 6 %, or \$8.50.

The **proceeds** = \$1000 − \$8.50 = \$991.50.

Notes. — 1. If the time given in the note were "three months" instead of "ninety days," the date of maturity would be Nov. 8, 1906, instead of Nov. 6, 1906, and the other answers would be changed accordingly.

2. In some states the term of discount would include both Sept. 16 and Nov. 6, making the term 52 days instead of 51 days.

3. In states that allow days of grace, the date of maturity would be 3 days later and the term of discount 3 days longer.

Local customs should govern in all these matters. The answers in this book, however, are based on the method shown in the above solution.

2. Find the date of maturity, the term of discount, the bank discount, and the proceeds of a 60-day note for \$5000, without interest, dated Mar. 4, discounted Mar. 7 at 6 %.

Find the date of maturity, the term of discount, the bank discount, and the proceeds of the following:

3. A 30-day note for \$5000, without interest, dated Aug. 27, discounted Aug. 27 at 6%.

4. A 15-day note for \$10,000, without interest, dated May 1, discounted immediately at 5%.

5. A 90-day note for \$7500, without interest, dated Nov. 18, discounted Nov. 19 at 6%.

6. A 10-day note for \$18,000, without interest, dated July 28, discounted the same day at 5%.

7. A 120-day note for \$3200, without interest, dated Sept. 30, discounted the same day at 7%.

8. A 60-day note for \$840, with interest at 6%, dated Oct. 12, discounted Nov. 2 at 6%.

9. A 30-day note for \$3000, with interest at 5%, dated July 3, discounted July 5 at 6%.

10. A 45-day note for \$60,000, with interest at 4%, dated Dec. 28, discounted Jan. 2 at 5%.

11. A 90-day note for \$8200, with interest at 6%, dated Feb. 1, 1907, discounted the same day at 7%.

12. A 60-day note for \$25,000, with interest at 6%, dated Feb. 1, 1908, discounted Mar. 1 at 4%.

13. A note for \$4000 on interest for 4 months at 6%, dated Jan. 1, 1907, discounted Jan. 10, 1907, at 6%.

14. A 60-day note for \$75,000, without interest, dated Apr. 7, discounted Apr. 7 at 3%.

15. A note for \$7200 on interest for 3 months at 6%, dated July 17, discounted Sept. 1 at 4%.

16. A 10-day note for \$1000, discounted at $3\frac{1}{2}$% on the day it was made.

17. The following represents a bank's purchases of notes on the first of November:

Note for	Dated	Due	Interest Rate	Discount Rate	Cost of Note
$3000	Nov. 1	Dec. 1	4%	$4\frac{1}{2}$%	$ . . . .
4200	Oct. 31	Nov. 30	5%	$4\frac{1}{2}$%	. . . .
5000	Nov. 1	Dec. 1	none	$4\frac{1}{2}$%	. . . .
6400	Oct. 28	Dec. 27	none	5%	. . . .
5000	Oct. 30	Nov. 9	none	4%	. . . .
7500	Oct. 31	Nov. 15	5%	4%	. . . .
12,000	Nov. 1	Jan. 30	none	6%	. . . .

Find the amount invested in notes that day.

18. Find the face of a 60-day note that, discounted at 6% on the day it is made, will realize $9900 in cash.

SUGGESTION.—The proceeds of $1 discounted for 60 days at 6% = $.99.

19. For what sum must I draw my note for 60 days to obtain $2975 in cash, if the rate of discount is 5%?

20. For what sum must a 90-day note be drawn to realize $19,775 in cash when the rate of discount is $4\frac{1}{2}$%?

SAVINGS BANK ACCOUNTS

564. Savings banks pay the depositors compound interest, compounding it either monthly, quarterly, or semiannually.

565. The interval between the dates at which interest is paid is called the **interest term.**

Quarterly interest terms begin Jan. 1, Apr. 1, July 1, and Oct. 1; semiannual terms begin Jan. 1 and July 1, or Apr. 1 and Oct. 1.

566. The bank books of depositors in savings banks must be presented both when deposits are made and when money is drawn out, the amounts being credited or charged as the case may be. Books should also be presented at the bank at the end of each interest term to have interest credited.

567. The most general custom with savings banks is to credit interest, at the end of every interest term, on the *smallest balance* on deposit during the entire term.

Usually no interest is computed on the cents of the balance.

Custom varies greatly with different banks and in different localities. Frequently interest is allowed for the rest of the term on deposits made on the first (or within a few days of the first) of any month of the term. On the other hand, some banks subtract withdrawals from the balance at the beginning of the term and credit interest only on the difference obtained, which may be less than the smallest balance during the term.

568. The following is an illustrative statement of deposits and withdrawals with interest compounded quarterly at 3% per annum.

STATEMENT

DATE	DEPOSITED	DRAWN OUT	INTEREST	BALANCE
1905				
Dec. 15	320			320
1906				
Jan. 1				320
Feb. 6		75		245
Apr. 1			1 83	246 83
June 10	40			286 83
July 1	25		1 84	313 67
Sept. 12		50		263 67
Sept. 20	125			388 67
Oct. 1			1 97	390 64

The statement shows that the deposit of Dec. 15, 1905, did not begin to draw interest until Jan. 1, 1906, and the interest for the quarter from Jan. 1 to Apr. 1 was computed on the *smallest* balance for the quarter, namely, $245. Similarly, $1.84 is the interest on $246 for 3 months, and $1.97 is the interest on $263 for three months.

NOTE. — The fractional part of a cent in the interest is usually dropped. Local customs, in regard to all points concerning which the usage of banks differs, should be followed. The answers in this book, however, are based on the method of solution just given.

WRITTEN EXERCISES

569. Arrange the following as in a savings bank book, and find the balance due Jan. 1, 1907:

1. Interest quarterly (Jan. 1, Apr. 1, etc.) at 4 %.

Deposits: Aug. 8, 1905, \$425; Oct. 2, 1905, \$100; June 30, 1906, \$150; Sept. 3, 1906, \$80.

Withdrawals: Sept. 18, 1905, \$35; Mar. 19, 1906, \$250.

2. Same as exercise 1, except interest semiannually (Jan. 1 and July 1).

3. Interest quarterly (Jan. 1, Apr. 1, etc.) at 4 %.

Deposits: Jan. 30, 1906, \$85; Feb. 18, 1906, \$45; Mar. 8, 1906, \$100; Mar. 29, 1906, \$50; June 2, 1906, \$125.

Withdrawals: Apr. 13, 1906, \$60; Sept. 1, 1906, \$80.

4. Same as exercise 3, except interest semiannually (Jan. 1 and July 1) at 3 %.

5. Same as exercise 3, except interest semiannually at $3\frac{1}{2}$ %.

6. Interest semiannually (Jan. 1 and July 1) at 3 %.

Deposits: Jan. 2, 1906, \$250; Jan. 15, 1906, \$75; Feb. 3, 1906, \$125; Mar. 12, 1906, \$175; June 4, 1906, \$60.

Withdrawals: Feb. 28, 1906, \$10; Apr. 2, 1906, \$50.

7. Interest quarterly (Jan. 1, Apr. 1, etc.) at $3\frac{1}{2}$ %. Balance, July 2, 1906, \$147.12.

Deposits: July 7, 1906, \$15.50; Sept. 12, 1906, \$25; Oct. 12, 1906, \$60; Nov. 10, 1906, \$22.50.

Withdrawals: Oct. 2, 1906, \$16; Dec. 12, 1906, \$35.

8. Interest semiannually (Jan. 1 and July 1) at $3\frac{1}{2}$ %.

Deposits: Jan. 2, 1902, \$400; Mar. 6, 1902, \$200; Aug. 5, 1902, \$150; Jan. 2, 1903, \$125; May 6, 1903, \$85; Oct. 1, 1903, \$110; Jan. 15, 1904, \$75; Sept. 23, 1904, \$120; Mar. 4, 1905, \$60; July 1, 1905, \$50; Dec. 2, 1905, \$12; Feb. 1, 1906, \$44; July 2, 1906, \$36.

Withdrawals: None.

EXCHANGE

570. Paying debts or collecting credits in distant places without actually transferring money is called **exchange.**

571. The various methods of exchange explained in the following pages are by **postal money order,** by **express money order,** by **bankers' association money order,** by **telegraphic money order,** by **check,** by **bank draft** (similar to a check), and by **foreign bill of exchange** (similar to a draft).

572. Exchange between two places in the same country is called **domestic, or inland, exchange;** exchange between two places in different countries is called **foreign exchange.**

DOMESTIC EXCHANGE

573. A **postal money order** is an order made by the postmaster in one place on the postmaster in another to pay to the person named therein a specified sum of money. It is *negotiable.*

In addition to the *face*, money orders cost as follows:

For orders for sums not exceeding	\$ 2.50	...... 3 cents.
Over \$ 2.50 and not exceeding	\$ 5.00	...... 5 cents.
Over \$ 5.00 and not exceeding	\$ 10.00	...... 8 cents.
Over \$10.00 and not exceeding	\$ 20.00	......10 cents.
Over \$20.00 and not exceeding	\$ 30.00	......12 cents.
Over \$30.00 and not exceeding	\$ 40.00	......15 cents.
Over \$40.00 and not exceeding	\$ 50.00	......18 cents.
Over \$50.00 and not exceeding	\$ 60.00	......20 cents.
Over \$60.00 and not exceeding	\$ 75.00	......25 cents.
Over \$75.00 and not exceeding	\$100.00	......30 cents.

NOTE. — The maximum amount for which a single order may be issued is \$100. When a larger sum is to be sent, additional orders must be obtained.

574. An **express money order** is similar to a **postal order**, the rates being the same. It is *negotiable*.

575. Telegraphic money order. — Telegraph companies make transfers in small amounts between their offices, subject to the following charges: 1% on all sums of $25 or over, and 25¢ for any smaller sum, a further charge being added for telegraphic service not to exceed double the usual rate for a 10-word message between the two places.

Banks and express companies transfer for their customers by telegraph when haste is necessary.

576. Exchange by check. — A personal check drawn on a bank in which the drawer has money deposited may be sent by mail. The payee gets it cashed at some bank and this bank collects it from the bank on which it was drawn, sometimes charging the payee a small fee, called **exchange**, for collecting.

577. Exchange by bank draft. — The following illustrates the ordinary **bank draft**, or **bill of exchange**, which is simply a check drawn by one bank on another:

No. 522

First National Bank

Fort Wayne, Ind., Dec. 7, 1906

Pay to the order of Henry Adams .... *$ 500.00*

Five hundred $\frac{00}{100}$ ---------------------------- *Dollars.*

To **Commercial Bank**
NEW YORK CITY

Ernest Rayburn,
Cashier.

The person who signs a draft is the **drawer**; the one who is directed to pay it, the **drawee**; and the one to whom it is to be paid, the **payee**.

578. Banks usually keep money on deposit in some bank, called a **correspondence bank,** in a large money center.

The draft shown on the previous page is an order on the Commercial Bank of New York to pay Henry Adams or his order $500, and to charge the amount to the account of the First National Bank of Ft. Wayne. By indorsing it Mr. Adams may get it cashed at any bank, which in turn may collect the cash from the New York bank or, what is more likely, deposit the draft as a credit to draw on in the New York bank. The latter may charge it to the Ft. Wayne bank's account, if it has one, or it may collect of the Ft. Wayne bank, or deposit the draft there as a credit. In the end, however, the Ft. Wayne bank must pay $500 in credit or money, though in the meantime it has had the use of the $500 paid in by the original purchaser, and has earned a fee charged for exchange.

579. **Bankers' association money orders** are bank drafts drawn by certain banks on a circuit of banks located in important money centers.

580. **Collecting by draft.** — Suppose that W. C. Atkins of Chicago owes John Best of Boston $500 on account, and that Mr. Best wishes to collect the debt at once. He may draw on Mr. Atkins by means of a **commercial draft**:

$500 $\frac{00}{100}$ *Boston, Mass., Jan. 6, 1906.*

At sight pay to the order of Merchants' Bank of Boston

Five hundred $\frac{00}{100}$ *Dollars.*

Value received, and charge to account of

To W. C. Atkins, *John Best.*

25 Federal St., Chicago, Ill.

The Merchants' Bank of Boston cashes this draft and sends it to some Chicago bank for collection. The Chicago bank either collects of Mr. Atkins and remits to the Boston bank, or if payment is refused, returns the draft to the Boston bank, which in turn notifies Mr. Best to "take up

the draft" and refund the money advanced on it. Some banks would not have cashed the draft until collected. In either case charges for collection would be deducted from the face of the draft.

Mr. Best might make the paper payable to "Self." It would then be merely a demand for payment, but would become a **sight draft** by his indorsement.

581. Suppose that Mr. Atkins' debt were not due for sixty days. Instead of "at sight" the draft would read "sixty days after sight." Such a commercial draft is called a **time draft.**

When presented to Mr. Atkins, if he intended to pay it, he would write across the face in red ink "Accepted" with the date, and sign his name underneath. The draft is then called an **acceptance** and is equivalent to a promissory note, due sixty days after the date when it was accepted.

An acceptance may be sold by indorsing it and discounting it for the time it has to run. In discounting, banks sometimes add several days to the discount term to allow for time to collect; they may also charge exchange.

Time drafts are sometimes made payable at a given time *after date.* Then the acceptor does not write the date of acceptance.

582. Exchange Market. — Bills of exchange are credits, and when indorsed may be bought and sold. The business of buying and selling them is carried on largely by banks and in the open market. Hence, as with any other commodity, when the supply offered on the market exceeds the demand, the price goes down and exchange is at a **discount,** or **below par**; but when the demand exceeds the supply, the price goes up and exchange is at a **premium,** or **above par.**

Thus, when the Chicago banks have large sums on deposit in New York banks and wish to use part of their money at home, they will sell sight drafts on New York at favorable rates, probably at par for small amounts, and slightly below par for large amounts.

When the deposits of Chicago banks in New York banks are not sufficient to meet the demands made upon them for New York drafts, they are obliged to send money to New York by express at considerable expense and they must charge something above par for their New York drafts.

583. How Exchange is Quoted. — The price of domestic exchange (for sight drafts) is quoted either at par, at a certain per cent premium or discount, or at a certain amount of money, premium or discount, for $1000.

The following, taken from a New York newspaper, are market quotations for exchange on New York at:

Boston	10 ¢ discount.
Chicago	10 ¢ premium.
New Orleans . .	commercial, par; bank, $1 premium.
Savannah . . .	buying, 50 ¢ discount; selling, 75 ¢ premium.
San Francisco .	sight, 3½ ¢ premium; telegraph, 7½ ¢ premium.

EXERCISES

584. 1. At the rates quoted above, how much will New York exchange for $5000 cost in Boston? in Chicago? In which city are the banks more eager to sell New York exchange?

2. Find the cost of a draft on a New York bank for $10,000, in New Orleans; in Savannah.

3. A cotton dealer in Savannah sold some cotton to a New York firm and drew on them at sight for $12,000. A Savannah bank bought the bill of him at a *discount* of 50¢ per $1000. How much did the bank pay for the bill?

4. At this time the Savannah bank had very little money in New York to draw against. Selling bank drafts on New York meant expressing money to meet them. Consequently, the bank charged a merchant who applied for a draft for $12,000, a *premium* of 75¢ per $1000. Find the selling price of this draft.

Find the cost of a postal or an express money order for:

5. $3.10	7. $ 5.75	9. $21.80	11. $50	13. $62.75
6. $8.50	8. $11.20	10. $17.65	12. $66	14. $99.80

15. How much will it cost Mr. Drew to telegraph $200 to his son in New Haven, the telegraphic rate being 25–2?

WRITTEN EXERCISES

585. 1. Find the cost of a draft for $750 at $\frac{1}{10}$% premium.

2. What will be the cost of a draft for $125,340 when exchange is at $\frac{1}{15}$% discount?

3. A. T. Stewart of Kalamazoo draws on W. Wetmore of New York for $3500. The bank in Kalamazoo charges $\frac{1}{8}$% for collection. Find the proceeds of the draft.

Write the draft, supplying necessary details.

4. What will be the proceeds of a draft for $2000, payable 30 days after date, discounted at 6%, if 6 days are allowed for the collection and the return of the money?

5. The Cleveland Electric Co. sent to Chas. Avery, their salesman in Nashville, a New York draft on the First National Bank for $200, dated June 27. The draft was cashed for Mr. Avery by the proprietor of a hotel, James Hanlon, who deposited it in the Cotton Exchange Bank. This bank forwarded it to the First National Bank of New York as credit. Write the draft and its indorsements.

6. Find the proceeds of a 60-day draft for $1885.50, discounted at 7% for the full term, if the bank charges 80¢ in addition for collection.

7. C. M. Clay of Charleston sold 200 barrels of tar @ $1.80, 50 casks of crude turpentine @ $3.20, and 250 barrels of D resin, @ $2.05, to Hines & Co., New York. Hines & Co. remitted with the order a sight draft for $275 on Colfax & Son, Charleston, and authorized C. M. Clay to draw at 60 days' sight for the balance. Write both drafts, supplying suitable dates.

8. C. M. Clay discounted the second draft for 63 days at 6% and was charged $\frac{1}{32}$% of the face of the draft for exchange. Find the net proceeds of the shipment.

9. A speculator in San Francisco sent his New York broker $4000 by telegraph at $7\frac{1}{2}$¢ premium. The message cost $1.35. Find the total cost.

10. A commission firm in Louisville received a car load of oranges, 396 boxes, from a dealer in New Orleans, paying $77 freight charges. The firm sold 200 boxes @ $2.50 and the rest @ $2.40. They remitted the proceeds, after deducting freight charges, 2% commission, $2\frac{1}{2}$¢ per box for storage, and $\frac{1}{8}$% on $1000 insurance, by New York exchange at par. What was the face of the draft?

11. A Cincinnati pork packer received an order from a New York customer for 75 barrels of mess pork @ $13.25, freight charges paid to New York. He shipped the pork, drew at sight on his customer for the whole amount less $54.64 for freight charges and insurance, and sold the draft at the bank at 30¢ discount. Find the net proceeds.

12. A merchant in Minneapolis received an order from a New York firm for 800 barrels of flour @ $3.25 on the cars at Minneapolis, terms 60 days' sight draft. He shipped the flour. On receiving the New York firm's acceptance, he discounted it at the bank for 66 days, at 6%, and was charged 15¢ per $1000 of the face of the acceptance for exchange. Find the net proceeds.

13. Express companies and the U. S. subtreasury charge 75¢ per $1000 to transfer money from New York to New Orleans, 50¢ to St. Louis, Chicago, and the West, and 25¢ to near-by Eastern cities. One week in October when money was in demand for moving the crops, New York banks transferred through these agencies $3,142,000 to New Orleans and the South, $7,495,000 to St. Louis, $3,280,000 to Chicago and the West, and $3,746,000 to near-by Eastern cities. What was the total cost of the week's exchange?

FOREIGN EXCHANGE

586. Foreign exchange does not differ in principle from domestic exchange.

A **foreign bill of exchange** is similar to a bank draft and is payable in the money of the country on which it is drawn. **Commercial drafts** are also drawn and accepted as in domestic exchange.

587. Foreign bills of exchange are usually written in duplicate, called a **set of exchange**, illustrated as follows:

1

EXCHANGE FOR *New York, U.S.A., Dec. 1, 1906.*
£ 200 . 8 . 5

Three days

after sight of this **First** *of Exchange (second unpaid)*

Pay to the order of Hiram Putnam

Two hundred Pounds 8/5 Sterling

Value received and charge the same to account of

To Brown, Shipley & Co.,
London, *Brown Brothers & Co.*
No. 527 *England.*

In the duplicate, "Second of Exchange (first unpaid)" is substituted for "First of Exchange (second unpaid)," and "2" for "1" in the left margin. When either one of the set is paid, the other becomes void.

588. Par of exchange between two countries is the value of the monetary unit of one expressed in that of the other.

The table in § 417 gives the par of exchange in the United States, on England, France, and Germany.

589. How foreign exchange is quoted. — Exchange on:

England, by giving the cost of a bill of exchange for £ 1; thus, 4.87 means that a bill for £ 1 costs $ 4.87.

France (and other countries using the same monetary system), by giving the number of francs of exchange that can be bought for $ 1: thus, 5.18 means that 5.18 fr. can be bought for $ 1.

Germany, by giving the number of cents that 4 marks of exchange cost; thus, $94\frac{3}{8}$ means that 4 marks cost $94\frac{3}{8}$ ¢.

Newspapers usually give exchange rates for cable transfers, demand bills, and sixty-day bills, thus:

	CABLES	DEMAND	60 DAYS
Sterling	$4.86\frac{1}{4}$	$4.85\frac{1}{4}$	4.82
Francs	$5.16\frac{7}{8}$	$5.18\frac{1}{8}$	$5.20\frac{5}{8}$
Marks	$95\frac{1}{4}$	$95\frac{1}{8}$	$94\frac{1}{2}$

590. Foreign exchange for small amounts is usually effected by means of **international** express and postal **money orders**, at fixed rates.

591. Letters of credit. — A person intending to travel abroad may deposit funds with an international banking house to draw against, and receive a **letter of credit** guaranteed by the bank.

The purchaser signs several signature blanks, one of which is sent to each correspondent bank. When he presents the letter at any one of these banks he is asked to sign a draft or check for the amount he wishes to draw. The cashier compares the signature with that on the signature blank, and if they correspond, the money is paid and charged on the letter, which is returned to the owner.

Letters of credit are usually written in sterling money, the holder paying London exchange when he purchases it, and a further exchange when he draws in any other country than England.

592. Travelers' checks. — These are guaranteed checks issued in denominations of $ 10, $ 20, $ 50, $ 100, and $ 200, by banks and express companies, at a fixed rate of $\frac{1}{2}$ % of the face value.

The purchaser signs them on the face when purchased, and on the face or back when cashed. Identification is by comparison of signatures. These checks are readily received at European hotels, railroads, and business places.

WRITTEN EXERCISES

593. 1. Find the cost of a cable transfer of 265 fr. at $5.16\frac{7}{8}$.

SUGGESTION. — One franc costs \$1 ÷ $5.16\frac{7}{8}$.

2. How much will a 60-day bill for 250 M. cost at $94\frac{1}{2}$?

SUGGESTION. — One mark costs $\frac{1}{4}$ of \$$.94\frac{1}{2}$.

3. Find the cost of a demand bill for £ 75 6*s*. 4*d*. at $4.85\frac{1}{4}$.

SUGGESTION. — See exercise 11, page 320.

Find the cost, to the nearest cent, of exchange for:

4. £ 1200 @ 4.8365
5. £ 1525 @ 4.8420
6. £ 95 12*s*. @ $4.87\frac{1}{4}$
7. £ 225 10*s*. 2*d*. @ $4.83\frac{5}{8}$
8. 10,000 fr. @ 5.18
9. 25,000 M. @ $95\frac{1}{16}$
10. 1224.75 fr. @ $5.19\frac{7}{8}$
11. 4520.65 M. @ $94\frac{3}{4}$

12. A cotton exporter in Mobile drew a 60-day bill on London for £ 1075 against a shipment of 100 bales of cotton, and sold the bill at the bank at $4.83\frac{7}{8}$. Find the proceeds.

13. An American sent to his family in Milan a bill for 2000 lire, exchange at 5.19. What was the cost of the bill?

14. Find the cost of £ 5 at 4.87, sent to Dublin at Christmas time by international money order, the fee being 30 ¢.

15. Find the cost of travelers' checks for \$4000, at $\frac{1}{2}$ %.

16. The value of these checks in foreign money was according to the following values, printed on a 20-dollar check:

£ *s*. *d*.	FRANCS	MARKS	LIRE	CROWNS	FLORINS
4 1 2	102.50	82.50	102.50	73.39	49.02

The purchaser cashed \$700 in checks in England, \$900 in France, \$400 in Germany, \$200 in Denmark (crowns), \$500 in Belgium (francs), \$200 in Holland (florins), and \$600 in Italy. Find the amount of money of each kind that he received and the amount refunded to him at the end of the trip.

17 An importer purchased a sight draft on London for £15,000 at $4.86\frac{1}{2}$. How much did it cost him?

18. Suppose that he had purchased the draft through a broker and had obtained a slightly lower rate, 4.8640. The brokerage being \$5 per £10,000, how much less would the draft have cost him?

19. Find the proceeds of a bill for 25,600 marks sold through a broker at $94\frac{1}{2}$, brokerage $\frac{1}{64}\%$.

20. A firm in Newark drew on a firm in Rio Janeiro at 60 days' sight for £25,000, representing a shipment of machinery, and sold the bill through a New York broker at $4.83\frac{7}{8}$, brokerage \$5 per £10,000. What were the proceeds?

21. A New York bank contracted with cotton shippers in the South for the purchase of £100,000 of bills of exchange for cotton at $4.81\frac{1}{2}$, and deposited them with its London correspondent for collection. The London bank charged $\frac{1}{2}\%$ for collection and credited the proceeds to the New York bank. The New York bank drew demand bills against this credit and sold them in the open market at $4.85\frac{7}{8}$. Find the profit.

22. On Nov. 24, when money was worth 5%, a banker invested £10,000 in 60-day bills on London at 4.80. How much interest could he have obtained for this money, if he had not bought the drafts?

23. He sent the bills to London for acceptance. They were accepted and returned to him. He held them until they became due, when he sold them as demand bills at 4.87. How much did he gain?

24. A grain shipper sold 22,500 centals of wheat (1 cental = 100 pounds) at \$1.60 per cental to a merchant in Havre, France, and drew on him for the amount at $5.20\frac{1}{4}$. Find the face of the bill.

STOCKS AND BONDS

594. Corporations. — When a number of persons wish to engage in a business requiring a large capital, they usually raise as much capital as they can among themselves, perhaps soliciting subscriptions from others, and organize a **stock company**, or **corporation**.

A corporation is authorized under the law; has certain powers and privileges; is subject to certain limitations; and is regarded by the law as a single person engaged in a stated business with a declared capital, or **stock**.

595. Shares. — The stock of a corporation is divided into equal parts, called **shares**.

A share is usually $100, but it may be more or less than that sum; unless stated to the contrary the face value of a share, in this book, means $100.

If the capital of a corporation is $400,000, it is divided, perhaps, into 4000 shares of $100 each.

596. Stockholders. — A person who becomes the owner of one or more of these shares is called a **shareholder**, or a **stockholder**.

Corporations are managed by officers elected by the stockholders, a stockholder having one vote for each share he owns.

A stockholder's liability for the debts of the corporation is usually limited to the original value of the stock he owns.

Stockholders of national banks, however, are responsible for *double* the original value of their stock.

597. Certificates of Stock. — Every stockholder receives a *certificate* showing the number of shares to which he is entitled and the *original*, or **par value**, of each.

598. Dividends. — When the business has been successful, the profits, after all expenses are paid and a working surplus is laid aside, are divided among the stockholders according to the number of shares each holds. This is called a **dividend.** It is usually a certain per cent of the par value of the stock.

Thus, if the capital stock is $100,000 and the net earnings are $5000, a 5% dividend may be declared, and a man who owns stock to the par value of $1000 will receive $50 as his part of the profits.

599. Assessments. — If the business has been unsuccessful, the dividend may be omitted, or *passed;* and sometimes the stockholders may be required to make up the deficiency by an **assessment.**

600. Corporations often issue two kinds of stock, called **preferred** and **common.**

The holders of preferred stock are generally entitled to a fixed rate of dividend that must be paid before the holders of common stock are entitled to participate in the profits.

601. Market value. — Stock is said to be **at par**, when it is worth its face value in the market; **above par**, or **at a premium**, when it is worth more than its face value; and **below par**, or **at a discount**, when it is worth less than its face value.

When the dividends are large enough to pay stockholders good interest on their investment, the stock will usually be at or above par; otherwise below par.

602. A **corporate bond** is a corporation's formal written promise under seal to pay a certain sum of money to the purchaser, on or before a specified time, with interest at regular intervals at a fixed rate.

Corporate bonds must be secured by a **mortgage**, which is an agreement that the holder of the bond may sell the property of the corporation, if the bond is not paid.

603. Government bonds.—Bonds are issued by the United States, or other countries, by states, cities, counties, and other political divisions of a country, to raise money for various purposes of government.

Government bonds are not secured by mortgage.

604. When bonds are recorded by number and in the name of the person owning them, they are called **registered bonds**.

Registered bonds cannot be transferred without indicating the transfer in the records of the bonds.

605. Bonds to which interest certificates, called **coupons**, are attached, are called **coupon bonds.**

Coupon bonds are payable to the bearer. When interest is due, a coupon is cut off and presented for payment at a bank or elsewhere.

606. Stocks and bonds compared.—Stockholders of a corporation are owners of the property; bondholders are lenders to the corporation. Bonds bear interest at a fixed rate; the income from stocks depends upon the prosperity of the corporation and the rate of dividend declared.

607. How stocks and bonds are quoted.—The following quotations were found in a newspaper:

Stocks		Bonds	
Am. Locom.	$61\frac{3}{4}$	U. S. 3s reg.	103
Am. Locom. pf.	113	U. S. 4s coup., 1907. . . .	$103\frac{3}{8}$
N. Y., N. H., & H.	$194\frac{3}{4}$	Cen. Pac. 1st 4s	99

These quotations in order mean: American Locomotive common stock, at $61\frac{3}{4}$% of its par value; American Locomotive preferred stock, at 113% of par; New York, New Haven, and Hartford railroad common stock, at $194\frac{3}{4}$%; United States registered bonds, paying 3% interest, at 103%; United States coupon bonds, paying 4% interest and due in 1907, at $103\frac{3}{8}$%; and Central Pacific railroad first mortgage bonds, paying 4% interest, at 99%.

608. A person whose business it is to buy and sell stocks and bonds for others is called a **stock broker**; the compensation he receives is called **brokerage.**

The customary brokerage, for 100-dollar shares, is $\frac{1}{8}$% of the *par value* for buying and the same for selling.

Dealings in stocks are usually by "blocks" of 100 shares, but a smaller number of shares may be bought.

EXERCISES

609. 1. Which of the stocks and bonds quoted in § 607 are above par? below par?

2. Bonds are usually issued for $1000 each. From the quotations find the cost of one of the U. S. 3s reg.

3. The capital stock of a bank is $100,000. How many shares are there? How much of the capital do I own, if I have 12 shares? 30 shares? 50 shares?

4. If on a capital of $100,000 there is a net gain of $5000, what per cent of dividend may be declared?

5. If the semiannual dividend is $3\frac{1}{2}$%, what amount of income do I receive yearly from 1 share? from 50 shares? from 100 shares?

6. What is the market value of 10 shares of stock when quoted at par? at 125? at 150? at 50?

7. If the par value of an industrial stock is $50, how much above par is it when a share sells for $75? for $100? How much below par, when it sells for $25? for $40?

8. A bank fails and the stockholders are assessed 25%. How much must I pay, if I own 10 shares? 50 shares?

9. If a bank stock is quoted at 300, how much must be paid for 1 share? for 20 shares?

10. If I received $5 dividend on every share of a stock that I hold, what was the entire dividend declared on a capital of $100,000?

11. At $\frac{1}{8}$% commission, what is the brokerage on the sale of 1 share of stock? 100 shares?

WRITTEN EXERCISES

610. 1. Find the cost of 500 shares of C. M. and St. P. R.R. common stock at $163\frac{7}{8}$, brokerage at the usual rate.

SOLUTION

Brokerage per share = $\frac{1}{8}$% of \$100, or \$$\frac{1}{8}$
Total cost per share = \$$163\frac{7}{8}$ + \$$\frac{1}{8}$ = \$164
Then 500 shares cost 500 times \$164, or \$82,000.

2. How much must be paid, including brokerage, for 25 shares of Canadian Pacific railroad stock at $174\frac{5}{8}$?

3. How much will it cost me to buy 25 1000-dollar U. S. 3s reg., at $2\frac{3}{4}$% premium, brokerage at $\frac{1}{8}$%?

4. If U. S. Steel preferred is worth $106\frac{3}{8}$, how much will 100 shares of it cost me, including brokerage?

5. Find the cost, including brokerage, of 200 shares of Commercial Cable Co. stock at $192\frac{3}{8}$.

6. How much must be paid, including brokerage, for 500 shares of N. Y. Central R.R. stock at $137\frac{1}{2}$?

7. Find the cost of 500 shares of an industrial stock at $127\frac{3}{4}$, brokerage $\frac{1}{4}$%, if the par value of a share is \$50.

8. A man purchased 100 shares of Republic Iron & Steel Co. pf. at $93\frac{1}{2}$ and sold them the same day at $95\frac{7}{8}$. Find his net gain, if he paid brokerage for both buying and selling.

9. A speculator bought 500 shares of railroad stock at $67\frac{1}{2}$ and sold them the same day at $68\frac{1}{4}$, in each case through a certain broker. How much did the broker receive? How much did the speculator gain?

10. A broker received an order to sell 200 shares of General Electric stock and to buy 400 shares of Illinois Central R.R. stock. With the order was a draft for \$7000. The broker sold the first at $166\frac{1}{2}$, bought the second at 170, and drew on his customer for the balance. Find the face of this draft.

11. When Baltimore & Ohio pf. is selling at 96, how many shares can be bought for \$5767.50, brokerage at the usual rate?

SOLUTION

Total cost per share = \$96 + \$$\frac{1}{8}$ = \$96.125
Number of shares = \$5767.50 ÷ \$96.125 = 60.

12. How many shares of Northern Pacific R.R. stock at 97$\frac{1}{8}$ can be bought through a broker for \$4862.50?

13. If Southern Ry. pf. stock can be bought at 100, how many shares of it can I buy through a broker for \$8010?

14. How many shares of Pressed Steel Car stock pf. at 92$\frac{3}{4}$ can be bought through a broker for \$40,865?

15. I purchased 24 shares of Consolidated Gas stock at 138 and, after receiving one quarterly dividend of 1$\frac{1}{2}$%, sold the stock at 141$\frac{1}{2}$. How much did I gain, allowing brokerage for both buying and selling?

SOLUTION

Paid for 1 share, \$138 + \$$\frac{1}{8}$, or \$138$\frac{1}{8}$
Paid for 24 shares, 24 times \$138$\frac{1}{8}$, or \$3315
Received for 1 share, \$141$\frac{1}{2}$ − \$$\frac{1}{8}$, or \$141$\frac{3}{8}$
Received for 24 shares, 24 times \$141$\frac{3}{8}$, or \$3393
Dividend on 24 shares = 1$\frac{1}{2}$% of \$2400 = \$36
Total amount received = \$3393 + \$36 = \$3429
Gain = \$3429 − \$3315 = \$114.

16. Mr. Brown bought 125 shares of N. Y. C. R.R. stock at 149$\frac{1}{4}$, and after receiving one quarterly dividend of 1$\frac{1}{4}$%, sold them at 152$\frac{1}{2}$. Allowing brokerage in each case, how much did he gain?

17. My money in the savings bank was paying me 3$\frac{1}{2}$% interest. I drew out \$2002 and invested it in railroad stock at 125, paying the usual brokerage. This stock yielded a semiannual dividend of 2$\frac{1}{2}$%. Was my semiannual income increased or diminished, and how much?

18. A man bought 300 shares of a certain stock at $87\frac{1}{2}$. Later he received 15 shares of the stock as a dividend on his 300 shares. Becoming alarmed because the dividend was not in cash, he sold his shares at $81\frac{7}{8}$. Find his loss, including brokerage for both buying and selling.

19. How much must be invested in Galveston Street Railway 5% bonds (denomination $100) at 112, usual brokerage, to secure to the purchaser an annual income of $1000?

SOLUTION

Since the income from each bond is $5, it will take as many bonds to give an income of $1000 as $5 is contained times in $1000, or 200 bonds.

At 112, each bond, including brokerage, will cost $$112\frac{1}{8}$, and 200 bonds will cost 200 times $$112\frac{1}{8}$, or $22,425. Therefore $22,425 must be invested to secure an annual income of $1000.

20. What sum must be invested in Imperial Japanese 6s, at $99\frac{5}{8}$, to give an annual income of $2400, allowing the usual brokerage?

21. What amount invested in Republic of Cuba 5s at $105\frac{3}{8}$, brokerage as usual, will secure an annual income of $500?

22. What sum must I invest in American Hide & Leather 6s at $99\frac{7}{8}$, allowing brokerage, to secure an income of $1200?

23. What per cent shall I make on my money, if I buy 4% bonds at 80?

SUGGESTION. $4 is what per cent of $80?

24. Find the per cent realized on an investment in 6% preferred stock at 75.

25. How much must I pay for 4% stock in order to realize 5% on my investment?

SUGGESTION. $4 is 5% of what must be paid for a share.

26. How much must be paid for 6% stock to realize 5% on the money invested?

27. Find the cost of 3% stock when the money invested in it yields 8%.

28. A man receives a quarterly dividend of \$50 on 25 shares of stock. Find the rate of dividend each quarter.

29. A company with a capital of \$300,000 pays its stockholders \$6000 quarterly. Find the quarterly dividend of a man who holds 40 shares of the stock.

30. During one year the net earnings of a company whose capital is \$360,000 were \$65,835. If it retained a surplus of \$5835 and distributed the rest in dividends, how much was received by a man owning 60 shares?

31. What annual income will a man receive who invests \$4835.25 in a 6% stock purchased at 115, brokerage at the usual rate?

32. A man bought at private sale 50 shares of a bank stock at 193, and afterward sold them through a broker at $191\frac{1}{2}$. How much did he lose?

33. How much must be paid, including brokerage, for a sufficient number of U. S. 4s at $133\frac{1}{2}$ to obtain an annual income of \$1000?

34. At one time during the war between Japan and Russia, Japan borrowed a large sum of money by means of an issue of 6% bonds. They sold at $93\frac{1}{2}$. What rate of interest, to the nearest .01%, did Japan actually pay on a par basis?

35. Several years before the war Japan had sold an issue of 5% bonds at $103\frac{5}{8}$. Find the true rate of interest paid.

36. A stock company declared a dividend of \$375,000 for one quarter. The stock consisted of \$12,500,000 preferred and \$12,500,000 common, the preferred stock being allowed 7% annually before the common stock could participate in the profits. Find the dividend of a stockholder who owned 50 shares of preferred and 10 shares of common stock.

RATIO AND PROPORTION

RATIO

611. 1. Compare 15, 20, 25, and 30 with 5 in such a way as to tell how many times each contains 5.

2. What number expresses the relation, or *ratio*, of 15 to 5? of 20 to 5? of 12 to 3? of \$50 to \$10? of 32 qt. to 8 qt.?

3. What number expresses the ratio of 3 to 6? of 2 to 6? of 3 to 12? of \$10 to \$40? of 7 days to 28 days?

612. The relation of one number to another of the same kind, expressed by the quotient of the first divided by the second, is called the **ratio** of the first number to the second.

All ratios are abstract numbers.

Thus, the ratio of 15 to 3 is $15 \div 3$, or $\frac{15}{3}$, or 5.

613. The **ratio sign** is a colon (:).

Thus, the ratio of 15 to 3 may be written 15 : 3.

614. The first number in a ratio is called the **antecedent**, and the second number, the **consequent**.

$$3 \div 4 = \frac{3}{4} = 3:4 = \frac{\text{dividend}}{\text{divisor}} = \frac{\text{numerator}}{\text{denominator}} = \frac{\text{antecedent}}{\text{consequent}}.$$

The antecedent and consequent of a ratio are called its **terms**.

615. Since a ratio may be written in the form of a fraction, and since multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction,

Multiplying or dividing both terms of a ratio by the same number does not change its value.

Thus, 4 : 8 reduced to lowest terms is 1 : 2; also 4 : 8 = 12 : 24.

EXERCISES

616. What is the ratio of

1. 14 to 7? 7 to 14? 18 to 6? 6 to 18?

2. 15 to 30? 12 to 60? 96 to 16? 100 to 20?

3. 2 to 1? 3 to 5? 7 to 2? 5 to 4? 17 to 3? 25 to 8?

4. 25 to 100? 40 to 100? 90 to 100? 120 to 100?

5. $12\frac{1}{2}$ to 100? $37\frac{1}{2}$ to 100? $62\frac{1}{2}$ to 100? $33\frac{1}{3}$ to 100? $112\frac{1}{2}$ to 100? 125 to 100?

6. 20% to 1? 50% to 1? 40% to 1? $87\frac{1}{2}$% to 1? 110% to 1?

Reduce to lowest terms the ratios expressed by:

7. 10 : 2	**10.** 3 : 27	**13.** $\frac{12}{24}$	**16.** $75 \div 100$
8. 12 : 6	**11.** 4 : 40	**14.** $\frac{16}{48}$	**17.** $60 \div 120$
9. 16 : 4	**12.** 9 : 72	**15.** $\frac{60}{80}$	**18.** $80 \div 240$

19. What is the ratio of \$70 to \$100? of \$600 to \$1000?

20. What is the ratio of 15 days to 30 days? of 21 days to 1 week?

21. What is the ratio of 1 rod to 1 mile? of 1 ounce to 1 pound?

22. What is the ratio of a long ton to a short ton?

WRITTEN EXERCISES

617. Find the value of the following ratios:

1. $\frac{4}{5} : \frac{2}{5}$	**4.** .7 : .8	**7.** 10 : 4.5	**10.** 4 ft. : 16 in.
2. $\frac{3}{4} : \frac{1}{2}$	**5.** .4 : 10	**8.** 72 : 8.5	**11.** 2 yd. : 18 in.
3. $\frac{5}{8} : \frac{1}{4}$	**6.** 7.5 : 12	**9.** $2\frac{1}{2} : 7\frac{1}{2}$	**12.** 1 Km. : 62.5 m.

13. The capital stock of a street railway company was \$7,500,000, the gross earnings for a year \$1,500,000, and the net earnings \$600,000. Find the ratio of gross earnings to capital stock; of net earnings to gross earnings; of net earnings to capital stock. Then express each ratio in per cent.

14. In 1880, 17% of the persons in the United States were illiterate; in 1890, 13.3%; in 1900, 10.7%. Express these per cents as ratios having 10,000 as a consequent.

SIMPLE PROPORTION

618. An equality of two ratios is called a **proportion.**

Thus, $4 : 6 = 8 : 12$ is a proportion; also $\frac{4}{6} = \frac{8}{12}$.

The double colon (::) is often used instead of the sign of equality.

The proportion $4 : 6 = 8 : 12$, or $\frac{4}{6} = \frac{8}{12}$, or $4 : 6 :: 8 : 12$ is read "the ratio of 4 to 6 is equal to the ratio of 8 to 12"; or more briefly, "4 is to 6 as 8 is to 12."

619. The first and last terms of a proportion are called the **extremes,** and the second and third terms the **means.**

In $4 : 6 = 8 : 12$, the extremes are 4 and 12; the means, 6 and 8.

620. 1. In the proportion $4 : 6 = 8 : 12$, what is the product of the extremes? of the means? How does the product of the extremes compare with the product of the means?

2. Compare the product of the extremes with the product of the means in $1 : 2 = 4 : 8$; in $1 : 2 = 5 : 10$; in $2 : 4 = 6 : 12$.

3. Form other proportions and compare the product of the extremes in each with the product of the means.

In the following compare the first ratio with the second to find whether or not they are equal; then compare the product of the extremes in each case with the product of the means.

4. Does $\frac{3}{6} = \frac{4}{8}$? $\frac{3}{6} = \frac{5}{10}$? $\frac{3}{6} = \frac{7}{14}$? $\frac{3}{6} = \frac{6}{12}$?

5. Does $2 : 4 = 5 : 10$? $3 : 6 = 9 : 18$? $4 : 8 = 5 : 10$?

6. Does $2 : 8 = 5 : 20$? $2 : 8 = 6 : 24$? $3 : 5 = 9 : 15$?

In any proportion the product of the extremes is equal to the product of the means.

WRITTEN EXERCISES

621. **1.** If $24 : 7 = 48 : x$, what is the value of x?

SOLUTION

Product of extremes = product of means

$$24x = 7 \times 48$$
$$x = 7 \times 2 = 14.$$

2. Solve the proportion $12 : 16 = x : 60$.

SOLUTION

Reducing $12:16$ to lowest terms, $3:4 = x:60$

Product of means = product of extremes

$$4x = 3 \times 60$$
$$x = 3 \times 15 = 45.$$

SUGGESTION. — Reduce ratios to lowest terms before solving.

Solve the following proportions:

3. $30 : 5 = 12 : x$

4. $10 : 5 = x : 7$

5. $15 : x = 3 : 6$

6. $15 : x = 6 : 2$

7. $15 : x = 10 : 2$

8. $5 : 20 = x : 32$

9. $\frac{1}{2} : \frac{3}{4} = x : 1$

10. $x : \frac{2}{3} = \frac{1}{2} : \frac{1}{4}$

11. $2 : 3 = \frac{1}{2} : x$

12. $\frac{7}{8} : \frac{3}{16} = x : \frac{5}{8}$

13. $50 : 75 = x : 3$

14. $24 : 72 = 3 : x$

15. Solve 3 tons : x tons = \$15 : \$40.

SUGGESTION. — Since $\frac{3 \text{ tons}}{x \text{ tons}} = \frac{3}{x}$ and $\frac{\$15}{\$40} = \frac{15}{40}$, the number of tons may be found from the proportion $\frac{3}{x} = \frac{15}{40}$, or $3:x = 15:40$.

Solve the following proportions:

16. $\frac{18 \text{ yd.}}{24 \text{ yd.}} = \frac{\$x}{\$3.60}$

17. $\frac{15 \text{ bu.}}{x \text{ bu.}} = \frac{\$37.50}{\$100}$

18. $\frac{4 \text{ men}}{12 \text{ men}} = \frac{5 \text{ days}}{x \text{ days}}$

19. $\frac{x \text{ ft.}}{5600 \text{ ft.}} = \frac{4}{1120}$

WRITTEN EXERCISES

622. 1. If 4 tons of coal cost \$17, how much will 14 tons cost at the same price per ton?

SOLUTION

The cost of the coal is proportional to the quantity; that is,

$$\left\{\begin{matrix}\text{Cost of}\\ \text{14 tons}\end{matrix}\right\} : \left\{\begin{matrix}\text{cost of}\\ \text{4 tons}\end{matrix}\right\} :: \text{14 tons} : \text{4 tons}$$

or, $$\$x : \$17 = \text{14 tons} : \text{4 tons}$$

Therefore, $$x : 17 = 14 : 4$$

$$4x = 238$$

$$\therefore x = 59\tfrac{1}{2}.$$

Hence the cost of 14 tons is \$59.50.

2. A man bought a 180-acre farm for \$11,000 and sold 30 acres of it at cost. How much did he receive for the part sold?

3. If 55 acres of land yield 1430 bushels of wheat, how many bushels will 132 acres yield at the same rate?

4. If sound travels 825 meters in $2\frac{1}{2}$ seconds, how long will it take the report of a cannon to travel 3.3 Km.?

5. If 24 pounds of maple sugar cost \$3.60, how much must be paid for 5 pounds?

6. A motor-boat race of 136.5 miles on the Hudson River was run at the rate of 52.5 miles in 2 hours. How long did it take to complete the course?

7. If it costs 66¢ to have a rug of $8\frac{1}{4}$ square yards cleaned, how much will it cost for a rug of $15\frac{1}{2}$ square yards?

8. A manufacturing plant turns out 7500 concrete building blocks in 6 days. How large an order for concrete blocks can it fill in 20 days?

9. A series of photographs for a moving picture was taken in the New York Subway at the rate of 500 in $33\frac{1}{3}$ seconds. How many photographs were there in the series, if it was completed in 7 minutes?

10. If 10 men can dig a ditch in 12 days, how many days will it take 15 men to dig it?

SOLUTION

Let x represent the number of days it will take 15 men.

Since 15 men can do more work per day than 10 men, they can dig a ditch in less time than 10 men can dig it.

Therefore the ratio x days : 12 days is less than 1.

The ratio, less than 1, between 10 men and 15 men is 10 men : 15 men.

Therefore, x days : 12 days = 10 men : 15 men

$$x : 12 = 10 : 15$$
$$15\,x = 120$$
$$\therefore x = 8$$

Hence 15 men can dig the ditch in 8 days.

Test. — There are 10×12, or 120, days' work to be done. 15 men can do this work in $(120 \div 15)$ days, or in 8 days.

11. If 16 men can do a piece of work in 15 days, how long will it take 20 men to do it?

12. If 16 men can do a piece of work in 15 days, how many men will be required to do it in 10 days?

13. If a force of men can do a certain piece of work in 20 days by working 8 hours a day, in how many days can they do the work by working 10 hours a day?

14. It is estimated that it will take 24 men 18 days to repair a portion of Second Street. The work must be done in 16 days. How many extra men must be employed?

15. A contractor agrees to do a piece of work in 60 days and employs 40 men for that purpose. Before beginning, however, the time is extended 15 days. How many men may he discharge?

16. If the interest on a sum of money for 6 months is $33, what will be the interest for 8 months?

17. By counting mile posts, a man found that the train upon which he was riding went 5 miles in 5 minutes 15 seconds. How many miles per hour was the train moving?

18. In a shipment by rail of 450 dozen eggs, 5 eggs out of every 3 dozen were broken. How many dozen eggs were left unbroken?

19. A owned a house assessed at $12,000, on which he paid a tax of $225 one year. The same year B's house, next door, was taxed at $157.50. For how much was B's house assessed?

20. The assessed valuation of the property in a town is $4,650,000, and the tax to be raised is $102,300. If I own property assessed at $23,250, how much tax must I pay?

21. A garrison of 150 soldiers consumed 26 barrels of flour in 9 weeks. At this rate how many days would the same amount of flour last a garrison of 210 soldiers?

22. On a freight train of 18 cars, 4050 barrels of flour are loaded. How many cars must be added so that the train may carry 5175 barrels?

PARTITIVE PROPORTION

623. The process by which a number is separated into parts proportional to two or more given numbers is called **partitive proportion.**

WRITTEN EXERCISES

624. **1.** Separate 225 into parts proportional to 2, 3, and 4.

SOLUTION

Since the parts are proportional to 2, 3, and 4, we may represent them by $2x$, $3x$, and $4x$, respectively.

Since the sum of the parts is the number itself,

$$2x + 3x + 4x = 225$$

$$\therefore x = 25$$

Hence, $2x = 50$, $3x = 75$, and $4x = 100$, the numbers.

Test. $50 + 75 + 100 = 225$; $50 : 75 : 100 = 2 : 3 : 4$.

2. Separate $24,000 into parts proportional to 3, 4, and 5.

3. Separate $36,000 into parts proportional to 3, 5, and 10.

4. The total receipts of a coal mining company one year were $16,725,000, and the expenses were to the net earnings as 13 is to 2. What were the expenses? the net earnings?

5. The daily ration of a German soldier in the field weighs 1300 grams and consists of bread, meat, rice, salt, and coffee proportional to 6, 3, 1, $\frac{1}{6}$, and $\frac{1}{5}$. Find the weight of each.

6. The freight earnings of two railroads on a train load of grain were $2160. One carried the grain 400 miles, the other 500 miles. Find the earnings apportioned to each road.

7. The annual earnings of a certain railroad company are $78,000,000. Find the amounts received from freight charges, from passenger service, and from other sources (such as mail, express, etc.), if they are proportional to 8, 4, and 1.

8. A quarterly dividend of $6412.50 was divided among the 8 shareholders of a corporation. The holdings of the shareholders were 30, 15, 24, 18, 48, 36, 42, and 72 shares, respectively. What sum did each shareholder receive?

PARTNERSHIP

625. When two or more persons agree to combine their money, goods, labor, or skill, in some business enterprise, and to share the profits and losses of the business in certain proportions, they become **partners**, thus forming a **partnership**.

The partners are collectively called a **firm**, or a **house**.

626. As a rule the legal liability of a partner in a firm is different from that of a stockholder in a company or corporation; for while a stockholder is liable, with few exceptions, for only the par value of his holdings, *a partner is usually liable for the entire indebtedness of the firm.*

627. The investment of a partner is called his **capital**.

The capital may be money or anything that has a money value in the business, as goods, labor, skill, experience, the "good will" of the trade, or some mercantile advantage, etc.

628. *The gains and losses of a firm are shared in proportion to the amount of capital invested by each, and the length of time such capital is invested in the business.*

WRITTEN EXERCISES

629. 1. A and B engaged in business as partners and gained \$4000. A's capital was \$10,000, and B's was \$6000. Find the profits apportioned to each.

SOLUTION

Entire capital $= \$16000$
A's share of the capital $= \frac{10000}{16000} = \frac{5}{8}$; $\therefore$ A's gain $= \frac{5}{8}$ of $\$4000 = \$2500$
B's share of the capital $= \frac{6000}{16000} = \frac{3}{8}$; $\therefore$ B's gain $= \frac{3}{8}$ of $\$4000 = \$1500$
Test. — A's gain + B's gain $= \$2500 + \$1500 = \$4000$, the entire gain.

2. A and B owned a strawboard factory, A's investment being \$75,000 and B's \$45,000. The net earnings for one year were \$11,200. How much of the earnings did each partner receive?

3. Apportion a loss of \$2400 to the three partners in a business, if their respective investments are \$11,000, \$15,000, and \$6000.

4. A business block worth \$28,000, owned by three men, was insured for $\frac{3}{5}$ of its value. One had \$16,000 invested, one \$7000, and one \$5000. The block was completely destroyed by fire. What was the amount of insurance due each man?

5. As the result of a damage suit, a judgment for \$4600 was obtained against the joint owners of a quarry. The owners' investments were \$10,000, \$3000, \$6500, and \$3500, respectively. How much was each owner obliged to pay?

6. Three men invested \$2200, \$1800, and \$2000, respectively, in a business. After several years, during which the business had grown to \$10,500, the first partner withdrew from the firm. How much was his share worth?

The other two bought his part in proportion to their holdings. How much did each pay?

7. Four partners with a capital of \$48,000, of which A, B, and C furnished \$10,000 each and D the rest, declared a 16% dividend and used the surplus profits, \$2880, to increase their capital. Find each partner's dividend and his increased capital.

8. A and B formed a partnership with a capital of \$8000, of which A furnished \$5000. After 18 months A withdrew \$1000, and at the end of 2 years the partnership was dissolved. If the gain for 2 years was \$7440, how much did each partner receive?

SOLUTION

A's capital,	\$5000 for 18 mo.	= \$ 90000	for 1 mo.
" "	\$4000 for 6 mo.	= \$ 24000	for 1 mo.
A's total capital		= \$114000	for 1 mo.
B's capital,	\$3000 for 24 mo.	= \$ 72000	for 1 mo.
Total investment for both		= \$186000	for 1 mo.

A's gain = $\frac{114000}{186000}$ of \$7440 = \$4560; B's gain = $\frac{72000}{186000}$ of \$7440 = \$2880.
Test. \$4560 + \$2880 = \$7440.

9. A and B were partners and divided \$3075 in profits. A's investment was \$5000 for 1 year; B's was \$5000 for 9 months and \$6000 for 3 months. How much did each receive?

10. A and B began business with a capital of \$10,000, $\frac{3}{5}$ of which A furnished. After 6 months C entered the firm with a capital of \$5000. After another 6 months they divided \$5500 in profits. How much did each partner receive?

11. A, B, and C were partners for 16 months. A contributed \$30,000, B \$20,000, and C \$40,000, \$15,000 of which he withdrew in 12 months. When they dissolved partnership they divided \$41,400 in profits. Find the profits allotted to each.

POWERS AND ROOTS

RAISING TO POWERS

630. 1. $2 \times 2 = 4$ $2 \times 2 \times 2 = 8$ $2 \times 2 \times 2 \times 2 = 16$

Or, $2^2 = 4$ $2^3 = 8$ $2^4 = 16$

What is the product when 2 is taken 2 times as a factor? 3 times? 4 times? 5 times? 6 times?

What is the *second power* of 2? the *third* power of 2? the *fourth* power? the *fifth* power? the *sixth* power?

2. What number is the second power of 3? the third power of 3? the fourth power?

3. Find the second power of 10; the third power.

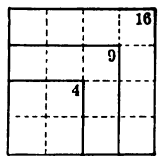
631. The number of times a number is to be used as a factor may be indicated by using an **exponent** (§ 104).

632. The product arising from using a number a certain number of times as a factor is called a **power** of that number.

4 is the second power of 2, for $4 = 2 \times 2$, or 2^2; 8 is the third power of 2, for $8 = 2 \times 2 \times 2$, or 2^3; 16 is the fourth power of 2; etc.

A number is regarded as the *first* power of itself.

633. If the side of a *square* is 2, the area is the *second* power of 2; if the side is 3, the area is the *second* power of 3; etc.



Therefore the *second* power of a number is called its **square**.

If the edge of a *cube* is 2, the volume is the *third* power of 2; if the edge is 3, the volume is the *third* power of 3; etc.

Therefore the *third* power of a number is called its **cube**.

EXERCISES

634. 1. Find the square of:

1 2 3 4 5 6 7 8 9 10 11 12

2. Find the cube of 1; of 2; of 3; of 4; of 5.

3. Find the fourth power of 1; of 2; of 3.

4. 10^2, 10^3, 10^4, 10^5, are read "ten square," "ten cube," "ten fourth power," and "ten fifth power," respectively. Tell the meaning and value of each indicated power.

5. Multiply $\frac{2}{3}$ by $\frac{2}{3}$, or square $\frac{2}{3}$.

6. Square $\frac{1}{2}$; $\frac{3}{4}$; $\frac{5}{6}$; $\frac{6}{5}$; .3; .5; .7; .12; 1.2.

7. Cube $\frac{1}{2}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{3}{4}$; .2; .3; .5; .1; .02; .05.

635. *The square of a fraction may be obtained by squaring both terms; the cube, by cubing both terms; etc.*

WRITTEN EXERCISES

636. Raise each of the following to the power indicated:

1. 20^2	5. 12^3	9. $(\frac{7}{8})^3$	13. 1.5^2
2. 25^2	6. 21^3	10. $(\frac{5}{12})^3$	14. 7.5^2
3. 45^2	7. 15^3	11. $(\frac{15}{28})^2$	15. 1.25^2
4. 52^2	8. 11^4	12. $(\frac{25}{32})^2$	16. $.111^2$

Find the area of a square whose side is:

17. 21 in.	20. 44.7 ft.	23. 0.25 ft.
18. 17 yd.	21. 50.4 ft.	24. 7.56 ft.
19. 32 rd.	22. 0.19 ft.	25. .842 mi.

Find the volume of a cube whose edge is:

26. 11 in.	28. 22 cm.	30. 3.4 ft.
27. 2.5 in.	29. 1.5 m.	31. 12.5 Dm.

32. The number of feet a body will fall, from rest, in any number of seconds is 16.08 times the square of the number of seconds. How far will a body fall in 4 sec.? in 15 sec.?

637. 1. Square 10 ; 2 tens ; 3 tens ; 4 tens ; 12 tens.

2. Any integer of two or more figures may be regarded as composed of tens and units. Express 35 as tens and units ; 45 ; 92 ; 106 ; 125 ; 432 ; 2563.

3. You already know the squares of the integers from 1 to 12, inclusive. The square of 13 may be obtained thus :

$$\begin{array}{r} 13 \\ \underline{13} \\ 39 \\ \underline{13} \\ 169 \end{array} \qquad \begin{array}{l} \phantom{100+{}}10+3 \\ \phantom{100+{}}\underline{10+3} \\ \phantom{100+{}}30+9 \text{ (Product of } 10+3 \text{ by } 3) \\ \underline{100+\ 30\qquad} \text{ (\ " \quad " \quad " \ by } 10) \\ 100+2\,(30)+9 = 10^2+2\,(10\times 3)+3^2 \end{array}$$

The square of any integer composed of tens and units is equal to the square of the tens, plus twice the product of the tens and the units, plus the square of the units.

EXERCISES

638. 1. Find the square of the number 14.

SOLUTION

$$14^2 = 10^2 + 2\,(10 \times 4) + 4^2 = 100 + 80 + 16 = 196$$

2. Complete the following table, then commit it to memory :

NUMBER	SQUARE	NUMBER	SQUARE	NUMBER	SQUARE	NUMBER	SQUARE	NUMBER	SQUARE
1	1	6	36	11	121	16	. . .	21	. . .
2	4	7	49	12	144	17	. . .	22	. . .
3	9	8	64	13	169	18	. . .	23	. . .
4	16	9	81	14	196	19	. . .	24	. . .
5	25	10	100	15	. . .	20	. . .	25	. . .

Square the following as in exercise 1, regarding each number as composed of tens and units :

3. 32	6. 35	9. 65	12. 64	15. 120
4. 41	7. 45	10. 75	13. 81	16. 125
5. 43	8. 55	11. 85	14. 92	17. 162

EXTRACTING ROOTS

639. 1. Since $25 = 5 \times 5$, what is one of the two equal factors of 25, or the *square root* of 25?

2. What is the square root of 36? of 49? of 8^2? of 10^2?

3. What is the *cube root* of $2 \times 2 \times 2$, or of 2^3? of 27, or of 3^3? of 64, or of 4^3? of 125, or of 5^3? of 10^3? of 12^3?

640. One of the equal factors of a number is called a **root** of the number.

One of the *two* equal factors of a number is its *second*, or *square*, root; one of the *three* equal factors, the *third*, or *cube*, root; one of the *four* equal factors, the *fourth* root; etc.

641. The **root sign**, or **radical sign**, is $\sqrt{\ }$. When placed over a number, it indicates that a root is to be taken.

A small figure, written in the opening of the radical sign, as in $\sqrt[2]{25}$, $\sqrt[3]{8}$, $\sqrt[4]{16}$, shows what root is to be taken. It is called the **index** of the root.

The index 2 is usually omitted. Thus, $\sqrt{25}$ means *the square root of* 25.

Roots by Factoring

EXERCISES

642. 1. Find the cube root of 125.

SOLUTION

$$\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5$$

In the same way find the value of each of the following indicated roots:

2. $\sqrt{16}$	6. $\sqrt[3]{8}$	10. $\sqrt{121}$	14. $\sqrt{169}$
3. $\sqrt{36}$	7. $\sqrt[3]{27}$	11. $\sqrt{144}$	15. $\sqrt{289}$
4. $\sqrt{49}$	8. $\sqrt[3]{64}$	12. $\sqrt{225}$	16. $\sqrt{400}$
5. $\sqrt{64}$	9. $\sqrt[4]{16}$	13. $\sqrt{625}$	17. $\sqrt[3]{1000}$

WRITTEN EXERCISES

643. **1.** Find the cube root of 216.

SOLUTION

By factoring, we find that $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$.
Arranging the factors to give three like groups,

$$216 = (2 \times 3) \times (2 \times 3) \times (2 \times 3)$$
$$\therefore \sqrt[3]{216} = 2 \times 3, \text{ or } 6.$$

Find the roots indicated:

2. $\sqrt{729}$	**6.** $\sqrt{1024}$	**10.** $\sqrt{11664}$	**14.** $\sqrt[5]{243}$
3. $\sqrt{784}$	**7.** $\sqrt{2025}$	**11.** $\sqrt[3]{46656}$	**15.** $\sqrt[3]{343}$
4. $\sqrt[3]{512}$	**8.** $\sqrt[3]{3375}$	**12.** $\sqrt[4]{20736}$	**16.** $\sqrt[7]{128}$
5. $\sqrt[3]{729}$	**9.** $\sqrt[5]{1024}$	**13.** $\sqrt[6]{15625}$	**17.** $\sqrt[4]{625}$

18. The area of a square is 324 sq. in. Find the side.

19. Find the edge of a cube whose volume is 8000 cu. in.

Square Root

644. The square of an integer or a fraction is called a **perfect square.**

645.

$\sqrt{1} = 1$	$\sqrt{100} = 10$	$\sqrt{10000} = 100$
$\sqrt{25} = 5$	$\sqrt{3600} = 60$	$\sqrt{160000} = 400$
$\sqrt{81} = 9$	$\sqrt{9801} = 99$	$\sqrt{998001} = 999$

How many figures are there in the square root of any perfect square that is expressed by not more than *two* figures? by *four* figures or by one less than four figures? by *six* figures or by one less than six figures?

The number of figures in the square root of a perfect square is the same as the number of periods of two figures each into which the number can be separated, beginning at units.

The left-hand period may contain only one figure.

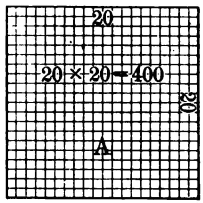
WRITTEN EXERCISES

646. 1. What is the square root of 576, or what is the side of a square whose area is 576 square units?

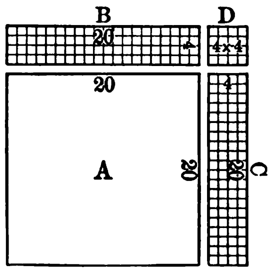
$$\begin{array}{lr|l} & 5'76 & \underline{20} \\ 20^2 = & 4\;00 & \underline{\;\;4} \\ 2 \times 20 = & 40)\overline{1\;76} & 24 \\ (40+4)\times 4 = & 1\;76 & \end{array}$$

Since the number of figures in the square root of a number may be determined by separating the number into periods of two figures each, beginning at units, the square root of 576 is seen to be composed of *tens* and *units*.

Since the square of tens is hundreds, 5 hundreds must be the square of at least 2 tens. 2 tens or 20 squared is 400, and 400 subtracted from 576 leaves 176. Therefore the root 20 must be increased by such an amount as will exhaust the remainder.



The square, A, already formed from the 576 square units is one whose side is 20 units, but inasmuch as the number of units has not been exhausted, such additions must be made to the square as will exhaust the units and keep the figure a square. The necessary additions are two equal rectangles, B and C, and a small square, D.



Since the square, D, is small, the area of the rectangles, B and C, is nearly 176 square units. This area, divided by the length of the rectangles, will give the width, which is 4 units. The width of the additions is 4 units, and the entire length, including the small square, is 44 units; therefore the area of all the additions is 4 times 44 units, or 176 square units, which is equal to the entire number of square units to be added. Hence the side of the square is 24 units, or the square root of 576 is 24.

2. Extract the square root of 3844.

$$\begin{array}{r|l} 38'44 & \underline{62} \\ 36 & \\ 2 \times 60 = 120 & 2\;44 \\ \underline{2} & \\ 122 & \underline{2\;44} \end{array}$$

Separating the number into periods, it is found that the root is composed of tens and units. Since the largest square in 38 is 6, the tens of the root cannot be greater than 6 tens, or 60. Writing 6 tens in the root, squaring, and subtracting from 3844, there is a remainder of 244.

Since the square of a number composed of tens and units is equal to (*the square of the tens*) + (*twice the product of the tens and the units*) + (*the square of the units*), when the square of the tens has been subtracted, the remainder, 244, is twice the product of the tens and the units, plus the square of the units, or only a little more than twice the product of the tens and the units.

Therefore 244 divided by twice the tens is approximately equal to the units. 2×6 tens, or 120, then, is a *trial*, or *partial*, *divisor*. Dividing 244 by the trial divisor, the units' figure is found to be 2.

Since twice the tens are to be multiplied by the units, and the units also are to be multiplied by the units to obtain the square of the units, to abridge the process the tens and units are first added, forming the *complete divisor* 122, and then multiplied by the units. Thus $(120 + 2)$ multiplied by $2 = 244$.

Therefore the square root of 3844 is 62.

Extract the square root of:

3. 2809	**8.** 5184	**13.** 3721	**18.** 3249
4. 2601	**9.** 1156	**14.** 7056	**19.** 2401
5. 6724	**10.** 6889	**15.** 2116	**20.** 9604
6. 1936	**11.** 1089	**16.** 9025	**21.** 6241
7. 8281	**12.** 8836	**17.** 5776	**22.** 9801

Find the side of a square whose area is:

23. 7225 sq. in.	**26.** 841 sq. ft.	**29.** 7744 sq. m.
24. 1444 sq. in.	**27.** 4225 sq. cm.	**30.** 4761 ares
25. 2209 sq. in.	**28.** 4624 sq. cm.	**31.** 1225 hektares

32. The area of a square field is 12.1 acres. How many rods of fence are required to inclose it?

33. I have a lot 144 feet long and 64 feet wide. What is its area in square feet? Find the side of a square lot that has the same area.

34. An officer has 5625 men and wishes to arrange them in a solid square. How many men shall he place on a side?

WRITTEN EXERCISES

647. 1. Extract the square root of 15625; of 1.5625.

	1′56′25	125
	1	
22	56	
	44	
245	12 25	
	12 25	

	1′.56′25	1.25
	1	
22	56	
	44	
245	12 25	
	12 25	

After two figures of the root in the first process have been obtained, we find that we have subtracted, in all, 10000 + 4400, or 14400, the square of 12 tens, *the part of the root already found*. Therefore, regarding 12 tens as the first part of the root, the units of the root are obtained in the usual way.

When there are decimal figures, as in the second process, they are pointed off into periods of two figures each, *beginning at the decimal point*. The process is then the same as for integers.

Separate the number into periods of two figures each, beginning at units or at the decimal point.

Find the greatest square in the left-hand period, and write its root for the first figure of the required root.

Square this root, subtract the result from the left-hand period, and annex to the remainder the next period for a dividend.

Double the root already found, for a partial divisor, and by it divide the dividend, disregarding the right-hand figure. The quotient, or quotient diminished, will be the second figure of the root.

Annex to the partial divisor for a complete divisor the figure last found, multiply this divisor by the figure of the root last found, subtract the product from the dividend, and to the remainder annex the next period for the next dividend.

Proceed in this manner until all the periods have been used. The result will be the square root sought.

1. When the number is not a perfect square, annex periods of decimal ciphers and continue the process.

2. The square root of a common fraction may be found by extracting the square root of numerator and denominator separately or by reducing it to a decimal and then extracting its root.

Extract the square root of:

2. 15129	7. 930.25	12. .555025
3. 24336	8. 655.36	13. .633616
4. 13689	9. 282.24	14. .994009
5. 30976	10. 11.0224	15. .00675684
6. 44521	11. 14.5924	16. .00767376

Extract the square root and express as a common fraction:

17. $\frac{20736}{30625}$ 18. $\frac{25921}{50625}$ 19. $32\frac{164}{625}$ 20. $87560\frac{521}{1024}$

Verify the following:

21. $\sqrt{2} = 1.414^+$ 22. $\sqrt{3} = 1.732^+$ 23. $\sqrt{5} = 2.236^+$

Extract the square root, to the nearest thousandth, of:

24. 8	27. $\frac{1}{3}$, or $\frac{3}{9}$	30. 0.1	33. .7854
25. 6	28. $\frac{1}{2}$, or $\frac{2}{4}$	31. 2.5	34. .41265
26. 7	29. $\frac{1}{5}$, or .2	32. 3.6	35. .7400063

36. Find the side of a square whose area is 1 acre.

SOLUTION

Let x = number of feet in one side of the square.
Then, the area $= x^2$ square feet. But 1 acre = 43,560 square feet.
Therefore, $x^2 = 43{,}560$
Extracting the square root of both members,
$x = 208.7+$, the number of feet in the side.

Note that a square 209 feet on a side is only a little more than an acre.

Find the side of a square whose area is:

37. 1.5 A. 38. 12,000 sq. ft. 39. 375,000 sq. ft.

40. A football field is 110 yd. long and 160 ft. wide. Find the side of a square field having the same area.

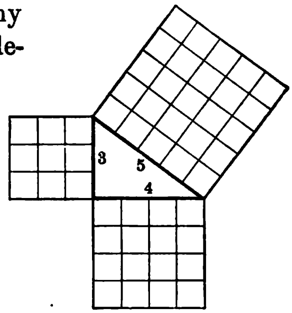
41. Find the dimensions, in rods, of a 20-acre rectangular field whose length is twice its width.

SUGGESTION. — Let x = number of rods in width. Then $2x$ = number of rods in length, and $x(2x)$, or $2x^2$, = number of square rods in area.

42. Find the dimensions, in rods, of a 90-acre rectangular field whose length is 4 times its width.

43. How much more fence is required to inclose the field mentioned in exercise 42 than a square field of equal area?

648. 1. Since the longest side, or **hypotenuse**, of this **right-angled triangle** is 5 units long, how many square units are there in the square described upon the hypotenuse?



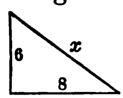
2. Since one of the other two sides, or **legs**, is 3 units long and the other 4 units long, how many square units are there in the square described upon each? in both these squares?

3. How does the number of units in the square of the hypotenuse compare with the number of units in the sum of the squares of the other two sides?

The square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.

WRITTEN EXERCISES

649. 1. Find the hypotenuse of this right-angled triangle.

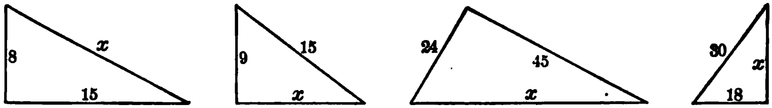


SOLUTION. $x^2 = 6^2 + 8^2 = 36 + 64 = 100$

$\therefore x = 10$

2. Draw a right-angled triangle whose legs are 5 in. and 12 in. Compute the hypotenuse. Measure it. Compare results.

3. In the following right-angled triangles find the length of each side marked x:



4. Draw an inch square and a straight line connecting two opposite corners, or a *diagonal.* Compute the length of the diagonal to the nearest .001 of an inch.

5. Draw a rectangle 4 in. by $7\frac{1}{2}$ in. Draw one of its diagonals and describe a square on it. Compute the area of the square.

6. Draw a 5-inch square and one diagonal. How does the area of the square on the diagonal compare with the area of the 5-inch square?

7. Measure the length and width of a rectangular room. Measure a diagonal on the floor as accurately as you can. Then compute the length of the diagonal to the nearest .001 of a foot. Which is the more accurate method of finding the length of the diagonal when the length and width are known exactly?

8. Next measure the height of the ceiling. Compute the distance from the lower corner of the room at one end of the diagonal on the floor to the upper corner at the other end.

9. A 40-foot ladder leans against a wall, with the foot 6 feet from the base of the wall. Draw a sketch and compute the height of the top of the ladder.

10. Two vessels sailed from the same point, one north at the rate of 15 knots an hour, the other east at the rate of 20 knots an hour. How far apart were they after 6 hours?

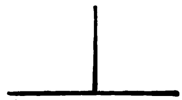
11. How far apart are the opposite corners of a square farm that contains 360 acres?

MENSURATION

PLANE FIGURES

650. The difference in direction of two lines that meet is called an **angle** (§ 212); the lines are called the **sides**, and the point where they meet, the **vertex** of the angle.

651. When a straight line meets another straight line forming two *equal* angles, each angle is called a **right angle** (§ 214).



Two Right Angles

652. A surface such that a straight line joining any two points of it lies wholly in the surface is a **plane surface.**

653. A plane surface can be measured in only two directions, and hence has only two *dimensions*, **length** and **breadth.**

654. A portion of a plane surface bounded by four straight lines is called a **quadrilateral.**

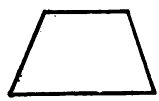
655. Lines in the same plane surface that cannot meet, however far they are extended, are called **parallel lines.**

Parallel Lines

656. What name is given to a quadrilateral whose opposite sides are parallel (§ 224)? to a right-angled parallelogram (§ 222)? to a rectangle whose sides are equal (§ 222)?

657. A quadrilateral that has only two sides parallel is called a **trapezoid.**

A parallelogram or a trapezoid has two bases known as the **lower base** and the **upper base.**



Trapezoid

658. The number of square units that any surface contains is called its **area.**

How do you find the area of a rectangle (§ 229)? a parallelogram (§ 231)? a triangle (§ 233)?

659. Find the area of each of these parallelograms:

	Base	Altitude		Base	Altitude
1.	14.5 in.	6 in.	4.	10 Km.	500 m.
2.	320 rd.	4 rd.	5.	$12\frac{3}{4}$ ft.	3 ft. 6 in.
3.	200 m.	9 m.	6.	24 rd.	4 rd. $5\frac{1}{2}$ ft.

7. What is the altitude of a parallelogram whose area is 51 square inches and whose base is $8\frac{1}{2}$ inches long?

Find the area of these triangles:

8. Base 24 ft., alt. 12 ft.

9. Base 45 rd., alt. 16 rd.

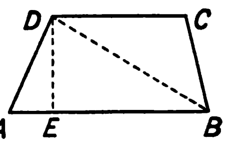
10. Base 44 m., alt. 26 m.

11. Base 8 Dm., alt. 5.2 m.

12. What is the altitude of a triangle whose area is 84 square feet and whose base is 16 feet?

13. Find the base of a triangle whose area is 147 square inches and whose altitude is 1 ft. 2 in.

660. To find the area of a trapezoid.



1. Draw a trapezoid $ABCD$; its altitude DE; and one of its diagonals DB.

2. What is the base of the triangle ABD? its altitude? its area?

3. If DC, the upper base of the trapezoid, is taken as the base of the triangle BCD, what is its altitude? its area?

4. Then, since $\frac{1}{2}\,DE \times AB$ represents the area of triangle ABD and $\frac{1}{2}\,DE \times DC$ the area of BCD, $\frac{1}{2}\,DE \times (AB + DC)$ represents the area of trapezoid $ABCD$. Hence,

661. *The area of a trapezoid is equal to one half the product of its altitude and the sum of its bases.*

For brevity, in this and subsequent similar statements, the *product of lines* means the product of the *numbers* that represent them. Dimensions must be expressed in like units.

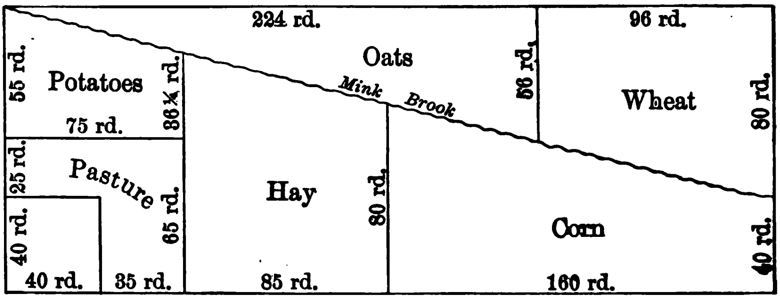
662. 1. What is the area of a trapezoid whose bases are 7 in. and 9 in., respectively, and whose altitude is $3\frac{1}{2}$ in.?

2. If the sum of the bases of a trapezoid is 32 cm. and its altitude 5 cm., what is its area?

3. A plank 28 ft. long is 1 ft. wide at one end and $1\frac{1}{2}$ ft. wide at the other end. How many square feet are there in one side of the plank? If it is 2 in. thick, how many board feet does it contain?

4. The area of a trapezoid is 36 sq. in. The altitude is 4 in. and the lower base 10 in. How long is the upper base?

One year Mr. Drake's rectangular farm was divided into fields as shown in the diagram:



Find the area, in acres and square rods, of the field devoted to:

5. Corn
6. Wheat
7. Oats
8. Hay
9. Potatoes
10. Pasture

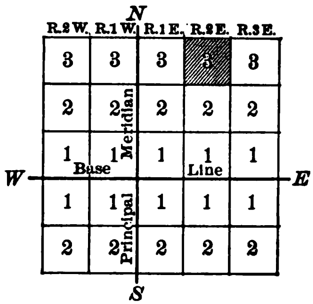
11. How many acres did the field in the lower left-hand corner contain?

12. Find the area of the farm by adding the areas of the different fields. Test your answer by finding the area of the whole farm from its length and width.

13. By applying § 648 find, to the nearest .01 of a rod, the length of Mink Brook that is within Mr. Drake's farm.

663. Government lands are usually surveyed into tracts bounded by lines that run north and south, and east and west.

664. Usually a **principal meridian** (north and south line) and a **base line** (east and west) are first established. Then other lines are run *six miles* apart each way as nearly as possible, forming **townships**.



665. A line of townships extending north and south is called a **range**.

Ranges are designated by their number east or west of the *principal meridian*, and the *townships* in each range by their number north or south of the *base line;* thus, in the first diagram, the township that is shaded is T. 3 N., R. 2 E., which means the third township north of the base line in the second range east of the principal meridian.

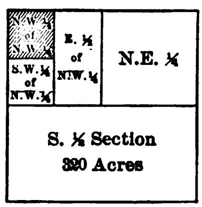
6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

TOWNSHIP

666. A township is divided into **sections**, each being *one mile* square, containing 640 acres, and numbered as in the second diagram.

667. Each section is then subdivided into *half sections*, *quarter sections*, *half-quarter sections*, and *quarter-quarter sections* as shown in the third diagram.

If this section is the shaded section of the second diagram, and that diagram in turn is the shaded township of the first diagram, the shaded part of this section would be designated thus: N.W. ¼ of N.W. ¼, Sec. 12, T. 3 N., R. 2 E., which means the northwest quarter of the northwest quarter of section 12 of the third township north of the base line in the second range east of the principal meridian.



SECTION

NOTE. — This subject is of most importance in the newer parts of the country, where land is laid out in this way.

668. 1. How many acres of land are there in a township? in a section? in a quarter-quarter section?

Write the description in full and give the number of acres in these tracts of land:

2. N.E. $\frac{1}{4}$, Sec. 14, T. 3 N., R. 5 W.

3. W. $\frac{1}{2}$ of N.W. $\frac{1}{4}$, Sec. 27, T. 5 S., R. 4 E.

4. S.E. $\frac{1}{4}$ of N.E. $\frac{1}{4}$, Sec. 36, T. 2 N., R. 1 W.

5. N.E. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 18, T. 4 S., R. 5 E.

6. S. $\frac{1}{2}$, and W. $\frac{1}{2}$ of N.W. $\frac{1}{4}$, Sec. 22, T. 3 S., R. 6 W.

7. Find the value of the tract of land mentioned in exercise 3, at $25.50 an acre.

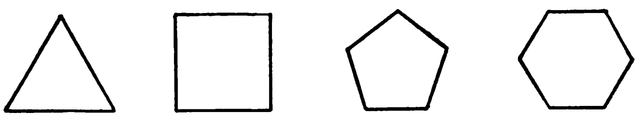
8. How many rods of fence are required to inclose the tract of land mentioned in exercise 4?

9. What is the value of a quarter section and a half-quarter section of land at $18.75 an acre?

Regular Polygons

669. Any plane figure bounded by straight lines is a **polygon.**

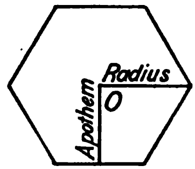
670. A polygon having all of its sides equal and all of its angles equal is called a **regular polygon.**



EQUILATERAL TRIANGLE SQUARE PENTAGON HEXAGON

671. A regular polygon of *three* sides is called an **equilateral triangle**; of *four* sides, a **square**; of *five* sides, a **pentagon**; of *six* sides, a **hexagon**; of *seven* sides, a **heptagon**; of *eight* sides, an **octagon**; of *nine* sides, a **nonagon**; of *ten* sides, a **decagon**; of *twelve* sides, a **dodecagon.**

672. A **straight line** drawn from the center of a regular polygon to any vertex is called the **radius** (plural *radii*) of the polygon.



673. A straight line drawn from the center of a regular polygon perpendicular to any side is called the **apothem** of the polygon.

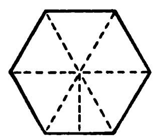
674. The line bounding a polygon, or the sum of its sides, is called its **perimeter**.

675. 1. What is the perimeter of a regular pentagon, each side of which is 5 inches long?

Find the perimeter of each of these regular polygons, having sides as given:

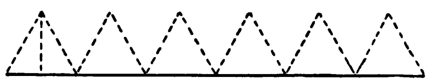
2. Hexagon, 8 in.
3. Heptagon, 4 in.
4. Decagon, $3\frac{1}{2}$ in.
5. Octagon, 2 dm.
6. Nonagon, 3 cm.
7. Dodecagon, $6\frac{1}{2}$ cm.

676. To find the area of a regular polygon.



1. Into how many triangles do the radii of a regular hexagon divide the hexagon?

2. How may the area of each triangle be found? the area of the six triangles, that is, the area of the hexagon?



3. How does the altitude of each triangle compare with the apothem of the polygon? the sum of the bases of the triangles with the perimeter of the polygon?

4. Any regular polygon may be divided in a similar manner into as many triangles as the polygon has sides. Hence,

677. *The area of a regular polygon is equal to one half the product of its perimeter and apothem.*

678. 1. If the side of a regular hexagon is 6 inches long and the apothem 5.2 inches, what is its area?

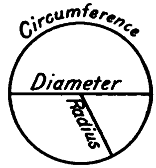
2. The side of a square is 16 inches long. How long is the apothem? Find the area by § 677 and compare with the result found in the usual way.

3. What is the area of a regular octagon whose side is 9 inches long and whose apothem is 10.86 inches?

4. The area of a regular pentagon is 61.8 square inches. Its apothem is 4.12 inches. What is the length of one side?

Circles

679. A plane figure bounded by a curved line every point of which is equally distant from a point within is called a **circle**; the point within is called the **center**; and the bounding line, the **circumference**.



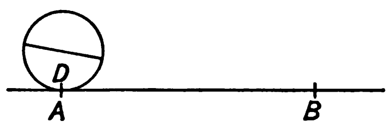
680. A straight line drawn from the center to the circumference of a circle is called a **radius**.

681. A straight line passing through the center of a circle and terminating at both ends in the circumference is called a **diameter**.

A diameter is equal in length to two radii.

682. To find the circumference of a circle.

1. Draw a circle on a piece of stiff cardboard and carefully cut it out.



2. Place one point of its circumference, as *D*, on the point *A* of a straight line and roll the cardboard along the line until *D* again touches the line as at *B*. Then *AB* is the length of the circumference.

3. Measure AB and divide its length by the length of the diameter. What is the quotient?

4. Do the same thing with other circles. The result in each case will be nearly $3\frac{1}{7}$.

683. The ratio of the circumference of a circle to its diameter cannot be expressed exactly, but it is proved in geometry to be more nearly 3.1416 than $3\frac{1}{7}$.

The symbol for this ratio is the Greek letter π (pī).

684. ***The circumference of a circle is π times the diameter, or 2π times the radius.***

Unless stated otherwise, the approximate value $3\frac{1}{7}$ will be used for π.

685. Find the circumference of a circle whose diameter is:

1. 35 in.	3. 63 yd.	5. 84 m.	7. 126 m.
2. 77 ft.	4. 49 rd.	6. 56 cm.	8. 168 mm.

Find the circumference of a circle whose radius is:

9. 28 ft.	11. 91 rd.	13. 13.3 Hm.	15. 24.5 ft.
10. 42 ft.	12. 11.9 m.	14. 5.88 cm.	16. 36.4 in.

Find the diameter of a circle whose circumference is:

17. 88 yd.	19. 12.1 in.	21. 25.3 m.	23. 990 rd.
18. 55 ft.	20. 3.74 cm.	22. 4.84 Dm.	24. 649 yd.

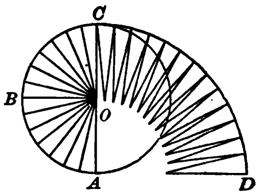
Using $\pi = 3.1416$, find the circumference of a circle whose diameter is:

25. 12 ft.	26. 20 m.	27. $15\frac{1}{2}$ yd.	28. $22\frac{3}{4}$ rd.

29. A wagon wheel is $3\frac{1}{2}$ feet in diameter. How far will the wagon have gone when the wheel has made 100 revolutions?

30. A merry-go-round has two rows of seats. One row is 7 feet from the center and the other $10\frac{1}{2}$ feet from the center. If John sits in an outer seat and Mary in an inner seat, how much farther than Mary does John ride each revolution?

686. To find the area of a circle.



From the accompanying figure, it is evident that a circle may be regarded as composed of a large number of triangles, the sum of whose bases forms the circumference, $ABCD$, of the circle, and whose altitude is the radius, AO, of the circle. Hence,

687. *The area of a circle is equal to one half the product of its circumference and radius.*

688. In § 684 the circumference of a circle is found to be 2π times the radius. Let r denote the radius; then,

$$\text{Circumference} = 2\pi r.$$

Multiplying $2\pi r$ by r and dividing by 2 (§ 687),

$$\text{Area} = \frac{2\pi r \times r}{2} = \pi r^2.$$

Therefore, if the radius of a circle is known its area may be found without finding the circumference.

689. *The area of a circle is equal to the square of the radius multiplied by π, or πr^2.*

690. Find the area of a circle whose radius is:

1. 14 in.	3. 2.8 rd.	5. 6.3 cm.	7. .07 ft.
2. 35 ft.	4. 4.2 m.	6. .91 dm.	8. 10.5 m.

9. If from a hose water can be thrown 42 feet, how many square feet of lawn can be watered from one position?

10. What is the area of a circular flower bed that is 21 feet in diameter?

11. Find the area of a table that is 22 feet in circumference.

12. A cow is tied to a stake with a rope $17\frac{1}{2}$ feet long. Over how many square feet of surface can she graze?

13. Find, to the nearest .01 of a rod, the radius of a circular field containing half an acre.

SOLIDS

691. Anything that can be measured in three directions is said to have three *dimensions*, — *length*, *breadth*, and *thickness*, — and is called a **solid**.

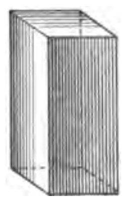
692. The surfaces that bound a solid are called its **faces** and their intersections its **edges**.

693. The number of cubic units that any solid contains is called its **volume**.

694. A solid having six rectangular faces is called a **rectangular solid** (§ 235); a rectangular solid whose faces are equal squares is called a **cube**.

695. A solid whose sides are parallelograms and whose two ends are equal polygons, parallel to each other, is called a **prism**.

TRIANGULAR PRISM



SQUARE PRISM

Prisms are named from the shape of their bases, — *triangular*, *square*, *rectangular*, *pentagonal*, etc.

696. The perpendicular distance between the bases of a prism is called its **altitude**.

697. The parallelograms taken together form the **convex surface** of the prism.

698. A solid bounded by a uniformly curved surface and having for its bases circles that are parallel to each other is called a **circular cylinder**.

CYLINDER

There are other kinds of cylinders, but in this book "cylinder" means "circular cylinder."

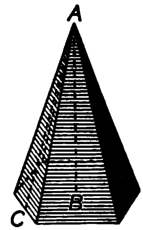
A cylinder may be thought of as a prism whose bases are circles and whose convex surface is made up of an infinite number of infinitely narrow parallelograms.

699. A solid whose base is a polygon and whose faces are triangles meeting at a point is called a **pyramid**.

The triangles form the **convex surface** of the pyramid, and the point where they meet is called the **vertex**.

Pyramids, like prisms, are named from their bases, as *triangular*, *square*, *hexagonal*, etc.

700. The perpendicular distance, as AB, from the vertex to the base of a pyramid is called its **altitude**.



PYRAMID

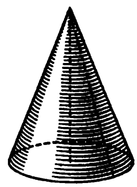
701. The altitude, as AC, of one of the triangles of a pyramid is called the **slant height** of the pyramid.

702. A solid whose base is a circle and whose surface tapers uniformly to a point, called the vertex, is a **circular cone**.

In this book "cone" means "circular cone."

The vertex of a cone is sometimes called its **apex**.

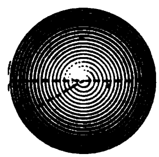
A cone may be thought of as a pyramid whose base is a circle and whose convex surface is made up of an infinite number of infinitely narrow triangles.


CONE

The **altitude** and **slant height** of a cone correspond to the altitude and slant height of a pyramid.

703. A solid bounded by a curved surface every point of which is equally distant from a point within, called the center, is a **sphere**.

704. A straight line passing through the center of a sphere and terminating at both ends in the surface is called its **diameter**.


SPHERE

705. One half the diameter of a sphere, or the distance from the center to the surface, is called its **radius**.

706. A circle of a sphere whose plane passes through the center is called a **great circle** of the sphere.

707. A great circle divides a sphere into two equal parts called **hemispheres**. The circle is the **base** of each hemisphere.

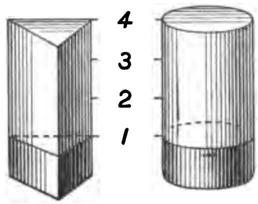
708. The circumference of a great circle of a sphere is called the **circumference** of the sphere.

The circumference of a sphere is the greatest distance around it.

Surfaces of Solids

709. To find the convex surface of a prism or a cylinder.

1. How many square inches are there in the convex surface of a prism or a cylinder 1 inch high, if the perimeter of its base is 6 inches?



2. What is the area of the convex surface, if the altitude of the solid is 2 inches? 3 inches? 4 inches?

3. How, then, may you find the area of the convex surface of a prism or a cylinder?

710. *The convex surface of a prism or a cylinder is equal to the product of its altitude and the perimeter of its base.*

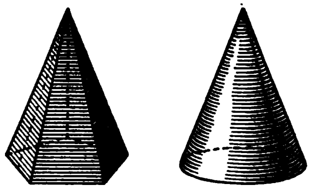
711. 1. What is the convex surface of a cylinder whose diameter is 2 feet and whose height is 5 feet?

2. Find the convex surface of a triangular prism whose base is 2 centimeters on each side and whose altitude is 4 centimeters.

3. What is the convex surface of a square prism whose sides are each $2\frac{1}{2}$ feet and whose altitude is 6 feet?

4. Find the entire surface (convex surface and surface of the two bases) of a cylinder that is 8 feet in height and has a base 3 feet in diameter.

712. To find the convex surface of a pyramid or a cone.



1. You have learned that the convex surface of a pyramid is composed of triangles, and that the convex surface of a cone may be assumed to be made up of an infinite number of triangles. The bases of these triangles form the perimeter of the base of the solid, and their altitude is the slant height of the solid.

2. How, then, may you find the area of the convex surface of a pyramid or a cone?

713. *The convex surface of a pyramid or a cone is equal to one half the product of its slant height and the perimeter of its base.*

714. 1. What is the convex surface of a rectangular pyramid whose base is 6 feet square, and whose slant height is 5 feet?

2. Find the convex surface of a cone having a base 5 centimeters in diameter and a slant height of 3 decimeters.

3. What is the convex surface of a cone whose base is 20 feet in diameter, and whose slant height is 20 feet?

4. At 30¢ per square yard, what is the cost of painting a church steeple, the base of which is an octagon 6 feet on each side, and whose slant height is 80 feet?

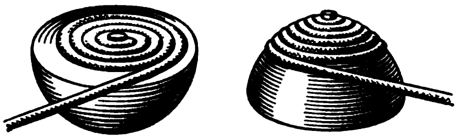
5. How many feet of convex surface are there on a cone, the base diameter of which is 6 feet, and whose slant height is $9\frac{1}{2}$ feet?

6. How many feet of convex surface are there on a pyramid whose base is 10 feet square, and whose slant height is 20 feet?

7. How many feet of convex surface are there on a cone whose base is 8 feet in diameter, and whose slant height is 6 feet?

715. To find the convex surface of a sphere.

1. The length of a waxed cord sufficient to cover the convex surface of a hemisphere, when carefully wound as shown in the picture, is just twice the length of cord required to cover the base of the hemisphere; that is, the area of the convex surface of a hemisphere is twice the area of its base.



2. Then how many times the area of a great circle of a sphere is the convex surface of the *whole* sphere?

3. It may be proved also by geometry that the convex surface of a sphere is equal to 4 great circles.

4. How does the radius of a great circle of a sphere compare with the radius of the sphere? What formula (§ 688) expresses the area of a circle? Then, what formula expresses the area of 4 great circles, or the convex surface of a sphere?

716. *The convex surface of a sphere is equal to* 4 *great circles, or to* $4\pi r^2$.

717. Denote the diameter by d. Then, since $4r^2 = (2r)^2 = d^2$, *the convex surface of a sphere is equal to the square of the diameter times* π, *or* πd^2.

718. 1. What is the convex surface of a baseball 3 inches in diameter?

2. Find the convex surface of a rubber ball having a radius of 12 centimeters.

3. What is the convex surface of a spherical cannon ball 8 inches in diameter?

4. Find the cost, at 12 cents a square foot, of gilding a sphere 28 inches in diameter.

Volume of Solids

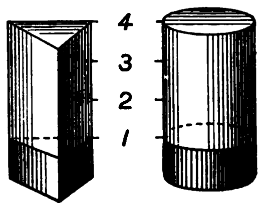
719. 1. What is meant by the volume of a solid?

2. How can you find the volume of a rectangular solid (§ 235) from its three dimensions?

3. Find the volume of a rectangular solid whose base is 5 feet by 4 feet and whose altitude is 5 feet.

720. To find the volume of a prism or a cylinder.

1. If a triangular prism with a base of 9 square inches were 1 inch high, how many cubic inches would it contain? if it were 2 inches high? 3 inches? 4 inches?



2. If a cylinder with a base 9 square inches in area were 1 inch high, how many cubic inches would it contain? if it were 2 inches high? 4 inches high?

3. How, then, can you find the volume of a prism or a cylinder from its altitude and the area of its base?

721. *The volume of a prism or a cylinder is equal to the product of its altitude and the area of its base.*

722. 1. What is the volume of a prism with a base 5 inches square, if its altitude is 3 inches?

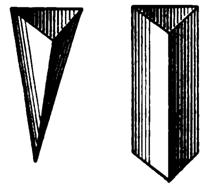
2. Find the volume of a cylinder whose diameter is 8 decimeters and altitude 2 meters.

3. What is the capacity in bushels of a bin 8 feet square and 9 feet high, inside measurements? (1 bu. $= 1\frac{1}{4}$ cu. ft.)

4. How many gallons of water will a cylindrical vat hold, if it is 7 feet in diameter and 8 feet high? (1 cu. ft. $= 7\frac{1}{2}$ gal.)

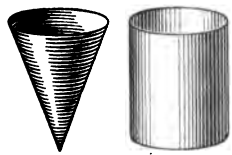
5. Find the value at 63¢ a bushel of the wheat that would fill a bin 15 feet square and 12 feet deep. (1 bu. $= 1\frac{1}{4}$ cu. ft.)

723. To find the volume of a pyramid or a cone.



1. Take a hollow pyramid and a hollow prism of the same base and altitude. Fill the pyramid with sand and empty it into the prism. How many times must you fill and empty the pyramid of sand to fill the prism?

What is the relation, then, of the volume of the pyramid to that of the prism?



2. Try the same experiment with a cone and a cylinder of the same base and altitude.

What is the relation of the volume of the cone to that of the cylinder?

3. It is proved also in geometry that the volume of a pyramid or a cone is, respectively, one third that of a prism or a cylinder having the same base and altitude.

4. Since the volume of a prism or a cylinder is equal to the product of its altitude and the area of its base, how can you find the volume of a pyramid or a cone?

724. *The volume of a pyramid or a cone is equal to one third the product of its altitude and the area of its base.*

725. 1. What are the solid contents of a cone, the diameter of whose base is 6 feet and whose altitude is 9 feet?

2. Find the solid contents of a pyramid whose base is 10 meters square and whose altitude is 20 meters.

3. If a cubic foot of granite weighs 165 pounds, what is the weight of a granite cone the diameter of whose base is 6 feet and whose altitude is 8 feet?

4. A pile of coal in the shape of a cone is 30 feet high and 132 feet in circumference. Find its volume.

5. What is the weight of a marble pyramid whose base is 4 feet square and whose altitude is 8 feet, if a cubic foot of marble weighs 171 pounds?

6. The pyramid of Cheops has a square base 746 feet on a side and an altitude of 480 feet. Find its volume.

726. To find the volume of a sphere.

1. As indicated in the figure, a sphere may be divided into a great number of figures that are essentially pyramids. The sum of the bases of these pyramids is the convex surface of the sphere, and the altitude of each pyramid is the radius of the sphere.

Then how may the volume of a sphere be found?

2. It is proved also in geometry that the volume of a sphere is equal to $\frac{1}{3}$ the product of its radius and its convex surface.

3. Since (§ 716) $4\pi r^2$ represents the convex surface of a sphere and r its radius, (§ 724) $\frac{1}{3}r \times 4\pi r^2$, or $\frac{4}{3}\pi r^3$, represents the volume of a sphere.

727. *The volume of a sphere is equal to one third the product of its radius and its convex surface, or $\frac{4}{3}\pi r^3$.*

728. 1. Find the volume of a sphere whose radius is $1\frac{1}{2}$ feet.

2. What is the volume of a sphere 21 centimeters in diameter?

3. The circumference of a sphere is 22 inches. Find its volume.

4. A spherical aquarium has a diameter of 14 inches, measured on the inside. How many cubic inches of water will it hold? Find the weight of water that it will hold.

5. Find the weight of a 6-inch cannon ball made of iron that has a specific gravity of 7.21. (Use 1 cu. ft. = $7\frac{1}{2}$ gal.)

Similar Surfaces

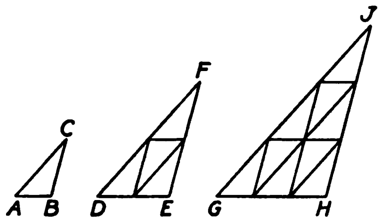
729. Figures that are of exactly the same shape though they differ in size are called **similar figures.**

All circles are similar; also all squares, and all regular polygons having the same number of sides. Two maps of the same country drawn to different scales are similar figures.

In order that polygons may be similar, for every angle of the one there must be a corresponding equal angle of the other and the sides about the equal angles must be proportional.

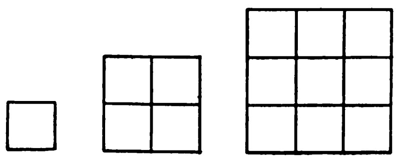
730. 1. Triangles ABC, DEF, and GHJ are similar. How do the sides of ABC compare in length with the corresponding sides of DEF? of GHJ?

How many triangles of the size of ABC are there in DEF? in GHJ?



2. The sides of the first two triangles are in the ratio of 1 to 2, and their areas are in the ratio of 1 to 4 (the squares of 1 and 2). The sides of the first and third triangles are in the ratio of 1 to 3, and their areas in the ratio of 1 to 9.

3. Show in the same way that the sides of these squares are proportional to 1, 2, and 3, and that their areas are proportional to 1, 4, and 9, the squares of the sides.



731. 1. *The corresponding sides or like dimensions of similar plane figures are proportional.*

2. *The areas of similar plane figures are proportional to the squares of their corresponding lines.*

732. 1. If a rectangle is 4 inches long and 3 inches wide, find the width of a similar rectangle that is 8 inches long.

SUGGESTION. $4 : 8 = 3 : x.$

2. The area of a circle is 5 square inches. Find the area of a circle whose diameter is twice the diameter of the first.

3. The side of a square is 5 inches. Find the side of another square that contains 4 times as much area.

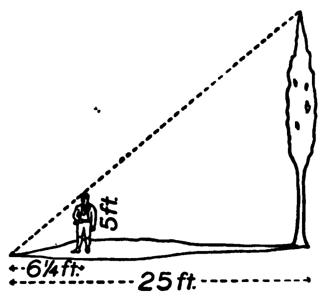
4. The sides of two regular octagons are as 1 to 3. What is the ratio of their areas?

5. A schoolroom has two square blackboards whose sides are 3 feet and 6 feet, respectively. What is the area of the first? Find the area of the second by applying the principle of similar figures.

6. A lady has two circular flower beds, one having a radius of 4 feet and the other a radius of 16 feet. How do they compare in area?

7. The sides of a triangle are 1 centimeter, 2 centimeters, and $2\frac{1}{2}$ centimeters. What are the sides of a similar triangle containing 25 times the area of the first?

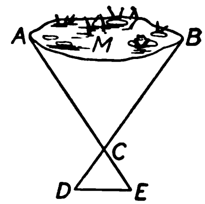
8. If the ratio of two similar triangles is 16, what is the ratio of their bases?



9. By the principle of similar figures, find the height of a poplar tree that casts a 25-foot shadow when a boy 5 feet tall casts a shadow $6\frac{1}{4}$ feet long.

10. When a telephone pole 30 feet high casts a shadow of 60 feet, what is the height of a church steeple that casts a shadow 300 feet long?

11. Suppose that A and B are two points on the opposite sides of the pond M, and we wish to find the distance between them.



Measure the distances AC and BC. Set a stake at E, a short distance from C in the line AC, and set another stake at D in the line BC, making the ratio of CD to CB equal to the ratio of CE to CA. Measure DE.

The triangles DCE and ACB are then similar and $AB : DE = AC : CE$.

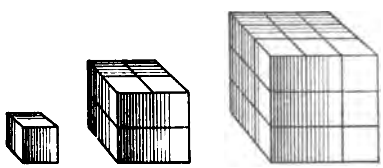
12. I wish to ascertain the distance between A and B on the opposite sides of a lake. From C, I measure the line AC, 2000 feet, and BC, 1500 feet. I set a stake at E in line with AC, 100 feet from C, and another at D in line with BC, 75 feet from C. The distance between D and E is 50 feet. What is the distance between A and B?

Similar Solids

733. Solids that have exactly the same shape though they differ in volume are **similar solids.**

The corresponding edges or other lines of similar solids are proportional.

734. 1. Draw three cubes as in these figures, having edges proportional to 1, 2, and 3.

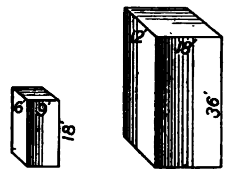


2. How many cubes the size of the first does the second contain? the third?

3. The volumes of cubes with edges proportional to 1, 2, and 3 are proportional to 1, 8, and 27, the cubes of the edges.

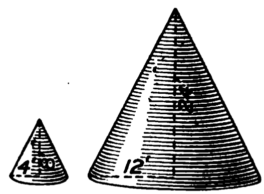
735. *The volumes of similar solids are proportional to the cubes of their corresponding lines.*

736. 1. In what ratio are the corresponding dimensions of the two similar prisms shown here? Find the volume of the first prism (§ 721). By what number must you multiply this to get the volume of the second prism? What is the volume of the second prism?



Find the volume of the second prism by § 721 and compare results.

2. In what ratio are the corresponding dimensions of these two cones? How can you find the volume of the second cone from that of the first? Find the volume of each cone.



3. If a prism 5 inches high contains 30 cubic inches, how many cubic inches will a similar prism 10 inches high contain?

4. The volume of a sphere is 96 cubic feet. What is the volume of a sphere having a diameter half as long?

5. If the altitude of a cone that weighs 10 pounds is 2 feet, what is the altitude of a similar cone of the same material that weighs 270 pounds?

6. If a bowl 1.2 decimeters in diameter holds a certain quantity of milk, how many times this quantity will a similar bowl 1.8 decimeters in diameter hold?

7. A ball weighs 10 pounds. Find the weight of a ball of the same material, if its diameter is 3 times as great.

8. If a column 2 decimeters in diameter contains 728 cubic decimeters, what will be the volume of a similar column 1 decimeter in diameter?

9. How many more gallons of water can be contained in a tank 21 feet in diameter and 60 feet high than in a similar tank 40 feet high?

GENERAL REVIEW

737. 1. Mr. Kirk purchased 60 shares of a Philadelphia stock company, engaged in the manufacture of infants' hose, at \$ $6\frac{3}{4}$ above par, brokerage $\frac{1}{8}$ %. How much did the stock cost him?

2. The company was capitalized at \$75,000. What per cent of the stock did Mr. Kirk own?

3. Mr. Kirk inspected the mill in detail. An order was received for 80 doz. pairs of infants' black hose size 4; 132 doz. size $4\frac{1}{2}$; 248 doz. size 5; 280 doz. size $5\frac{1}{2}$; 236 doz. size 6; and 112 doz. size $6\frac{1}{2}$. How many dozen pairs were ordered?

4. He found the weights per dozen pairs to be as follows:

Size	**4**	**$4\frac{1}{2}$**	**5**	**$5\frac{1}{2}$**	**6**	**$6\frac{1}{2}$**
Weight (ounces)	8	9	$10\frac{1}{2}$	$11\frac{3}{4}$	14	$15\frac{5}{8}$

Find the total weight of the yarn used for these stockings, allowing a waste in manufacture of $\frac{1}{2}$ oz. per dozen pairs.

5. The price of the yarn for this order ($832\frac{1}{2}$ lb.) was \$ $1.21\frac{1}{2}$ per pound and for dyeing it $6\frac{1}{2}$ ¢ per pound. If these two items were $62\frac{1}{2}$ % of the total cost of the stockings, what was the cost?

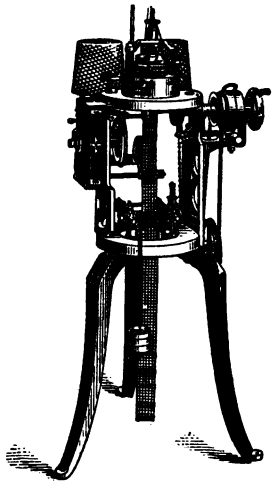
6. The purchaser was given his choice of buying the 1088 dozen pairs at \$1.90 per dozen, or buying size 4 at \$1.60 per dozen, $4\frac{1}{2}$ at \$1.70, 5 at \$1.80, $5\frac{1}{2}$ at \$1.90, 6 at \$2, and $6\frac{1}{2}$ at \$2.10. Which was the better offer and how much?

7. He accepted the second offer and paid \$2038 less 5 % discount. How much did the mill gain, the cost being \$1704.96?

8. The hose for this order were the product of $3\frac{2}{5}$ days' work. What was the daily output of the mill?

9. At the rate of 320 dozen per day, how many dozen pairs of hose would be turned out during August, if there are 4 Sundays? how many with 4 Saturday half-holidays?

10. In the knitting room, 48 "ribbers" were used for making the legs. $\frac{1}{4}$ of them could make 810 dozen of sizes 4 or $4\frac{1}{2}$, in a week of 6 days; $\frac{1}{2}$ of them, 1392 dozen of sizes 5 or $5\frac{1}{2}$; and the rest, 612 dozen of sizes 6 or $6\frac{1}{2}$. Find the daily capacity of a single machine on each size.



RIBBER

11. At a winding machine a woman in one day wound enough yarn for 100 dozen pairs of stockings, size 4 (8 oz. plus $\frac{1}{2}$ oz. waste per doz.), and $2\frac{7}{8}$ lb. over. How many pounds did she wind?

12. How much did she receive for winding 56 lb. at $2\frac{1}{2}$¢ per pound?

13. The 45 girls employed to make and knit the feet to the legs, that day footed 330 dozen pairs, earning in all $59.40. How many dozen did each girl foot? at what price? how much did each earn?

14. One of the girls employed to mend stockings in the finishing room averaged 104 dozen per day and received $1\frac{3}{8}$¢ per dozen. Find her wages for a year, deducting 8 days less than 20 % of the year for Sundays and holidays.

15. The mill and stock were insured for $45,000 at $1\frac{3}{8}$ %. What was the premium?

16. The year's sales, in dozens of pairs, were:

Size	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	6	$6\frac{1}{2}$
White	4256	5248	4912	4032	1312	1024
Black	1664	4288	8896	9744	8120	3696
Colored	1552	2624	3840	2432	2256	912

How many dozen of each size were sold? how many white? how many black? how many colored? Find the total number.

17. Referring to exercise 4, how many pounds of yarn were used for all the white hose, adding $\frac{1}{2}$ oz. per dozen for waste?

18. This yarn, 14,062 lb., was $1\frac{1}{4}\%$ less than the total amount bought for white hose, the shrinkage being due to loss in weight during bleaching. How much of this yarn was purchased?

19. Find the total cost of the 14,240 lb. of yarn for white stockings at an average price of \$1.21 per pound, and 4¢ per pound additional for bleaching.

20. How much yarn was used in the manufacture of the black and the colored hose, allowing as before $\frac{1}{2}$ oz. waste per dozen?

21. With this yarn (37,938 lb.) there was no shrinkage. Find its cost at \$1.21 per pound and $6\frac{1}{2}$¢ per pound for dyeing.

22. What was the cost of materials for all hose sold?

23. The cost of labor in the manufacture of stockings was 35¢ per dozen; the cost of boxes, holding $\frac{1}{2}$ dozen, was $1\frac{1}{2}$¢ each. Find the cost of these items for the 70,808 dozen made during the year; the cost with \$66,170.95 for materials added.

24. These expenses, \$93,077.99, were \$915.21 more than 85% of the entire expenses for the year, including supplies and repairs. What were the company's total expenses?

25. The average price at which these infants' hose were sold during the year was \$1.90 per dozen pairs. If the discount allowed was 5%, find the net receipts from the year's sales.

26. The only other source of income was the sale of waste, amounting to $\frac{1}{2}$ oz. per dozen pairs of hose. How much money was received by selling the waste at 12¢ per pound?

27. The company's expenses being \$108,426.80 and income \$128,073.97, find the gain; the gain per cent, to the nearest tenth.

28. From the net earnings ($19,647.17) a surplus of $1647.17 was laid aside for future needs, and the remainder was divided among the stockholders. What was the rate of dividend (capital $75,000)?

29. Find the amount of dividend on Mr. Kirk's 60 shares.

30. As he had purchased the shares at $6\frac{3}{4}$% premium, find, to the nearest tenth, the per cent of profit on his investment.

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31. If a German mason earned 51 marks per week, find the amount he received per day (10 hours); per hour.

32. A British report states that women in a Japanese cotton mill receive 5*d.* per day, men $7\frac{1}{2}$*d.* How much more does a man earn in 20 days than a woman?

33. One year the average weekly wage of the English laborer was 31 shillings and of the American laborer $11.679. Which laborer was the better paid, and how much better?

34. The English laborer expended 14 shillings per week for food, the American $3.893. Which expended the greater amount, and how much greater? What part of his wages did each expend for food?

35. A coal miner's pay in England is 43*d.* per ton of coal raised to the surface, and the average amount raised per year in a certain coal district is 260 tons per man. Find the yearly wage in U. S. money.

36. What is the cost of painting the front, sides, and back of a house 22 feet wide, 32 feet deep, and 24 feet high, one coat, at 30¢ per square yard, no allowance being made for doors and windows?

37. How many pencils 7 inches long can be made from a block of red cedar 7 in. by $10\frac{1}{2}$ in. by $2\frac{1}{4}$ in., if the block is sawed into strips $3\frac{1}{2}$ in. wide and $\frac{3}{16}$ in. thick, each strip making the halves of 6 pencils?

38. The largest paper mill in the world has a roof area of $5\frac{1}{2}$ acres. How many square feet does it cover?

39. The mill uses 225 cords of wood for 200 tons of paper daily. How many cords of wood are used in 310 working days? How much paper is made?

40. A paper machine in this mill turns out a strip of paper 500 feet long and 125 inches wide each minute. Find the area of paper turned out in an hour.

41. The average spruce tree yields enough wood for 500 pounds of paper. The editions of 9 books published recently used 2,000,000 pounds of paper. How many trees were necessary to produce the paper for these editions?

42. In a recent year 1,233,150,000 pounds of paper were used in the United States, 77.6 % of which was used for newspapers, 16.4 % for books and periodicals, and the remainder in job printing. How much was used in each branch?

43. The per capita value of paper used in the United States per year is the greatest in the world, $1.66. When the population was 76,303,387, what was the value of paper used in a year?

44. A large oil company declared a dividend of $43,875,000 on a capital of $97,500,000. What per cent was paid?

45. The National City Bank, N.Y., one year paid a dividend of $3\frac{3}{4}$ % on its capital of $25,000,000. How much was paid?

46. A salesman for a typewriter company received $10 per week salary and 15 % commission on all sales. Find his earnings for one week, if he sold 1 machine for $95 and 3 for $87 apiece.

47. The assessed valuation of property in St. Louis one year was $428,510,340, and the tax rate was $1.47 per $100. Find the amount collected by taxation that year.

48. Mr. Gibson, a real estate agent, leased some property for 8 years at $1800 per year. His commission was $2\frac{1}{2}$ % the first year and 1 % each succeeding year. Find his total commission.

49. A rice miller sold 560 sacks (100 lb.) of "choice" rice, and received $220.50 as his commission at $7\frac{1}{2}$ %. For how much did he sell the rice, and at what price per pound?

50. A set of carpenter's tools listed at $85 is subject to discounts of 60 % and 10 %. Find the net cost of the tools.

51. A grocer paid $57.60 for 2 cases of cocoa. There were 12 boxes per case, each box holding 6 pounds of cocoa in $\frac{1}{4}$-pound tins. He sold the cocoa at 13¢ per tin. What was his total gain? his per cent of gain? his gain per pound?

52. Using the following ingredients, a baker made 252 lb. of vanilla-cream biscuit, which he sold at 16¢ per pound:

1 bbl. flour @ $3.85	$3\frac{1}{2}$ gal. milk at 6¢ per qt.
20 lb. butter @ 26¢	2 qt. glycerine @ 75¢
16 lb. lard @ $11\frac{1}{2}$¢	10 oz. soda at 8¢ per lb.
64 lb. powdered sugar @ $7\frac{3}{4}$¢	Ammonia, salt, vanilla, 96¢

2 gal. eggs (10 eggs to a pint) at 24¢ per doz.

Find the cost; amount received; gain; per cent of gain. What part of the total cost was the cost of the eggs?

53. An engine for exhibition at St. Louis reached the grounds on 21 cars, the load of each averaging $34\frac{2}{7}$ tons. Find the entire weight of the engine.

54. A camera has been invented that takes 1100 images of insects in flight every second. How many images can be obtained in 2 min. 31 sec.?

55. A dragon fly's wings beat 28 times per second. The number of beats per second made by a bee's wings is 22 more than 6 times as many as this, and is 8 less than $\frac{3}{5}$ of the number of beats per second made by a house fly's wings. At what rate do a bee's wings beat? a house fly's wings?

56. The projectile from a $9\frac{1}{2}$-inch Krupp gun penetrated 25 cm. of armor plate 4500 m. away. At a distance $33\frac{1}{3}\%$ less, the penetration was 20% deeper. Find this distance, and the depth penetrated.

57. Four cannon on board a Japanese warship fire 2 shots apiece per minute. If used continuously for $\frac{1}{4}$ of an hour, they would discharge shot to the value of \$48,000. How much does each shot cost?

58. The charges for wireless telegraphing from the coast of England to passing vessels, by the Marconi system, are $6\frac{1}{2}$*d.* per word. Find the cost in United States money of a message of 18 words.

59. The British Admiralty pays £96 10*s.* per year per instrument for a period of 15 years for Marconi wireless telegraph instruments, after which it will own them. Find the cost, in U. S. money, of one instrument.

60. Some of the records of winners in the Olympic games at Athens one year were:

Event	Name and Country of Winner	Record
Throwing the discus	Sheridan, U. S.	41 m. 46 cm.
Throwing the stone	Giorgantas, Greece	19 m. $92\frac{1}{2}$ cm.
Running broad jump	Prinstein, U. S.	7 m. 20 cm.
Single hand lifting	Steinbach, Austria	76.55 Kg.
100-meter dash	Hahn, U. S.	$11\frac{1}{5}$ sec.
1500-meter race	Lightbody, U. S.	4 min. $19\frac{3}{5}$ sec.
400-meter swimming race . . .	Scheff, Austria	6 min. $23\frac{4}{5}$ sec.

Express the first three records in feet and inches to the nearest $\frac{1}{4}$ inch; the fourth, in pounds to the nearest pound.

How much greater is 100 meters than 100 yards?

How many feet less is 1500 meters than 1 mile?

How much greater or less is 400 meters than $\frac{1}{4}$ mile?

61. In Edinburgh 25 million gallons of water per day are supplied to a population of 455,000. Find, to the nearest gallon, the daily supply per person.

62. From the 1200 persons who travel between Berlin and Hamburg every day, 19,200 marks are collected in fares. What is the average fare in our money?

63. German railways issue three classes of mileage books for 1000 Km. each. The first is sold for 60 marks, the second for 40 marks, and the third for 25 marks. What is the price per mile in pfennigs and in cents for each class?

64. The municipal restaurants in Freiburg, Germany, serve breakfast and supper at 5¢ each and dinner at $6\frac{1}{2}$¢. How much must be paid in German money for 6 days' board?

65. How many of the 10,622 miles of railway in Hungary does the government operate, if it operates 83%?

66. The copper conductor of the cable between Ireland and Newfoundland weighs $633\frac{3}{4}$ tons, which is 650 pounds per mile. How long is the cable?

67. A cable message was sent at 2:33 P.M. from Washington (75° W.) to London (0°), where it was received $6\frac{3}{4}$ seconds later. At what time was it received?

68. Mexico has one of the tallest steel chimneys in America, 230 feet high. What is its height expressed in meters?

69. The load of a camel is about $4\frac{1}{2}$ metric quintals. Find the cost of transporting 21 camel loads of goods in Turkey from Samsun to Kharput at $4.34 per metric quintal.

70. The propeller shafts of the steamship *Deutschland* are 40 m. in length and 630 mm. in diameter, and terminate in bronze propellers 7 m. in diameter. What are these dimensions in feet and inches?

71. The temperature of the Gulf Stream averages about 28° C. What is its temperature in the Fahrenheit scale?

72. To build the new turbine Cunard steamships, the British Government lent the Cunard Co. £2,600,000 with interest at $2\frac{3}{4}\%$, the term of interest to begin 1 year after completion of the vessels, which occurred 26 months after the loan was made. Find the value of the use of this money at $2\frac{3}{4}\%$ until it began to draw interest.

73. This loan is payable in 20 equal yearly payments. Find the interest for a year on the sum due after the 10th payment.

74. The United States Government pays $4 per statute mile for the transatlantic transportation of mail by American vessels. Over route number 57, from New York to Southampton, the contract price for 47 trips is $690,483.20. What is the distance, to the nearest tenth of a mile?

75. One year the weight of the first-class mail over this route was 350,810 pounds, and of other mail 2,384,406 pounds. When not under contract American vessels receive the postage rate, 5¢ per $\frac{1}{2}$ ounce for first-class mail, and 1¢ per 2 ounces for other mail. How much was saved that year by contracting for the carriage at $4 per statute mile?

76. The first-class mail matter carried to Great Britain one year was 165,148,403 g. This was 29% of all the first-class transatlantic mail. What was the entire amount?

77. A refrigerator car costs about $800. Reckoning interest at 4%, depreciation at 8%, repairs at $3\frac{3}{4}\%$, and general expenses at $4, find the annual cost of operating a car.

78. The monthly rental for the car received by the owners from the railroads is $19. What is the annual income at this rate? What per cent of income is obtained on the cost of the car?

79. The ice for these cars costs $2.50 per ton loaded on the cars. An average price per car on one railroad for icing from New Orleans to Chicago, 923 miles, is $23. Find the number of tons of ice used, and the cost of icing per mile.

80. One firm in the private-car and meat-packing business on a capital of $110,500,000 cleared $47,727,412 in one year. What per cent, to the nearest .01 %, did the stock pay?

81. The cargo of a sailing vessel carrying 100,000 bushels of wheat worth 72¢ per bushel, from New York to Glasgow, is insured at $1\frac{3}{4}$ %, while a steam vessel's cargo of the same value is insured at $\frac{5}{8}$ %. What is the difference in the premiums?

82. Gold bullion is insured between London and New York for 25¢ per hundred pounds (avoirdupois). What is the premium on a shipment of $1,000,000 in gold, worth $26.87 per avoirdupois ounce?

83. A cargo of grain worth $80,000 in an American iron-hull steamship sailing between San Francisco and Liverpool is insured at $1\frac{1}{4}$ %. What is the premium?

84. The premium on a British vessel carrying an equivalent cargo between these ports is $200 less. What is the rate for the British steamship?

85. Find the insurance premium on a shipment of coffee consisting of 3750 bags of 60 Kg. each, if the value is 16¢ per kilogram and the insurance $\frac{7}{16}$ %.

86. The cost of shipping cotton goods from New York to Shanghai is $6.87 per 40 cubic feet of space (1 ton). What would be the cost of a shipment occupying 25,000 cubic feet?

87. The charge for piloting a boat from Sandy Hook to New York is $4.13 per foot of the vessel's draught. What are the charges for piloting the *Deutschland*, which draws 29 feet of water?

88. A wholesale carpet dealer imported 24,000 yards of Brussels carpet, 27 inches wide, paying 83¢ a yard for it. What was the total cost, including duty at 28¢ per square yard and 40 % ad valorem?

89. The duty on French plate glass is 30¢ per square foot. How much duty must be paid on a shipment of 9 plates, each $8\frac{1}{2}$ ft. by 11 ft.?

90. One year a large city levied a tax of $38,403,761.18 upon property assessed at $2,016,947,622. What was the tax rate per dollar, to the nearest hundredth of a mill?

91. The assessed valuation was about 40% of the true value. What was the tax of a man owning property worth $250,000?

92. A German department store whose receipts were 450,000 marks was taxed $.8\frac{4}{45}$ mark per hundred. Find the amount of the tax in marks; in United States money.

93. A harbor tax of 2% on the general cargo of a vessel is charged for the use of the harbor at Schiedam, Holland. What was the tax on a vessel carrying a cargo worth $163,180?

94. Elevators in some New York buildings move at the rate of 700 feet per minute. Find the ratio of this speed to that of surface cars that average 10 miles per hour.

95. A double eagle weighs $21\frac{1}{2}$ pennyweights. The ingots from which double eagles are stamped in one mint weigh 72 ounces apiece. What is the value of such an ingot?

96. From one ingot 65 double eagles are stamped, the remaining metal being recast into another ingot. How many ounces from each ingot are recast?

97. The sweepings of the San Francisco mint one year brought $\$.0064\frac{1}{4}$ per ounce avoirdupois. The entire receipts from this source were $14,257. How many pounds were sold?

98. The average price of silver in London recently was $26\frac{3}{8}d.$ per ounce (troy). Find the value of 25 pounds of silver in U. S. money.

99. A pile of 9600 new silver dollars weighed $687\frac{1}{2}$ pounds (troy). How many grains did each dollar weigh?

100. The earthquake that destroyed San Francisco in 1906 occurred at 5:13 A.M., the news being received in New York by telegraph at 9:15 A.M. How long was this after the event?

101. The brokerage for buying or selling London exchange is \$5 per £10,000. What is the brokerage on £3500?

102. In London a broker's commission is $\frac{1}{64}$ % on continental exchanges. Find in U. S. money a London broker's commission on 48,000 marks.

103. The report of 147 joint stock, fire-insurance companies for a period of five years showed a premium collection of \$196,532,866, 8.61 % of which was gain. Find the gain.

104. With U. S. 3s reg. at $102\frac{5}{8}$, what is the annual return on an investment of \$35,962.50, brokerage $\frac{1}{8}$ %?

105. How much must I invest in Imperial Japanese $4\frac{1}{2}$s at $91\frac{5}{8}$ to yield an annual income of \$1350, brokerage $\frac{1}{8}$ %?

106. One issue of 1000-dollar New York City 3 % bonds amounting to \$3,125,000 sold at a premium of \$45,877. What was the selling price per bond?

107. For one issue of \$3,555,000 of New York City bonds $112\frac{1}{7}$ was paid, the highest price paid up to that time for municipal bonds. How much did the entire sale bring?

108. When the shadow of a man 6 feet tall was $4\frac{1}{2}$ feet long, the shadow of a tree beside which he stood was 54 feet long. How tall was the tree?

109. A pencil 4 inches long, held 3 feet from the eye, just covers a tree 45 feet high. How far distant is the tree?

110. A balloon will remain in air as long as its own weight and that of the gas it contains is less than the weight of air displaced. Air weighs 31 grains per 100 cu. in. How many cubic feet of gas are required to support a balloon weighing 1674 lb., if the gas used is $\frac{1}{8}$ as heavy as air?

APPENDIX

GREATEST COMMON DIVISOR

738. What is meant by a *common divisor* (§ 121) of two or more numbers? by their *greatest common divisor?*

WRITTEN EXERCISES

739. 1. Find the greatest common divisor of 63, 105, and 231.

SOLUTION. — Factoring, we find that $63 = 3 \times 3 \times 7$, $105 = 3 \times 5 \times 7$, and $231 = 3 \times 7 \times 11$; hence, the *common* divisors of 63, 105, and 231 are 3, 7, and 3×7, or 21, and their *greatest* common divisor is 21.

$$\begin{array}{r|rrr} 3 & 63 & 105 & 231 \\ \hline 7 & 21 & 35 & 77 \\ \hline & 3 & 5 & 11 \end{array}$$

$\text{g. c. d.} = 3 \times 7 = 21$

The work may be conveniently arranged as in the margin, the common prime factors being taken out successively until the quotients contain no common factor.

The greatest common divisor of two or more numbers is equal to the product of all their common prime factors.

Find the greatest common divisor of:

2. 24 and 120	7. 210 and 350	12. 16, 24, and 40
3. 96 and 168	8. 135 and 225	13. 48, 60, and 96
4. 60 and 270	9. 330 and 495	14. 120, 210, and 345
5. 33 and 154	10. 352 and 384	15. 216, 360, and 432
6. 42 and 252	11. 232 and 496	16. 126, 294, and 462

17. The sides of a triangular lot are 108 ft., 132 ft., and 156 ft. long. How many boards of the greatest length possible are needed to fence it with a fence 4 boards high?

740. When the numbers cannot be factored readily, the method given on next page may be used.

WRITTEN EXERCISES

741. **1.** Find the greatest common divisor of 221 and 494.

```
221)494(2
    442
     52)221(4
        208
         13)52(4
            52
```

The greatest common divisor cannot be greater than the smaller number; therefore 221 is the g. c. d., if it is exactly contained in 494. By trial it is found that 221 is not an exact divisor of 494, for there is a remainder of 52. Therefore 221 is *not* the g. c. d.

Since 494 and 442, which is 2 times 221, are each divisible by the g. c. d. (§ 99, 10), their difference, 52, must be divisible by the g. c. d. (§ 99, 11); therefore the g. c. d. cannot be greater than 52. 52 is the g. c. d., if it is exactly contained in 221; for, if it is contained in 221, it is contained in 442 (§ 99, 10), and in 52 *plus* 442, or 494 (§ 99, 11). By trial we find that 52 is not an exact divisor of 221, for there is a remainder of 13. Therefore 52 is *not* the g. c. d.

Since 221 and 208, which is 4 times 52, are each divisible by the g. c. d., their difference, 13, must contain the g. c. d.; therefore the g. c. d. cannot be greater than 13. 13 is the g. c. d., if it is exactly contained in 52; for, if it is contained in itself and 52, it is contained in 208 and in 13 *plus* 208, or 221, also in 494, which is the sum of 52 and 442, the latter being 2 times 221. By trial we find that 13 is an exact divisor of 52. Hence 13 is the g. c. d. of 221 and 494.

To find the greatest common divisor of two numbers:

Divide the greater number by the less, and if there is a remainder, divide the less number by it, then the preceding divisor by the last remainder, and so on until there is no remainder. The last divisor will be the greatest common divisor.

NOTE. — If more than two numbers are given, find the greatest common divisor of any two, then of this divisor and another number, and so on.

Find the greatest common divisor of:

2. 286 and 338	**6.** 663 and 1053	**10.** 119, 255, and 357
3. 391 and 460	**7.** 714 and 1683	**11.** 146, 365, and 219
4. 406 and 551	**8.** 836 and 2432	**12.** 430, 602, and 989
5. 496 and 899	**9.** 925 and 5439	**13.** 627, 741, and 817

LEAST COMMON MULTIPLE

742. What is meant by a *common multiple* (§ 136) of two or more numbers? by their *least common multiple?*

743. *The least common multiple of two or more numbers is equal to the product of all their different prime factors, each factor used the greatest number of times that it occurs in any of the numbers.*

WRITTEN EXERCISES

744. 1. Find the least common multiple of 20, 28, and 55.

SOLUTION. — Factoring, we find that $20 = 2 \times 2 \times 5$, $28 = 2 \times 2 \times 7$, and $55 = 5 \times 11$; then the factors of the l. c. m. are 2, 2 (the greatest number of 2's found in any number) and 5, 7, 11 (the only factors of any of them not already taken).

Hence the l. c. m. $= 2 \times 2 \times 5 \times 7 \times 11 = 1540$.

$$\begin{array}{r|rrr} 2 & 20 & 28 & 55 \\ \hline 2 & 10 & 14 & 55 \\ \hline 5 & 5 & 7 & 55 \\ \hline & 1 & 7 & 11 \end{array}$$

$$2 \times 2 \times 5 \times 7 \times 11 = 1540$$

The work may be conveniently arranged as in the margin, the l. c. m. being obtained by dividing the given numbers by any prime factor common to two or more of them, until the quotients are prime to each other, and then finding the product of these divisors and the last quotients.

NOTE. — If any of the given numbers are factors of any of the others, they may be disregarded; thus, the common multiples of 4, 8, 16, 32, 80, and 128 are the same as the common multiples of 80 and 128.

Find the least common multiple of:

2. 16, 20, 48, and 60
3. 18, 21, 27, and 36
4. 20, 35, 40, and 45
5. 36, 40, 48, and 126
6. 45, 75, 135, and 180
7. 126, 36, 48, and 66
8. 16, 60, 140, and 210
9. 57, 36, 231, and 330
10. 126, 140, 154, and 280
11. 132, 144, 288, and 324

12. Find the capacity of the smallest vessel whose contents can be measured by using either a 3-quart, a 4-quart, a 5-quart, or a 6-quart measure.

13. What is the shortest length that can be measured by either of four sticks that are respectively 10 in., 15 in., 27 in., and 30 in. long?

14. The fore wheel of a carriage was 12 feet in circumference and the hind wheel 15 feet. A rivet in the tire of each was up when the carriage started, and when it stopped the same rivets were up together for the 640th time. How many miles had the carriage traveled?

745. When the prime factors of the given numbers cannot be discovered by inspection, they may be found by the method of finding the greatest common divisor under such circumstances.

WRITTEN EXERCISES

746. 1. Find the least common multiple of 255 and 357.

$$\begin{array}{r|rr} 51 & 255 & 357 \\ \hline & 5 & 7 \end{array}$$

$$51 \times 5 \times 7 = 1785$$

By the method of § 741, the g. c. d. of the numbers is found to be 51.

Dividing each of the given numbers by 51, the quotients 5 and 7 are obtained, which are prime to each other. Therefore $51 \times 5 \times 7$, or 1785, is the l. c. m. of the numbers.

Find the least common multiple of:

2. 282 and 354	7. 431 and 573	12. 777 and 1110
3. 348 and 638	8. 570 and 969	13. 1007 and 1855
4. 289 and 425	9. 665 and 760	14. 2232 and 2352
5. 414 and 529	10. 720 and 868	15. 3230 and 2550
6. 468 and 923	11. 871 and 938	16. 4136 and 1504

CUBE ROOT

747. The cube of an integer or a fraction is called a **perfect cube.**

What is the cube root (§ 640) of a number? How may the cube root of a number be found by factoring (§§ 642, 643)?

748. Verify these cube roots by the factoring method:

$\sqrt[3]{1} = 1$	$\sqrt[3]{1000} = 10$	$\sqrt[3]{1000000} = 100$
$\sqrt[3]{27} = 3$	$\sqrt[3]{46656} = 36$	$\sqrt[3]{47045881} = 361$
$\sqrt[3]{729} = 9$	$\sqrt[3]{970299} = 99$	$\sqrt[3]{997002999} = 999$

How many figures are there in the cube root of a perfect cube that is expressed by not more than *three* figures? by *six* figures or by one or two less than six figures? by *nine* figures or by one or two less than nine figures?

749. *The number of figures in the cube root of a perfect cube is the same as the number of periods of three figures each into which the number can be separated, beginning at units.*

The left-hand period may contain one, two, or three figures.

750. Observe this form of the cube of 24, or 20 + 4:

$$\begin{array}{rlr}
24 = & 20 + 4 & \\
24 = & 20 + 4 & \\
\hline
480 = & 20^2 + 20 \times 4 & \text{(Product by 20)} \\
96 = & \quad + 20 \times 4 + 4^2 & \text{(Product by 4)} \\
\hline
576 = & 20^2 + 2\,(20 \times 4) + 4^2 & \\
24 = & 20 + 4 & \\
\hline
11520 = & 20^3 + 2\,(20^2 \times 4) + \quad (20 \times 4^2) & \text{(Product by 20)} \\
2304 = & \quad + \quad (20^2 \times 4) + 2\,(20 \times 4^2) + 4^3 & \text{(Product by 4)} \\
\hline
13824 = & 20^3 + 3\,(20^2 \times 4) + 3\,(20 \times 4^2) + 4^3 &
\end{array}$$

Hence, if the tens of a number are represented by t and the units by u, the cube of a number consisting of tens and units will be the cube of $(t + u)$, or $t^3 + 3\,t^2u + 3\,tu^2 + u^3$.

WRITTEN EXERCISES

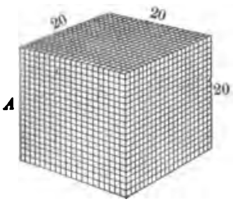
751. 1. What is the cube root of 13824, or what is the edge of a cube whose volume is 13824 cubic units?

$$
\begin{array}{rrl}
 & & 13'824\,(20+4=24 \\
20^3 = & & 8\,000 \\
3 \times 20^2 = & 1200 & \overline{5\,824} \\
3 \times 4 \times 20 = & 240 & \\
4^2 = & \underline{16} & \\
 & 1456 & 5\,824
\end{array}
$$

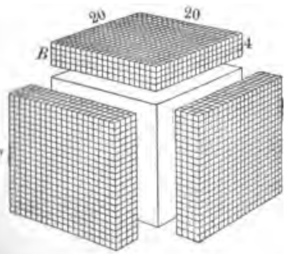
Since (§ 749) the number of figures in the cube root of a number may be determined by separating the number into periods of three figures each, beginning at units, the cube root of 13824 is seen to be composed of *tens* and *units*.

The tens in the cube root of the number cannot be greater than 2, for the cube of 3 tens is 27000. 2 tens, or 20, cubed is 8000, which, subtracted from 13824, leaves 5824; therefore the root, 20, must be increased by a number such that the additions will exhaust the remainder.

A



B



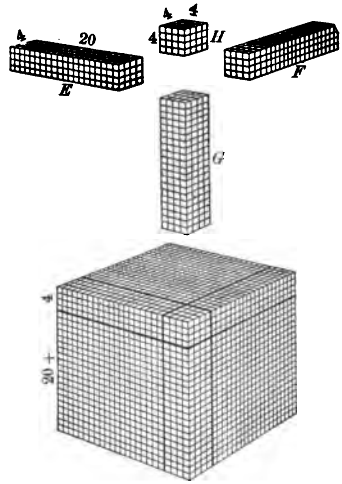
C D

The cube A already formed from the 13824 cubic units is one whose edge is 20 units in length. The additions to be made, keeping the figure formed a perfect cube, are 3 equal rectangular solids, B, C, and D; 3 other equal rectangular solids, E, F, and G; and a small cube, H. Inasmuch as the solids B, C, and D comprise much the greatest part of the additions, their volume will be *nearly* 5824 cubic units, the whole volume to be added.

Since the volume of these three equal solids is nearly equal to 5824 cubic units, and the area of a side of each is 20×20, or 400, square units, if we divide 5824 by 3 times 400, or 1200, we shall obtain the approximate thickness of the additions, which is 4 units. 3×20^2, or 1200, then, is a *trial*, or *partial*, *divisor*.

Since all the additions have the same thickness, if this area, 1200 square units, is multiplied by 4, the result will be the volume of the additions B, C, and D.

Each of the solids E, F, and G is 20 units long and 4 units wide; consequently the area of one side of each is 4×20, or 80, square units, and since there are 3 of these solids, 3×80, or 240, square units is the area which multiplied by 4 will give the volume of the additions E, F, and G. The area of one side of the cube H is 4×4, or 16, square units, which multiplied by 4 gives the volume added by the cube H.



But, in the process, instead of multiplying each of the areas represented by 1200, 240, and 16 separately by 4, it is more convenient to find their sum, 1456, which multiplied by 4 gives 5824, the total number of cubic units in the volume of the additions.

This completes the cube and exhausts the remaining cubic units.

Therefore the edge of the completed cube is $20 + 4$, or 24, units in length, or the cube root of 13824 is 24.

We may further explain the process (rewritten below) by reference to the form, $t^3 + 3t^2u + 3tu^2 + u^3$ (§ 750), the cube of a number composed of tens and units.

$$\begin{array}{rr|l}
 & & 13'824(24 \\
t^3 = 20^3 = & & 8\,000 \\ \cline{3-3}
3t^2 = 3 \times 20^2 = & 1200 & 5\,824 \\
3tu = 3 \times (20 \times 4) = & 240 & \\
u^2 = 4 \times 4 = & 16 & \\ \cline{2-2}
3t^2 + 3tu + u^2 = & 1456 & \\
u \times (3t^2 + 3tu + u^2) = & & 5\,824
\end{array}$$

Taking out the cube of the tens ($t^3 = 8000$) there remains 5824 which contains $3t^2u + 3tu^2 + u^3$, or ($3 \times$ the tens^2 $\times$ the units) + ($3 \times$ the tens $\times$ the units^2) + (the units^3).

Each of these parts contains the *units* (u) as a factor; hence 5824 is the product of two factors, one of which is the units (u) and the other is $3 \times$ the tens^2 + $3 \times$ the tens $\times$ the units + the units^2 ($3t^2 + 3tu + u^2$).

Since $3 \times$ the tens^2 is much greater than the rest of the factor, if 5824 is divided by $3 \times$ the tens^2, or 1200, the quotient is about equal to the units or other factor. It is found to be 4. 1200, then, is a *trial*, or *partial*, *divisor*.

The divisor *completed* is therefore $3 \times 20^2 + 3 \times 20 \times 4 + 4^2$, which is equal to $1200 + 240 + 16$, or 1456. This multiplied by 4 gives the product 5824. Therefore the cube root of the number is 24.

When the number consists of more than two periods of figures, the root may be found in the same manner by considering, each time, the root already found as *tens* and the next figure of the root as *units*.

2. What is the cube root of 48228544?

		48′228′544	364
$3^3 =$		27	
Partial divisor, $3 \times 30^2 =$	2700	21228	
$3 \times 30 \times 6 =$	540		
$6^2 =$	36		
Complete divisor,	3276	19656	
		1572544	
Partial divisor, $3 \times 360^2 =$	388800		
$3 \times 360 \times 4 =$	4320		
$4^2 =$	16		
Complete divisor,	393136	1572544	

3. What is the cube root of 22.906304?

		22.906′304	2.84
$2^3 =$		8	
$3 \times 20^2 =$	1200	14906	
$3 \times 20 \times 8 =$	480		
$8^2 =$	64		
	1744	13952	
		954304	
$3 \times 280^2 =$	235200		
$3 \times 280 \times 4 =$	3360		
$4^2 =$	16		
	238576	954304	

In pointing off decimal periods, begin at the decimal point.

When the number of figures in the root is more than two, the following method materially abridges the process.

4. What is the cube root of 4 to four decimal places?

			4 ⌊1.5874+
			1
	$3 \times 10^2 =$	300	3000
	$3 \times 10 \times 5 =$	150	
$u^2 =$	$5^2 =$	25	
$3t^2 + 3tu + u^2 =$		475	2375
$3tu + u^2 =$		175	625000
$3(t^2 + 2tu + u^2) =$	$3 \times 150^2 =$	67500	
	$3 \times 150 \times 8 =$	3600	
	$8^2 =$	64	
		71164	569312
		3664	55688000
	$3 \times 1580^2 =$	7489200	
	$3 \times 1580 \times 7 =$	33180	
	$7^2 =$	49	
		7522429	52657003
		33229	3030997000
	$3 \times 15870^2 =$	755570700	3022282800

Since the root of the number is not a whole number, periods of decimal ciphers are annexed, and the required number of decimal places found.

After two figures of the root have been found, the partial divisors may be found as follows: Add together 3 times the product of the tens by the units and the square of the units, then add this sum to the complete divisor, plus the square of the units, and the result, with two ciphers annexed, will be the next partial divisor.

Thus, in the example solved, to obtain the partial divisor for the third figure of the root add 150 and 25, and place their sum immediately below the complete divisor. Then add together 175, 475, and 25, and to the sum annex two ciphers. The result will be the next partial divisor.

After several decimal places have been found a *few* more may be found by ordinary division.

It will be seen by examining the solution that the numbers added together are equal to $3t^2 + 6tu + 3u^2$, or $3(t^2 + 2tu + u^2)$, or $3(t + u)^2$, that is, the sum is 3 times the square of (the tens plus the units).

Separate the number into periods of three figures each, beginning at units.

Find the greatest cube in the left-hand period, and write its root for the first figure of the required root.

Cube this root, subtract the result from the left-hand period, and annex to the remainder the next period for a dividend.

Take three times the square of the root already found, considered as tens, for a partial divisor, and by it divide the dividend. The quotient or the quotient diminished will be the second part of the root.

To this partial divisor add three times the product of the first part of the root, considered as tens, by the second part, and also the square of the second part. Their sum will be the complete divisor.

Multiply the complete divisor by the second part of the root, and subtract the product from the dividend.

Continue thus until all the figures of the root have been found.

1. When there is a remainder, after subtracting the last product annex periods of decimal ciphers, and continue the process. The figures of the root obtained after the ciphers are annexed will be decimals.

2. Decimals are pointed off into periods of three figures each, beginning at tenths and passing to the right.

3. The cube root of a common fraction is found by extracting the cube root of both numerator and denominator separately, or by reducing the fraction to a decimal and then extracting its root.

Extract the cube root of:

5. 54872
6. 175616
7. 405224
8. 857375
9. 3048625
10. 43614208
11. 130323843
12. 1865409391
13. 4065356736
14. 95256152263
15. 491916472984
16. 13312.053
17. 28.094464
18. .000166375
19. .000001953125

20. What is the cube root of 2 to four decimal places?

21. What is the cube root of 6 to five decimal places?

22. What is the cube root of $\frac{4}{9}$? $\frac{6859}{19683}$? $\frac{166375}{34965783}$?

752. Since the volume of a cube is the product of the three equal factors that represent its edges, it is evident that the cube root of the volume gives the length of the edge.

753. From § 735 it may be inferred (proved in geometry) that:

The corresponding dimensions of similar solids are proportional to the cube roots of their volumes.

WRITTEN EXERCISES

754. 1. A cubical box contains 54,872 cubic inches. What is its depth?

2. How deep is a cubical cistern containing 2744 cu. ft.?

3. What is the number of square inches in one face of a cubical block whose volume is 185,193 cubic inches?

4. What are the dimensions of a cubical box that contains as much as a rectangular box 5 ft. 4 in. long, 4 ft. 6 in. wide, and 2 ft. 8 in. deep?

5. Find, to the nearest tenth of a foot, the depth of a cubical bin that will contain 1200 bu. (Use 1 bu. = 2150.42 cu. in.)

6. A cubical cistern holds 400 barrels of water. How many feet deep is it? (Use 1 gal. = 231 cu. in.)

7. A bin that is just twice as long as it is wide or high holds 200 bu. of grain. What is its length? (Use 1 bu. = $1\frac{1}{4}$ cu. ft.)

8. If a globe 4 inches in diameter weighs 8 lb., what will be the diameter of a similar one that weighs 125 lb.?

SUGGESTION. $4 : x = \sqrt[3]{8} : \sqrt[3]{125}.$

9. If a haystack 18 feet in diameter contains 27 tons of hay, what is the diameter of a similar stack that contains 64 tons?

10. A bushel measure is in the form of a cylinder $18\frac{1}{2}$ in. in diameter, and 8 in. deep. What will be the dimensions of a peck measure of similar shape?

TABLES

Measures of Length

12 inches = 1 foot
3 feet = 1 yard
$16\frac{1}{2}$ feet = 1 rod
320 rods = 1 mile (statute)

1 mi. = 1760 yd. = 5280 ft. = 63,360 in.

A **nautical mile (knot)** = 6080.27 ft., or approximately 1.15 mi.

A **furlong** = $\frac{1}{8}$ mi.; a **fathom**, used in measuring the depth of water, is 6 ft.; a **hand**, used in measuring the height of horses, is 4 in.

Measures of Surface

144 square inches = 1 square foot
9 square feet = 1 square yard
$30\frac{1}{4}$ square yards = 1 square rod
160 square rods = 1 acre

1 acre = 43,560 sq. ft.

An acre of land in the form of a square is very nearly 209 ft. on a side.

A tract of land 1 mile square is often called a **section**.

100 sq. ft. of roofing, flooring, or slating is called a **square**.

Measures of Volume

1728 cubic inches = 1 cubic foot
27 cubic feet = 1 cubic yard

A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high, or 128 cu. ft. of wood, is called a **cord**.

Surveyors' Linear Measures

100 links = 1 chain
80 chains = 1 mile

This chain, called **Gunter's chain**, is 4 rd., or 66 ft., long. 1 **link** = 7.92 in.

Links are written as hundredths of a chain.

Surveyors' Square Measures

10 square chains = 1 acre
640 acres = 1 square mile

The convenience of this system lies in the easy reduction of square chains to acres, by moving the decimal point one place toward the left.

Liquid Measures

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon

1 gal. = 231 cu. in.; 1 cu. ft. = $7\frac{1}{2}$ gal., approximately. A gallon of water weighs about $8\frac{1}{3}$ lb.; a cubic foot of water weighs about $62\frac{1}{2}$ lb., or 1000 oz.

In measuring the capacity of cisterns, etc., $31\frac{1}{2}$ gal. = 1 **barrel**.

Dry Measures

2 pints = 1 quart
8 quarts = 1 peck
4 pecks = 1 bushel

1 bu. = 2150.42 cu. in., or approximately $1\frac{1}{4}$ cu. ft.

Our bushel is the **Winchester bushel**. In form it is a cylinder $18\frac{1}{2}$ in. in diameter and 8 in. deep. This has been displaced in England by the **imperial bushel** of 2218.192 cu. in.

Avoirdupois Weight	Troy Weight
16 ounces = 1 pound	24 grains = 1 pennyweight
100 pounds = 1 hundredweight	20 pennyweights = 1 ounce
2000 pounds = 1 ton	12 ounces = 1 pound
1 **long** or **gross ton** = 2240 pounds	1 troy lb. = 5760 gr. = $\frac{5760}{7000}$ av. lb.
1 av. lb. = 7000 gr.; 1 av. oz. = $437\frac{1}{2}$ gr.	1 troy oz. = 480 gr., or about 1.1 av. oz.

Apothecaries' Weight

This is used to some extent in filling prescriptions. The **grain, ounce,** and **pound** are the same as in troy weight, but the ounce is divided differently.

20 grains (gr.)	= 1 scruple . . .	sc. or ℈
3 scruples	= 1 dram . . .	dr. or ʒ
8 drams	= 1 ounce . . .	oz. or ℥
12 ounces	= 1 pound . . .	lb. or ℔

Apothecaries' Liquid Measures

60 drops (gtt.) or minims (♏)	= 1 fluid dram . . .	*f*ʒ
8 fluid drams	= 1 fluid ounce . . .	*f*℥
16 fluid ounces	= 1 pint	O.
8 pints	= 1 gallon	Cong.

Measures of Time

60 seconds = 1 minute	Thirty days have September,
60 minutes = 1 hour	April, June, and November.
24 hours = 1 day	All the rest have thirty-one,
7 days = 1 week	Save February, which alone
365 days = 1 year	Has twenty-eight, and one day more
366 days = 1 leap year	We add to it one year in four.

10 years = 1 **decade**; 100 years = 1 **century**.

The earth revolves around the sun in 365 days 5 hours 48 minutes 46 seconds. This is the **solar year**, and is nearly $365\frac{1}{4}$ days.

To correct the errors in the calendar, made by disregarding the fraction of a day over 365 days, centennial years divisible by 400 and other years divisible by 4 are lengthened 1 day, Feb. 29. These years are **leap years**.

Measures of Angles and Arcs

60 seconds (″)	= 1 minute (′)
60 minutes	= 1 degree (°)
360 degrees	= 4 right angles or 1 circumference

90° of angle = 1 **right angle**; 90° of arc = 1 **quadrant**.

Counting Table

2	=	1 pair
20	=	1 score
12	=	1 dozen
12 dozen	=	1 gross
12 gross	=	1 great gross

Stationers' Measures

24 sheets = 1 quire
20 quires = 1 ream

Paper is quite generally sold by the 100, 500, and 1000 sheets; also by the pound.

Metric System

The principal simple units in the metric system are the **meter**, the **gram**, and the liter, together with their decimal parts and multiples named by means of the following prefixes:

deci means	.1	**deka** means	10
centi means	.01	**hekto** means	100
milli means	.001	**kilo** means	1000

The unit of area is the **square meter.** In forming its parts and multiples, 100 square units of any order make 1 of the next higher order.

The unit of volume is the **cubic meter.** In forming its parts and multiples, 1000 cubic units of any order make 1 of the next higher order.

1 cu. cm. of water weighs 1 gram; 1 cu. dm., or 1 liter, of water weighs 1 kilogram; 1 cu. m. of water weighs 1000 kilograms, or 1 **metric ton.**

In land measures, a square dekameter is called an **are**; .01 of an are = 1 **centare**; 100 ares = 1 **hektare,** the commonest unit.

In measuring wood, a cubic meter is called **a stere.**

Equivalents

Common to Metric		Metric to Common	
1 yd.	= .9144 m.	1 m.	= 39.37 in.
1 mi.	= 1.60935 Km.	1 Km.	= .62137 mi.
1 sq. yd.	= .836 sq. m.	1 sq. m.	= 1.196 sq. yd.
1 A.	= .4047 Ha.	1 Ha.	= 2.471 A.
1 cu. yd.	= .765 cu. m.	1 cu. m.	= 1.308 cu. yd.
1 qt. (dry)	= 1.1012 l.	1 l.	= .908 qt. (dry)
1 qt. (liq.)	= .94636 l.	1 l.	= 1.0567 qt. (liq.)
1 bu.	= .35239 Hl.	1 Hl.	= 2.8377 bu.
1 oz. (troy)	= 31.10348 g.	1 Kg.	= 32.1507 oz. (troy)
1 lb. (av.)	= .45359 Kg.	1 Kg.	= 2.2046 lb. (av.)
1 T.	= .90718 M. T.	1 M. T.	= 1.1023 T.

INDEX

Above par, 425, 434.
Abstract number, 39.
Acceptance, 425.
Accident insurance, 364, 366.
Accounts, 113–115, 285–287.
Accurate interest, 387.
Acute angle, 123.
Acute-angled triangle, 126.
Ad valorem duty, 362.
Addends, 26.
Addition, 9, 25–31.
 defined, 25.
 of common fractions, 72–76.
 of decimal fractions, 98–100, 154–156.
 of denom. numbers, 181, 183.
 sign of, 9.
Agent, 249.
Aggregation, signs of, 58.
Agricultural problems, *see* Industrial problems.
Algebraic equations, 296–314, 402, 403.
Aliquot parts, 108–112, 283, 284.
 interest by, 376–378.
Altitude, 126, 471, 472.
Amount, in adding, 26.
 in interest, 376.
 in percentage, 201.
Analysis, 288–314.
Angle, defined, 123, 462.
 measures of, 123–125, 507.
Annual interest, 393.
Antecedent, 441.
Apex of cone, 472.
Apothecaries' liquid measures, 507.
Apothecaries' weight, 218, 507.
Apothem, 467.
Arabic notation, 18–22.
 defined, 18.
Arabic numerals, 18.
Arc, defined, 124.
 measures of, 123–125, 507.
Area, 126–130, 184, 462–470.
Assessment of stockholder, 434.
Assessors, 358.
Atlantic time, 327.
At par, 434.
Avoirdupois weight, 218, 315, 507.

Balance of account, 114, 285.
Bank, of deposit, 412.
 savings, 412, 419–421.
Bank bill, 412.
Bank discount, 416–419.
Bank draft, 422, 423.
Bank note, 412.
Bankers' association money order, 422, 424.
Bankers' method of finding interest, 379.
Banking, 412–421.
Base, in figures, 126, 462, 473.
 in percentage, 201.
Base line, 465.
Belgian money, 318.
Below par, 425, 434.
Bill of exchange, 422, 423, 429.
Bills and accounts, 113–115, 256.
Blank indorsement, 406.
Board feet, 222–224.
Bonds, 434–440.
Borrowing money, 416.
Braces, 58.
Brackets, 58.
Brickwork, 345, 346.
Broker, 249, 435.
Brokerage, 249–252, 435, 436.
Bunch of shingles, 227.
Bundle of laths, 225.
Burglar insurance, 365.

Cablegrams, 49.
Canadian money, 318.
Cancellation, 62, 63, 162.
Cancellation method of simple interest, 383.
Capacity, common measures of, 119, 216–219, 315, 338, 506, 508.
 metric measures of, 336, 337, 338, 508.
Capital, 433, 434, 449.
Carpeting, 228, 229, 344, 345.
Center of circle, 124, 468.
Centigrade thermometer, 342, 343.
Central angle, 124.
Central time, 327.
Certificate of stock, 433.
Check, 413, 422, 423, 430.
Check book, 413.
Checking results, 293.
Circle, 124, 468–470.
Circular measures, 125, 315, 507.
Circumference, 124, 468, 469, 473, 507.
Clearing equations of fractions, 303, 312.
Closing an account, 285.
Coefficient, 300.
Collecting by draft, 424.
Collector of taxes, 358.
Combination of processes, 58.
Commercial bank, 412.
Commercial discount, 203–205, 253–256, 399.
Commercial draft, 424, 425, 429.
Commission and brokerage, 249–252.
Commission merchant, 249.
Common denominator, 70.
Common divisor, 66, 495.
Common factor, 66.
Common fractions, 12, 13, 64–93, 134–149.
 addition of, 72–76.
 defined, 13, 64, 96.
 division of, 88–93, 141–148.
 multiplication of, 76–88, 138–141.
Common fractions, notation and numeration of, 64, 65.
 reduction of, 66–72.
 subtraction of, 72–76.
Common multiple, 70, 71, 497.
Common stock, 434.
Comparison, 14, 146–148.
 analysis by, 288.
 of fractions, 146–148.
Complete divisor, 457, 458, 502, 504.
Complex fraction, 144, 145.
Composite number, 59.
Compound denominate number, 177.
Compound fraction, 138.
Compound interest, 394–396.
 table of, 396.
Concrete number, 39.
Cone, circular, 472, 474, 477.
Consequent, 441.
Convex surface, 471–475.
Corporate bond, 434.
Corporation, 433.
Correspondence bank, 424.
Counting table, 120, 121, 508.
Coupon bond, 435.
Credit, 285.
Credit insurance, 365.
Creditor, 285.
Cube, 471.
Cube of number, 451, 452, 499.
Cube root, 499–505.
 by factoring, 454, 455.
Cubic common units, 118, 216, 218, 219, 338, 506, 508.
Cubic metric units, 334, 335, 338, 508.
Customhouse, 362.
Customs, 358, 362–364.
Cylinder, circular, 471, 473, 476.

Date line, 326.
Dates, time between, 182, 183, 276, 316, 380.
Daybook, 285.
Days of grace, 406.
Debit, 285.
Debtor, 113, 285.
Decagon, 466.
Decimal fractions, 13, 20–22, 30, 31, 34–36, 96–107, 150–164.
 addition of, 30, 31, 98–100, 154–156.
 defined, 13, 96.
 division of, 104–107, 159–163.
 multiplication of, 101–104, 156–158.
 notation and numeration of, 20–22, 150–152.
 reduction of, 96, 97, 152–154.
 subtraction of, 34–36, 98–100, 154–156.
Decimal notation, 18.
Decimal point, 20.
Degree, 123, 125, 320, 322, 342, 507.

Demand note, 405.
Denominate numbers, 177-187, 315-329.
addition of, 181, 183, 316, 320.
defined, 177.
division of, 185, 186, 316.
multiplication of, 184, 185, 316.
reduction of, 177-180, 315, 316, 319, 320, 397.
subtraction of, 182, 183, 316, 320.
Denominator, 64.
Deposit, bank of, 412.
Deposit slip, 414.
Diagonal, 124, 461.
Diameter, 468, 472.
Difference, in percentage, 201.
in subtraction, 32.
Digits, 60.
Dimensions, 462, 471.
Discount, bank, 416-419.
commercial, 203-205, 253-256, 399.
defined, 203.
exchange, 425.
on stocks, 434.
true, 391, 392.
Dividend, in division, 50.
on insurance, 367.
on stocks, 434.
Divisibility, tests of, 59-61.
Division, 11, 12, 50-57.
by 10, 100, 1000, etc., 53-56, 105, 106.
defined, 50.
of common fractions, 88-93, 141-148.
of decimal fractions, 104-107, 159-163.
of denominate numbers, 185, 186, 316.
sign of, 11, 51.
Divisor, 50.
Dodecagon, 466.
Domestic exchange, 422-428.
Draft, 422-425.
Drawee, 423.
Drawer, 404, 423.
Dry measures, common, 119, 216, 315, 333, 506, 508.
metric, 336, 338, 508.
Duties or customs, 358, 362-364, 400.

Eastern time, 327.
Edge of solid, 471.
Elimination by addition and subtraction, 309-314, 402, 403.
Endowment policy, 366.
English money, 318-321.
Equality, sign of, 9.
Equations, analysis by, 296-314, 402, 403.
in x, 296-308, 402, 403.
in x and y, 309-314, 402, 403.
Equator, 322.
Equilateral triangle, 466.
Equivalents, common measures, 216-220, 333, 506, 508.
metric and common measures, 333, 508.
money, 319.
Estimating results, 298.
Even number, 59.
Evolution, 454-461, 499-505.
Exact divisor, 59.
Exact interest, 387.
Excavating, 230, 345.
Exchange, 422-432.
Exchange market, 425.
Expectation of life, 366.
Exponent, 61, 451.
Express money order, 422, 423, 430.
Extracting roots, 454-461, 499-505.
Extremes, 443.

Face, of check, 413.
of note, 404.
of policy, 364.
of solid, 471.
Factor, defined, 59.
Factoring, 61, 62.
roots extracted by, 454, 455.
Factors and divisors, 59-63.
Fahrenheit thermometer, 220, 221, 342, 343.
Figures, notation by, 18-22.
Fire insurance, 364, 365.
Firm, 448.
Flooring, 224, 344.
Footing of bills and accounts, 118, 286.
Foreign bill of exchange, 422, 429.
Foreign exchange, 422, 429-432.
Foreign money, 318-321.
Fractional relations, 83-85, 92, 93, 145-148.
Fractions, common, 12, 13, 64-93, 134-149.
decimal, 13, 20-22, 30, 31, 84-86, 96-107, 150-164.
Freight, *see* Transportation.
French money, 318, 319, 321.
Full indorsement, 407.

General review, 483-494.
German money, 318, 319, 321.
Government bond, 435.
Government land, 465, 466.
Government revenue, 358-364, 400.
Grace, days of, 406.
Great circle, 472.
Greatest common divisor, 66, 67, 495, 496.
Greek money, 318.
Gross amount of bills, 255.
Grouping addends, 27-30.

Health insurance, 364, 366.
Hemisphere, 473, 475.
Heptagon, 466.
Hexagon, 466.
Holder of note, 406.
House, 448.
Hundredweight, 507.
Hypotenuse, 460.

Imperial bushel, 506.
Improper fraction, 12, 69.
Index of root, 454.
Indirect analysis, 288.
Indorsement, 406-409, 413, 416.
Indorser of note, 406.
Industrial problems:
Beet sugar, 371, 372.
Broom corn, 206, 207.
Cablegrams, 49.
Cattle and hogs, 272-274.
Cherries, 174, 175.
Coal mining, 352, 353.
Coffee, 262, 263.
Cotton, 373-375.
Cuts of beef, 274.
Freight, *see* Transportation.
Gardening, 166, 167.
India rubber, 265, 266.
Lumbering, 350, 351.
Manufacturing, 483-486.
Milk, butter, and cheese, 271.
Mushrooms, 207, 208.
Oil, 268-270.
Oysters, 208-210.
Peanuts, 165.
Pears, 294.
Renting, 261.
Rice, 210, 211.
Silk, 266, 267.
Strawberries, 168.
Sugar, 369-372.
Tea, 263, 264.
Telegraph, 43, 44, 305.
Telephone, 44, 176.
Transportation, 16, 17, 55, 56, 57, 119, 148, 164, 173, 174, 175, 217, 259, 260, 270, 375, 490, 491, 492.
Violets, 212.
Wheat, 348, 349.
Inland exchange, 422-428.
Insurance, 364-368, 399.
Integer, or integral number, 10.
Interest, annual, 393.
compound, 394-396.
exact, or accurate, 387.
problems in, 388-391, 401, 402.
simple, 376-392, 401, 402.
tables of, 384, 396.
Interest term, 419.
Internal revenue, 362.
International date line, 326.
International express and postal money orders, 430.
Involution, 451-453.
Italian money, 318.

Knot, 347, 506.
Known number, 296.

Land measures, 118, 333, 334, 338, 465, 466, 506, 508.

Laths, 225.
Least common denominator, 70–72.
Least common multiple, 70, 71, 497, 498.
Ledger, 285.
Ledger accounts, 285–287.
Leg, of right triangle, 460.
Lending money, 416.
Length, 462.
common measures of, 118, 338, 506, 508.
metric measures of, 330–332, 338, 508.
Letter of credit, 430.
Letters, notation by, 24, 296–314.
Liability insurance, 364, 366.
Life insurance, 364, 366–368, 399.
Like numbers, 26.
Like terms, 300.
Limited payment life policy, 366.
Linear measures, 118, 330–332, 338, 506, 508.
Liquid measures, common, 119, 216, 218, 219, 315, 338, 506, 508.
metric, 336, 338, 508.
List price, 208.
Long division, 56, 57.
Longitude and time, 322–329.
Lower base, 462.
Lowest terms, 66, 67.
Lumber measure, 222–224, 343.

Maker, of check, 418.
of note, 404.
Making change, 26, 27, 33.
Map measurements, 132.
Marine insurance, 364, 365.
Market value of stock, 434.
Marking goods, 257, 258.
Maturity of note, 406.
Means, 443.
Measurements, 13, 118–133, 218–232, 342–347, 397, 398.
practical, 220–232, 342–347, 398.
Measures, tables of, 121, 122, 125, 218, 216, 219, 331–338, 506–508.
Member of equation, 296.
Mensuration, 125–131, 184, 185, 186, 462–482.
Mercantile rule, for partial payments, 411.
Meridian, 322, 465.
Methods of solution, 288–314.
Metric system, 330–341, 401, 508.
Minuend, 32.
Miscellaneous problems, 14–17, 94, 95, 116, 117, 132, 133, 148, 149, 164–168, 173–176, 187, 194, 195, 202, 206–212, 230–232, 237, 238, 259–274, 279–282, 316–318, 339–341, 342–347, 348–353, 355–357, 369–375, 397–403, 483–494.

Mixed decimal, 20.
Mixed number, 11, 20, 68.
Money order, 422, 423, 424, 430.
Mortgage, 434.
Mountain time, 327.
Multiple, 70.
Multiplicand, 39.
Multiplication, 10, 37–49.
by 10, 100, 1000, etc., 45, 46, 102, 103.
defined, 39.
of common fractions, 76–88, 138–141.
of decimal fractions, 101–104, 156–158.
of denominate numbers, 184, 185, 316.
sign of, 40.
table, 37.
Multiplier, 39.

National bank, 412.
Nautical mile, 347, 506.
Negative unit, 300.
Negotiable check, 418.
Negotiable note, 406.
Net amount of bills, 255.
Net price, 208.
Net proceeds, 249.
Non-negotiable note, 406.
Nonagon, 466.
Notation and numeration, 18–24, 64, 65, 150–152.
of decimals, 20–22, 150–152.
of fractions, 64, 65.
of integers, 18, 19, 21, 22.
of United States money, 23.
Roman, 24.
Notes, promissory, 404–411.
Number, defined, 64.
symbol, 286.
Numeration, defined, 18.
Numerator, 64.

Obtuse angle, 128.
Obtuse-angled triangle, 126.
Octagon, 466.
Odd number, 59.
Orders of units, 18, 20.
tables of, 19, 150.
Ordinary life policy, 366.

Pacific time, 327.
Painting, 225, 226, 344.
Papering and carpeting, 228, 229, 344, 345.
Par of exchange, 429.
Par value of stock, 433, 434.
Parallel lines, 125, 462.
Parallelogram, 125, 129, 463.
Parentheses, 58.
Partial divisor, 457, 458, 500, 501, 502, 504.
Partial payments, 409–411.
Partition, 50.
Partitive proportion, 447, 448.
Partner, 448.
Partnership, 448–450.

Paving, 230, 345.
Payee, 404, 418, 423.
Pentagon, 466.
Per cent, 188.
Percentage, 188–205, 233–258, 354–368, 398, 399.
Perfect cube, 499.
Perfect square, 455.
Perimeter, 467.
Periods of figures, 19.
Perpendicular lines, 126.
Personal insurance, 364, 366–368, 399.
Personal property, 358.
Pi (π), 469.
Plane figures, 462–470.
Plane surface, 462.
Plastering, 225, 226, 230, 344.
Plate glass insurance, 365, 366.
Policy, 364.
Poll tax, 358.
Polygons, regular, 466–468.
Positive unit, 300.
Postal money order, 422, 428.
Posting, 285.
Power, defined, 451.
Powers and roots, 451–461, 499–505.
Practical measurements, 220–232, 342–347, 398.
Preferred stock, 434.
Preliminary review, 9–17, 169–176, 275–282, 397–403.
Premium, on exchange, 425.
on policy, 364.
on stocks, 434.
Present worth, 391, 392.
Price list, 208.
Prime factor, 61, 62.
Prime meridian, 322.
Prime number, 59.
Principal, 376.
Prism, 471, 473, 476.
Private bank, 412.
Problems in interest, 388–391, 401, 402.
Proceeds, net, 249.
of note, 417.
Product, 39.
Profit and loss, 245–248.
Promissory notes, 404–411.
Proper fraction, 12, 69.
Property insurance, 365, 366, 399.
Proportion, 443–450.
Protest, 407.
Public land, 465, 466.
Pyramid, 472, 474, 477, 478.

Quadrilateral, 462.
Qualified indorsement, 407.
Quotations, of exchange, 426 430.
of stocks and bonds, 435.
Quotient, 50.

Radical sign, 454.
Radius, 467, 468, 472.
Range, 465.

Rate, of discount, 208.
of insurance, 364, 366.
of interest, 376.
of taxation, 359.
Rate per cent, 201, 286.
Ratio, 441–443.
Ratio and proportion, 441–450.
Ratio sign, 441.
Real estate, 358.
Receipt, 118, 418.
Receipting bills, 118.
Reciprocal of a fraction, 90.
Rectangle, 125–128, 184.
Rectangular solid, 181, 185, 471.
Reduction, defined, 66.
of common fractions, 66–72.
of decimal fractions, 96, 97, 152–154.
of denominate numbers, 177–180, 315, 316, 319, 320, 397.
Registered bond, 435.
Regular polygons, 466–468.
Remainder, in division, 50.
in subtraction, 32.
Restrictive indorsement, 407.
Revenue, government, 358–364, 400.
Review, 9–17, 94, 95, 116, 117, 118–120, 132, 133, 134–137, 165–168, 169–176, 206–212, 233–238, 259–274, 275–282, 315–318, 348–353, 354–357, 369–375, 397–403, 483–494.
general, 483–494.
industrial problems for, 16, 17, 165–168, 174–176, 206–212, 259–274, 348–353, 369–375, 483–486.
preliminary, 9–17, 169–176, 275–282, 397–403.
Right angle, 123, 125, 462, 507.
Right-angled triangle, 126, 460.
Roll of paper, 228.
Roman notation, 24.
Roofing, 226, 227.
Roots, 454–461, 499–505.
Root sign, 454.

Savings banks, 412, 419–421.
Selling price, 257.
Set of exchange, 429.
Share of stock, 433.
Shareholder, 433.
Shingles, 227.
Shipping, *see* Transportation.
Sides of angle, 123, 462.
Sight draft, 425.
Significant figures, 18.
Similar figures, 479.
Similar fractions, 70.
Similar solids, 481, 482, 505.
Similar surfaces, 479–481.
Simple denominate number, 177.
Simple interest, 376–392, 401, 402.
Simple proportion, 443–447.
Simplification of complex fractions, 144, 145.
Six per cent interest method, 379–382.
Slant height, 472.
Solids, 181, 185, 471–478, 481, 482, 505.
similar, 481, 482, 505.
Solution, methods of, 288–314.
Solving an equation, 296.
Spanish money, 318.
Specific duty, 362.
Specific gravity, 219, 220, 340, 342, 346, 347.
Sphere, 472, 475, 478.
Square, of numbers, 451–453.
of roofing and flooring, 226, 506.
(rectangle), 125, 466.
Square common units, 118, 338, 506, 508.
Square metric units, 332–334, 338, 508.
Square root, 454–461.
Standard time, 327–329.
State bank, 412.
Stationers' measures, 122, 508.
Sterling money, 318.
Stock broker, 435.
Stock company, 433.
Stockholder, 433.
Stocks and bonds, 433–440.
Stub of check, 418.
Subtraction, 9, 10, 32–36.
Austrian method of, 34.
defined, 32.
of common fractions, 72–76.
of decimal fractions, 34–36, 98–100, 154–156.
of denominate numbers, 182, 183, 316, 320.
sign of, 9.
Subtrahend, 32.
Sum, 26.
Surfaces, common measures of, 118, 338, 506, 508.
metric measures of, 332–334, 338, 508.
Surfaces, of solids, 473–475.
similar, 479–481.
Surveyors' linear and square measures, 506.
Swiss money, 318.

Tables, interest, 384, 396.
of common measures, 121, 122, 125, 213, 216, 219, 338, 506–508.
of days between dates, 380.
of insurance premiums, 367.
of longitudes, 324.
of metric measures, 331–338, 508.
of money, 23, 318, 319.
of multiplication, 37.
Tare, 362.
Tariff, 362.
Taxes, 358–361, 400.
Telegrams, 43, 44, 305.
Telegraphic money order, 422, 423.
Telephone, 44, 176.
Temperature, measures of, 220, 221, 342, 343.
Term of discount, 417.
Term policy, 366.
Terms, of fraction, 65.
of ratio, 441.
Thermometer, 220, 221, 342, 343.
Time, and longitude, 322–329.
standard, 327–329.
Time belts, 327.
Time draft, 425.
Time measures, 119, 315, 507.
Tornado insurance, 365.
Trade, or commercial discount, 203–205, 253–256, 399.
Transportation problems, 16, 17, 55, 56, 57, 119, 148, 164, 173, 174, 175, 217, 259, 260, 270, 375, 490, 491, 492.
Transposition of terms, 301.
Trapezoid, 462–464.
Travelers' check, 430.
Trial divisor, 457, 500, 501.
Triangle, 126, 130, 184, 186, 460, 461, 468.
Troy weight, 213, 315, 507.
True discount, 391, 392.
Trust company, 412.

Unit, 64.
Unit of measure, 177.
Unitary analysis, 288.
United States money, 23.
United States rule for partial payments, 409, 410.
Unknown number, 296.
Unlike numbers, 26.
Upper base, 462.
Usury, 407.

Value, of a fraction, 69.
Venezuelan money, 318.
Vertex, of angle, 124, 462.
of cone, 472.
of pyramid, 472.
Vinculum, 58.
Volume, 181, 185, 471, 476–478, 481, 482, 505.
common measures of, 118, 216, 218, 219, 338, 506, 508.
metric measures of, 334, 335, 338, 508.

Water, weight of, 218–220, 506.
Weight, common measures of, 120, 213–215, 338, 507, 508.
metric measures of, 337, 338, 508.
Winchester bushel, 216, 338, 506, 508.
Wood measures, common, 216, 506.
metric, 335, 508.

ANSWERS

Page 14. — **1.** 121 yr. **2.** 44¢. **3.** 20¢; $\frac{5}{8}$¢.

Page 15. — **4.** 3¢. **5.** $3.35. **6.** 2 each and 4 over. **7.** 25¢. **8.** $2\frac{1}{2}$ qt. **9.** $1; $3.86. **10.** 195. **11.** 2 hr. 27 min.

Page 16. — **12.** $4.25. **13.** $4.20. **14.** $8.45. **15.** $2120. **16.** $682. **17.** $91. **18.** $495. **19.** $14.15.

Page 17. — **20.** $1902.20. **21.** $5304.35. **22.** $270. **23.** $297. **24.** $126. **25.** $6.03. **26.** $.76. **27.** $121.50. **28.** $821.29. **29.** $14\frac{3}{10}$ mi. **30.** 1 hr.

Page 30. — **2.** $47.57. **3.** 51,434. **4.** $303.18. **5.** 431.60. **6.** 274.233. **7.** 749,192. **8.** $6669.44. **9.** $651.70. **10.** 8142.881. **11.** 24,869,041.

Page 31. — **1.** 13,668 eggs. **2.** 103,284. **3.** $5128.38. **4.** 190.91 mi. **5.** $1879.20. **6.** $6988.75. **7.** 238.529 rd. **8.** 13,518,339. **9.** 812,870.

Page 34. — **2.** $4.23. **3.** 2889. **4.** $32.44. **5.** 32.32. **6.** 31,935. **7.** $4.69. **8.** 2395. **9.** $34.48. **10.** 81.59. **11.** 15,875. **12.** $3.14. **13.** 6132. **14.** $47.76. **15.** 27.38. **16.** 13,271.

Page 35. — **17.** $207.58. **18.** 177.79. **19.** 208,045. **20.** 2,195,778. **21.** $156.32. **22.** 156.55. **23.** 278,887. **24.** 1,685,092. **25.** $482.89. **26.** 645.38. **27.** 217,305. **28.** 4,759,777. **29.** $285.74. **30.** 264.87. **31.** 678,877. **32.** 30,566.48. **33.** $59.113. **34.** 19.046. **35.** 378.489. **36.** 29,062.49. **37.** $34.488. **38.** 62.089. **39.** 265.938. **40.** 38,957.59. **41.** $40.317. **42.** 47.057. **43.** 86.555. **44.** 3655.585. **45.** $34.265. **46.** 35.952. **47.** 147.389. **48.** 3288.234. **49.** $17.257. **50.** 35.219. **51.** 554.322. **52.** 3868.828.

Page 36. — **1.** 5290. **2.** 67 yr. **4.** 521 tickets. **5.** $1827. **6.** $9989.90. **7.** $327.49. **8.** 6669 girls. **9.** 5927 mi. **10.** 642,764. **11.** 16.155. **12.** 26 hr. **13.** 90.5 acres.

Page 43. — **1.** $1.08. **2.** 86¢. **3.** 22¢. **4.** 15¢. **5.** 37¢; 31¢. **6.** 46¢; 33¢. **7.** 76¢; 54¢. **8.** 53¢; 39¢. **9.** 62¢; 41¢. **10.** 61¢; 44¢. **11.** 67¢; 48¢. **12.** 70¢; 50¢. **13.** 89¢; 79¢. **14.** 95¢; 85¢. **15.** $1; 70¢; **16.** $1.16; $1.06. **17.** 92¢; 74¢. **18.** $1.20; 95¢. **19.** $1.40; $1.30. **20.** $2.60; $2.00. **21.** 96¢; 77¢. **22.** $1.40; $1.10. **23.** $1.08; 86¢. **24.** $1.10; 88¢. **25.** $1.30; $1.04. **26.** $1.49; $1.49. **27.** 1.63; $1.63. **28.** $1.55; $1.40.

Page 44. — **29.** 65¢. **30.** 55¢. **31.** 45¢. **32.** $1.25. **33.** $2.25. **34.** 55¢. **35.** $1.20. **36.** $1.40. **37.** $1.05. **38.** $2.25. **39.** $2.30.

Page 45. — **18.** 46,800. **19.** 87,000. **20.** 326,400. **21.** 684,000. **22.** 760. **23.** 4060. **24.** 1470. **25.** 6750. **26.** $5060. **27.** $11,160. **28.** $50,000. **29.** $316,000. **30.** $297,600. **31.** $262,400. **32.** $61,600. **33.** $94,080. **34.** 495,000. **35.** 2,583,000. **36.** 3,432,000. **37.** 1,440,000. **38.** 5,586,000. **39.** 708,000. **40.** 2340 sq. ft. **41.** 7840 lb.

Page 46. — **42.** 4320. **43.** 320 cars. **44.** 6000 tons.

Page 47. — 1. 320; 1600; 32,000; 16,000; 3200. **2.** 430; 2150; 43,000; 21,500; 4300. **3.** 490; 2450; 49,000; 24,500; 4900. **4.** 345; 1725; 34,500; 17,250; 3450. **5.** 355; 1775; 35,500; 17,750; 3550. **6.** 435; 2175; 43,500; 21,750; 4350. **7.** 450; 2250; 45,000; 22,500; 4500. **8.** 1100; 5500; 110,000; 55,000; 11,000. **9.** 800; 4000; 80,000; 40,000; 8000. **10.** 640; 3200; 64,000; 32,000; 6400. **11.** 885; 4425; 88,500; 44,250; 8850. **12.** 1730; 8650; 173,000; 86,500; 17,300. **13.** 1010; 5050; 101,000; 50,500; 10,100. **14.** 4070; 20,350; 407,000; 203,500; 40,700. **15.** 3675; 18,375; 367,500; 183,750; 36,750. **16.** 4580; 22,900; 458,000; 229,000; 45,800. **17.** 2485; 12,425; 248,500; 124,250; 24,850. **18.** 1590; 7950; 159,000; 79,500; 15,900. **19.** 4950; 24,750; 495,000; 247,500; 49,500. **20.** 4350; 21,750; 435,000; 217,500; 43,500. **21.** 5625; 28,125; 562,500; 281,250; 56,250. **22.** 17,435; 87,175; 1,743,500; 871,750; 174,350. **23.** 24,830; 124,150; 2,483,000; 1,241,500; 248,300. **24.** 49,995; 249,975; 4,999,500; 2,499,750; 499,950. **25.** $9000. **26.** 7000 lb. **27.** 24,800 lb. **28.** 28,000 lb.

Page 48. — **5.** 82,800. **6.** 422,730. **7.** 379,290. **8.** 935,728. **9.** 391,600. **10.** 586,088. **11.** 404,640. **12.** 1,210,788. **13.** 18,405. **14.** 53,720. **15.** 47,520. **16.** 55,250. **17.** 170,280. **18.** 164,920. **19.** 211,280. **20.** 163,930. **21.** 184,528. **22.** 305,140. **23.** 837,176. **24.** 536,004. **25.** 251,288. **26.** 1,005,237. **27.** 826,568. **28.** 898,900. **29.** 9,503,340. **30.** 39,586,720. **31.** 20,562,750. **32.** 35,087,500. **33.** 360,350,400. **34.** 176,841,000. **35.** 279,266,400. **36.** 288,055,656. **37.** $4092.00. **38.** $2038.32. **39.** $6444.13. **40.** $1245.84. **41.** $10,189.00. **42.** $89,443.14. **43.** $198,993.76. **44.** $1,475,269.65.

Page 49. — **2.** 7000 lb. **3.** 63,000 lb. **4.** $90,000. **5.** 5550 doz. **6.** 31,000 trees. **7.** $1259.25. **8.** $296.45. **9.** $36.25. **10.** $112.50. **11.** $792. **12.** $1181.25. **13.** 2,300,000 lb. **14.** $641.25. **15.** $3.75. **16.** $23.94. **17.** $26.88. **18.** $32.56.

Page 53. — 1. $91\frac{1}{3}$. **2.** $1207. **3.** 112 yd. **4.** 4400. **5.** $807. **6.** $$9.49\frac{2}{3}$. **7.** 4490. **8.** $1.01. **9.** $12.49. **10.** $6.40. **11.** $8.20. **12.** $9.99. **13.** $985\frac{1}{3}$. **14.** 364. **15.** $2018. **16.** 24. **17.** 22. **18.** 110. **19.** 160 yd. **20.** 660 ft. **21.** 480. **22.** $805\frac{1}{5}$. **23.** 375 mi. **24.** $3613\frac{4}{12}$.

Page 54. — **4.** 310. **5.** 71. **6.** 31. **7.** 41. **8.** 71. **9.** 81. **10.** $91\frac{5}{80}$. **11.** $2\frac{71}{600}$. **12.** $6\frac{511}{4000}$. **13.** $121\frac{3}{70}$. **14.** $9\frac{37}{200}$. **15.** $9\frac{313}{5000}$. **16.** $12\frac{7}{90}$. **17.** $12\frac{33}{800}$. **18.** $11\frac{919}{9000}$. **20.** $171\frac{13}{20}$; $114\frac{13}{30}$; $85\frac{33}{40}$; $68\frac{33}{50}$; $57\frac{13}{60}$; $49\frac{3}{70}$; $42\frac{73}{80}$; $38\frac{13}{90}$. **21.** $63\frac{11}{20}$; $42\frac{11}{30}$; $31\frac{31}{40}$; $25\frac{21}{50}$; $21\frac{11}{60}$; $18\frac{11}{70}$; $15\frac{71}{80}$; $14\frac{11}{90}$. **22.** $312\frac{17}{20}$; $208\frac{17}{30}$; $156\frac{17}{40}$; $125\frac{7}{50}$; $104\frac{17}{60}$; $89\frac{27}{70}$; $78\frac{17}{80}$; $69\frac{47}{90}$. **23.** $236\frac{18}{20}$; $157\frac{28}{30}$; $118\frac{18}{40}$; $94\frac{38}{50}$; $78\frac{58}{60}$; $67\frac{48}{70}$; $59\frac{18}{80}$; $52\frac{58}{90}$. **24.** $291\frac{7}{20}$; $194\frac{7}{30}$; $145\frac{27}{40}$; $116\frac{27}{50}$; $97\frac{7}{60}$; $83\frac{17}{70}$; $72\frac{67}{80}$; $64\frac{67}{90}$. **25.** $163\frac{11}{20}$; $109\frac{1}{30}$; $81\frac{31}{40}$; $65\frac{21}{50}$; $54\frac{31}{60}$; $46\frac{51}{70}$; $40\frac{71}{80}$; $36\frac{31}{90}$. **26.** $3003\frac{17}{20}$; $2002\frac{17}{30}$; $1501\frac{37}{40}$; $1201\frac{27}{50}$; $1001\frac{17}{60}$; $858\frac{17}{70}$; $750\frac{77}{80}$; $667\frac{47}{90}$. **27.** $4850\frac{1}{20}$; $3233\frac{11}{30}$; $2425\frac{1}{40}$; $1940\frac{1}{50}$; $1616\frac{41}{60}$; $1385\frac{51}{70}$; $1212\frac{41}{80}$; $1077\frac{71}{90}$. **28.** $37\frac{128}{200}$; $25\frac{28}{300}$; $18\frac{328}{400}$; $15\frac{28}{500}$; $12\frac{328}{600}$; $10\frac{528}{700}$; $9\frac{328}{800}$; $8\frac{328}{900}$. **29.** $34\frac{181}{200}$; $23\frac{81}{300}$; $17\frac{181}{400}$; $13\frac{481}{500}$; $11\frac{381}{600}$; $9\frac{681}{700}$; $8\frac{581}{800}$; $7\frac{681}{900}$. **30.** $213\frac{41}{200}$; $142\frac{41}{300}$; $106\frac{241}{400}$; $85\frac{141}{500}$; $71\frac{41}{600}$; $60\frac{641}{700}$; $53\frac{241}{800}$; $47\frac{341}{900}$. **31.** $192\frac{7}{200}$; $128\frac{7}{300}$; $96\frac{7}{400}$; $76\frac{407}{500}$; $64\frac{7}{600}$; $54\frac{607}{700}$; $48\frac{7}{800}$; $42\frac{607}{900}$. **32.** $365\frac{22}{200}$; $243\frac{122}{300}$; $182\frac{222}{400}$; $146\frac{22}{500}$; $121\frac{422}{600}$; $104\frac{222}{700}$; $91\frac{222}{800}$; $81\frac{122}{900}$. **33.** $327\frac{11}{200}$; $218\frac{11}{300}$; $163\frac{211}{400}$; $130\frac{411}{500}$; $109\frac{11}{600}$; $93\frac{311}{700}$; $81\frac{611}{800}$; $72\frac{611}{900}$. **34.** $1780\frac{101}{200}$; $1187\frac{1}{300}$; $890\frac{101}{400}$; $712\frac{101}{500}$; $593\frac{301}{600}$; $508\frac{501}{700}$; $445\frac{101}{800}$; $395\frac{601}{900}$. **35.** $3507\frac{7}{200}$; $2338\frac{7}{300}$; $1753\frac{207}{400}$; $1402\frac{407}{500}$; $1169\frac{7}{600}$; $1002\frac{7}{700}$; $876\frac{607}{800}$; $779\frac{307}{900}$.

Page 55. — **36.** $104. **37.** $40\frac{4}{30}$ hr. **38.** 17 tons. **39.** 561 bu. **41.** $51.04. **42.** $126. **43.** $127.05. **44.** $109.82. **45.** $423. **46.** $353.28. **47.** $292.38; $1329. **48.** $1552.80.

Page 56. — **49.** $256.50. **50.** $910. **51.** $143. **52.** $45. **53.** $41.80. **4.** 356. **5.** 41. **6.** 756. **7.** 922. **8.** 707. **9.** $805. **10.** $10.86. **11.** $9.09. **12.** $1208. **13.** 48. **14.** $.36. **15.** $2.47. **16.** $2.31. **17.** 384. **18.** 473. **19.** 801. **20.** 905. **21.** 809.

Page 57. — **22.** 904; $760\frac{512}{782}$; $1199\frac{128}{496}$; $676\frac{628}{879}$; $580\frac{882}{1025}$; $130\frac{2082}{4560}$; $75\frac{5782}{7854}$. **23.** $481\frac{212}{658}$; 405; $638\frac{262}{496}$; $360\frac{270}{879}$; $308\frac{1010}{1025}$; $69\frac{2070}{4560}$; $40\frac{2550}{7854}$. **24.** $478\frac{151}{658}$; $402\frac{311}{782}$; $634\frac{211}{496}$; $357\frac{872}{879}$; 307; $69\frac{85}{4560}$; $40\frac{515}{7854}$. **25.** $685\frac{134}{658}$; $576\frac{432}{782}$; 909; $512\frac{816}{879}$; $439\frac{889}{1025}$; $98\frac{3984}{4560}$; $57\frac{3186}{7854}$. **26.** $545\frac{22}{658}$; $458\frac{476}{782}$; $723\frac{24}{496}$; 408; $349\frac{907}{1025}$; $78\frac{2952}{4560}$; $45\frac{5202}{7854}$. **27.** $1413\frac{286}{658}$; $1189\frac{242}{782}$; $1875\frac{40}{496}$; $1058\frac{58}{879}$; $907\frac{865}{1025}$; $203\frac{4360}{4560}$; $118\frac{3268}{7854}$. **28.** 1204; $1013\frac{66}{782}$; $1597\frac{120}{496}$; $901\frac{258}{879}$; $772\frac{932}{1025}$; $173\frac{3852}{4560}$; $100\frac{6882}{7854}$. **29.** $1289\frac{308}{658}$; 1085; $1710\frac{310}{496}$; $965\frac{285}{879}$; $827\frac{795}{1025}$; $186\frac{310}{4560}$; $108\frac{238}{7854}$. **30.** $10{,}027\frac{8}{658}$; $8437\frac{40}{782}$; $13{,}301\frac{478}{496}$; 7506; $6436\frac{874}{1025}$; $1446\frac{4014}{4560}$; $840\frac{414}{7854}$. **31.** $5144\frac{448}{658}$; $4328\frac{704}{782}$; 6825; $3851\frac{171}{879}$; $3302\frac{650}{1025}$; $742\frac{1680}{4560}$; $431\frac{126}{7854}$. **32.** $7257\frac{126}{658}$; $6106\frac{340}{782}$; $9627\frac{240}{496}$; $5432\frac{504}{879}$; $4658\frac{782}{1025}$; $1047\frac{912}{4560}$; 608. **33.** $12{,}855\frac{168}{658}$; $10{,}816\frac{646}{782}$; $17{,}053\frac{470}{496}$; $9623\frac{141}{879}$; $8252\frac{458}{1025}$; $1854\frac{4518}{4560}$; 1077. **34.** 175 cars. **35.** 11 bales 20 lb. **36.** $.85. **37.** 4000. **38.** 125 lb. **39.** 6 tons. **40.** 14 days. **41.** $8. **42.** $1.21. **43.** 4375 crates. **44.** 310 car loads.

Page 58. — **1.** 43. **2.** 15. **3.** 22. **4.** 4. **5.** 6. **6.** 20. **7.** 62. **8.** 9. **9.** 10. **10.** 18. **11.** 6. **12.** 10.

Page 62. — **4.** $2^4, 3^2$. **5.** $2^2, 5, 13$. **6.** $3^2, 5, 7$. **7.** $2^6, 3^2$. **8.** $3^4, 11$. **9.** $2^5, 3, 7$. **10.** $2, 3, 5^2, 7$. **11.** $2, 3^2, 7^2, 11$. **12.** $2^2, 3, 5, 7, 11$. **13.** $2^7, 5, 101$. **14.** $2^3, 3^3, 11, 13$. **15.** $2^5, 5^3, 11$.

Page 63. — **2.** 8. **3.** $4\frac{1}{2}$. **4.** 2. **5.** 2. **6.** 168. **7.** 448. **8.** 2. **9.** $1\frac{1}{5}$. **10.** 168. **11.** 264.

Page 67. — **2.** $\frac{24}{48}$; $\frac{32}{48}$; $\frac{36}{48}$; $\frac{8}{48}$; $\frac{42}{48}$; $\frac{20}{48}$; $\frac{27}{48}$. **3.** $\frac{20}{60}$; $\frac{15}{60}$; $\frac{24}{60}$; $\frac{50}{60}$; $\frac{18}{60}$; $\frac{35}{60}$; $\frac{32}{60}$; $\frac{33}{60}$. **5.** $\frac{1}{2}$. **6.** $\frac{5}{9}$. **7.** $\frac{3}{5}$. **8.** $\frac{8}{15}$. **9.** $\frac{1}{5}$. **10.** $\frac{3}{10}$. **11.** $\frac{2}{3}$. **12.** $\frac{1}{4}$. **13.** $\frac{3}{8}$. **14.** $\frac{1}{6}$. **15.** $\frac{20}{40}$; $\frac{30}{40}$; $\frac{16}{40}$; $\frac{15}{40}$; $\frac{28}{40}$; $\frac{17}{40}$; $\frac{19}{40}$; $\frac{27}{40}$. **16.** $\frac{25}{100}$; $\frac{60}{100}$; $\frac{90}{100}$; $\frac{55}{100}$; $\frac{56}{100}$; $\frac{80}{100}$; $\frac{28}{100}$. **17.** $\frac{48}{72}$; $\frac{54}{72}$; $\frac{60}{72}$; $\frac{68}{72}$; $\frac{40}{72}$; $\frac{66}{72}$; $\frac{57}{72}$; $\frac{50}{72}$; $\frac{49}{72}$; $\frac{47}{72}$; $\frac{48}{72}$. **18.** $\frac{32}{48}$, $\frac{27}{48}$; $\frac{64}{96}$, $\frac{54}{96}$; $\frac{96}{144}$, $\frac{81}{144}$; $\frac{160}{240}$, $\frac{135}{240}$; $\frac{256}{384}$, $\frac{216}{384}$; $\frac{352}{528}$, $\frac{297}{528}$.

Page 68. — **2.** $\frac{47}{2}$. **3.** $\frac{57}{4}$. **4.** $\frac{112}{3}$. **5.** $\frac{97}{6}$. **6.** $\frac{225}{8}$. **7.** $\frac{222}{5}$. **8.** $\frac{308}{4}$. **9.** $\frac{191}{3}$. **10.** $\frac{263}{5}$. **11.** $\frac{491}{6}$. **12.** $\frac{751}{8}$. **13.** $\frac{783}{10}$. **14.** $\frac{581}{12}$. **15.** $\frac{982}{15}$. **16.** $\frac{1161}{16}$. **17.** $\frac{2081}{24}$. **18.** $\frac{571}{4}$. **19.** $\frac{1879}{5}$. **20.** $\frac{3871}{6}$. **21.** $\frac{6615}{8}$.

Page 70. — **2.** $25\frac{2}{3}$. **3.** $19\frac{3}{5}$. **4.** 21. **5.** $20\frac{5}{6}$. **6.** $36\frac{3}{4}$. **7.** 49. **8.** $18\frac{5}{6}$. **9.** $28\frac{7}{8}$. **10.** $20\frac{5}{6}$. **11.** 19. **12.** $12\frac{1}{3}$. **13.** $9\frac{1}{8}$. **14.** 16. **15.** $19\frac{3}{4}$. **16.** $23\frac{7}{16}$.

Page 71. — **2.** $\frac{15}{20}$, $\frac{4}{20}$. **3.** $\frac{4}{24}$, $\frac{9}{24}$. **4.** $\frac{10}{18}$, $\frac{15}{18}$. **5.** $\frac{49}{56}$, $\frac{64}{56}$. **6.** $\frac{10}{16}$, $\frac{4}{16}$, $\frac{7}{16}$. **7.** $\frac{8}{24}$, $\frac{21}{24}$, $\frac{6}{24}$. **8.** $\frac{24}{30}$, $\frac{20}{30}$, $\frac{27}{30}$. **9.** $\frac{30}{40}$, $\frac{25}{40}$, $\frac{32}{40}$. **10.** $\frac{15}{30}$, $\frac{18}{30}$, $\frac{5}{30}$. **11.** $\frac{27}{36}$, $\frac{8}{36}$, $\frac{15}{36}$. **12.** $\frac{16}{40}$, $\frac{5}{40}$, $\frac{22}{40}$. **13.** $\frac{28}{32}$, $\frac{7}{32}$, $\frac{18}{32}$.

Page 72. — **2.** $\frac{45}{60}$, $\frac{35}{60}$, $\frac{27}{60}$. **3.** $\frac{6}{15}$, $\frac{9}{15}$, $\frac{8}{15}$. **4.** $\frac{49}{56}$, $\frac{36}{56}$, $\frac{48}{56}$. **5.** $\frac{8}{30}$, $\frac{23}{30}$, $\frac{24}{30}$. **6.** $\frac{42}{72}$, $\frac{33}{72}$, $\frac{52}{72}$. **7.** $\frac{20}{144}$, $\frac{135}{144}$, $\frac{114}{144}$. **8.** $\frac{12}{32}$, $\frac{10}{32}$, $\frac{24}{32}$, $\frac{7}{32}$. **9.** $\frac{15}{60}$, $\frac{40}{60}$, $\frac{48}{60}$, $\frac{30}{60}$, $\frac{50}{60}$. **10.** $\frac{15}{36}$, $\frac{6}{36}$, $\frac{12}{36}$, $\frac{5}{36}$, $\frac{27}{36}$.

Page 73. — **2.** $1\frac{1}{2}$. **3.** $1\frac{2}{3}$. **4.** $2\frac{1}{5}$. **5.** $2\frac{7}{16}$. **6.** $2\frac{1}{4}$. **7.** $2\frac{1}{16}$. **8.** $2\frac{7}{12}$. **9.** $2\frac{5}{48}$. **10.** $2\frac{4}{5}$. **11.** $1\frac{7}{8}$. **12.** $2\frac{5}{8}$. **13.** $3\frac{1}{8}$.

Page 74. — **15.** 80. **16.** $138\frac{13}{40}$. **17.** $160\frac{11}{12}$. **18.** $212\frac{11}{12}$. **19.** $227\frac{1}{4}$. **20.** $242\frac{3}{4}$. **21.** $146\frac{3}{5}$. **22.** $240\frac{1}{5}$. **23.** $181\frac{47}{120}$. **24.** $264\frac{1}{10}$. **26.** $\frac{7}{16}$. **27.** $\frac{11}{20}$. **28.** $\frac{5}{12}$. **29.** $\frac{11}{20}$. **30.** $\frac{5}{24}$. **31.** $\frac{7}{30}$. **32.** $\frac{13}{30}$. **33.** $\frac{19}{24}$. **34.** $\frac{1}{5}$. **35.** $\frac{25}{48}$. **36.** $\frac{7}{12}$. **37.** $\frac{7}{20}$.

Page 75. — **39.** $15\frac{3}{4}$. **40.** $63\frac{2}{3}$. **41.** $28\frac{5}{8}$. **42.** $34\frac{5}{9}$. **43.** $15\frac{1}{2}$. **44.** $33\frac{5}{12}$. **45.** $54\frac{3}{4}$. **46.** $37\frac{5}{6}$. **47.** $27\frac{5}{6}$. **48.** $41\frac{1}{10}$. **49.** $22\frac{11}{12}$. **50.** $17\frac{5}{12}$. **51.** $29\frac{17}{24}$. **52.** $18\frac{13}{20}$. **53.** $27\frac{23}{24}$. **54.** $17\frac{19}{28}$. **55.** $90\frac{5}{6}$. **56.** $188\frac{1}{4}$. **57.** $266\frac{9}{10}$. **58.** $178\frac{14}{15}$.

1. $\frac{2}{5}$ sec. **2.** $1\frac{13}{20}$ tons. **3.** $\frac{7}{24}$. **4.** $211\frac{1}{5}$. **5.** $36\frac{1}{5}$ tons. **6.** $79\frac{3}{5}$. **7.** $36\frac{7}{8}$ bu.; $66\frac{1}{4}$ bu.

Page 76. — **8.** $235\frac{3}{4}$ bu. **9.** $3\frac{37}{40}$¢; $4\frac{4}{5}$¢; $5\frac{19}{80}$¢; $7\frac{11}{20}$¢; $8\frac{3}{10}$¢.

Page 77. — **2.** 27. **3.** $28\frac{1}{2}$. **4.** 64. **5.** $64\frac{2}{3}$. **6.** 45. **7.** $71\frac{1}{5}$. **8.** $77\frac{1}{2}$. **9.** $60\frac{3}{4}$. **10.** $17\frac{1}{4}$. **11.** 36. **12.** $48\frac{1}{3}$. **13.** $23\frac{3}{5}$. **14.** 18. **15.** $85\frac{3}{4}$. **16.** $25\frac{1}{2}$. **17.** $38\frac{1}{3}$. **18.** 40. **19.** $41\frac{2}{3}$. **20.** $31\frac{1}{2}$. **21.** $54\frac{2}{3}$. **22.** 57. **23.** $97\frac{1}{3}$. **24.** $315\frac{3}{4}$. **25.** $374\frac{4}{5}$. **26.** $463\frac{3}{4}$. **27.** 745. **28.** $206\frac{1}{2}$. **29.** $211\frac{1}{4}$. **30.** $326\frac{1}{4}$. **31.** $561\frac{3}{5}$. **32.** $657\frac{1}{3}$. **34.** $656\frac{3}{4}$. **35.** $1180\frac{2}{3}$. **36.** $2300\frac{2}{3}$. **37.** $3015\frac{1}{6}$. **38.** $5350\frac{4}{5}$. **39.** $19{,}069\frac{1}{8}$. **40.** 11,021. **41.** 4891. **42.** $31{,}116\frac{1}{4}$. **43.** $26{,}374\frac{2}{3}$. **44.** $26{,}502\frac{1}{4}$. **45.** $46{,}147\frac{1}{3}$. **46.** $19{,}204\frac{1}{2}$. **47.** $63{,}044\frac{3}{4}$. **48.** $43{,}480\frac{2}{3}$. **49.** $106{,}889\frac{3}{4}$. **50.** $257{,}354\frac{2}{5}$. **51.** $170{,}056\frac{5}{6}$. **52.** $477{,}042\frac{1}{2}$. **53.** $211{,}881\frac{3}{5}$. **54.** $657{,}870\frac{1}{4}$.

Page 79. — **1.** $\$18\frac{3}{4}$. **2.** $\$347\frac{1}{2}$. **3.** $\$40$. **4.** $\$9180\frac{1}{2}$. **5.** $\$325\frac{1}{2}$. **6.** $\$841$. **7.** $\$222.95$. **8.** $\$3101.54$. **9.** $45\frac{1}{2}$ bu. **10.** $\$8.91$. **11.** $172\frac{4}{5}$ tons. **12.** $\$2120$. **13.** $131\frac{1}{4}$ ft. **14.** 216 pages. **15.** $\$2030\frac{2}{5}$. **16.** $1732\frac{1}{2}$ lb. **17.** $\$1792$.

Page 82. — **2.** $2\frac{1}{5}$. **3.** $5\frac{5}{8}$. **4.** $5\frac{7}{9}$. **5.** $4\frac{1}{4}$. **6.** $2\frac{5}{8}$. **7.** $5\frac{7}{10}$. **8.** $5\frac{1}{2}$. **9.** $7\frac{2}{9}$. **10.** $76\frac{1}{2}$. **11.** $18\frac{2}{3}$. **12.** $19\frac{1}{5}$. **13.** $18\frac{3}{4}$. **14.** $2\frac{5}{8}$. **15.** $14\frac{1}{4}$. **16.** 48. **17.** $34\frac{4}{5}$. **19.** $961\frac{1}{3}$. **20.** $5392\frac{1}{5}$. **21.** $1486\frac{1}{2}$. **22.** $4219\frac{1}{2}$. **23.** $8275\frac{1}{2}$. **24.** $14{,}420\frac{1}{4}$. **25.** $26{,}272\frac{1}{6}$. **26.** $81{,}196\frac{4}{5}$. **27.** $77{,}949\frac{1}{2}$. **28.** $94{,}248\frac{3}{4}$. **29.** $341{,}279\frac{2}{5}$. **30.** $334{,}467\frac{3}{4}$.

1. $\$1.19$. **2.** $\$1.34$. **3.** $\$7.50$. **4.** $\$56\frac{1}{4}$. **5.** $\$8.94$. **6.** $\$36.07$. **7.** $1562\frac{1}{2}$ lb. **8.** $3203\frac{1}{5}$ tons.

Page 83. — **9.** $960\frac{3}{4}$ mi. **10.** $\$337\frac{1}{2}$. **11.** $2314\frac{1}{2}$ bu. **12.** 1527. **13.** $\$1396\frac{1}{2}$. **14.** $\$15{,}941\frac{1}{4}$.

Page 84. — **1.** $\frac{3}{4}$. **2.** $\frac{1}{4}$. **3.** $\frac{3}{4}$. **4.** $\frac{3}{4}$. **5.** $\frac{7}{12}$. **6.** $\frac{1}{3}$. **7.** $\frac{2}{5}$. **8.** $\frac{2}{3}$. **9.** $\frac{7}{16}$. **10.** $\frac{2}{3}$. **11.** $\frac{9}{25}$.

Page 85. — **12.** $\frac{1}{3}$. **13.** $\$920$. **14.** 516 mi. **15.** $\$2016$. **16.** $\$1020$. **17.** $\$376.80$.

Page 86. — **2.** $\frac{11}{48}$. **3.** $\frac{4}{21}$. **4.** $\frac{3}{35}$. **5.** $\frac{2}{21}$. **6.** $\frac{1}{32}$. **7.** $\frac{1}{40}$. **8.** $\frac{1}{48}$. **9.** $\frac{1}{48}$. **10.** $\frac{1}{32}$. **11.** $\frac{1}{90}$. **12.** $\frac{1}{40}$. **13.** $\frac{1}{72}$.

Page 87. — **15.** $4\frac{2}{3}$. **16.** $1\frac{5}{6}$. **17.** $3\frac{9}{16}$. **18.** $10\frac{3}{4}$. **19.** $80\frac{1}{2}$. **20.** $59\frac{3}{10}$. **21.** $147\frac{5}{7}$. **22.** $90\frac{5}{8}$. **23.** $673\frac{7}{12}$. **24.** $579\frac{3}{8}$. **25.** $587\frac{31}{72}$. **26.** $1260\frac{6}{7}$. **28.** $15\frac{5}{12}$. **29.** $24\frac{9}{10}$. **30.** $43\frac{7}{8}$. **31.** $14\frac{3}{5}$. **32.** $17\frac{17}{18}$. **33.** $17\frac{21}{40}$. **34.** $2\frac{11}{60}$. **35.** $12\frac{4}{7}$. **36.** $46\frac{25}{32}$. **37.** $89\frac{7}{15}$. **38.** $156\frac{19}{24}$. **39.** $157\frac{17}{18}$.

Page 88. — 1. $\frac{7}{60}$. 2. $\frac{3}{25}$. 3. $\frac{3}{8}$. 4. $\frac{2}{45}$. 5. $\frac{7}{10}$. 6. $62\frac{1}{18}$. 7. $72\frac{25}{36}$. 8. $19\frac{7}{8}$. 9. $21\frac{7}{30}$. 10. $21\frac{3}{7}$. 11. $699\frac{1}{6}$. 12. $610\frac{1}{4}$. 13. $936\frac{3}{5}$. 14. $126\frac{3}{8}$. 15. $15\frac{1}{12}$. 16. \$$5\frac{1}{4}$. 17. \$$2\frac{3}{10}$. 18. \$$3\frac{3}{20}$. 19. \$$3\frac{13}{20}$. 20. \$$8\frac{9}{10}$. 21. \$$34\frac{3}{4}$. 22. \$$26\frac{1}{2}$. 23. \$$19\frac{1}{5}$. 24. \$$24\frac{1}{4}$. 25. \$$19\frac{3}{10}$. 26. $\frac{4}{5}$ mi. 27. $\frac{1}{4}$ mi. 28. \$$37\frac{3}{4}$. 29. \$$3\frac{1}{2}$. 30. $7\frac{1}{4}$ qt.

Page 90. — 2. $22\frac{1}{2}$. 3. 32. 4. 70. 5. 42. 6. $52\frac{1}{2}$. 7. $103\frac{1}{3}$. 8. $65\frac{1}{7}$. 9. $76\frac{1}{2}$. 10. $86\frac{2}{3}$. 11. $194\frac{2}{5}$. 12. $101\frac{1}{8}$. 13. $87\frac{1}{2}$. 14. $262\frac{1}{2}$. 15. $957\frac{3}{5}$. 16. $116\frac{1}{4}$. 17. $97\frac{1}{3}$.

Page 91. — 18. $6\frac{2}{5}$. 19. $4\frac{2}{3}$. 20. $17\frac{1}{3}$. 21. $6\frac{3}{5}$. 22. $13\frac{1}{3}$. 23. $15\frac{3}{5}$. 24. 48. 25. $18\frac{2}{3}$. 26. $67\frac{5}{7}$. 27. $66\frac{2}{3}$. 29. $12\frac{15}{16}$. 30. $13\frac{3}{4}$. 31. $15\frac{9}{11}$. 32. 16. 33. $41\frac{3}{7}$. 34. $30\frac{6}{13}$.

1. 68. 2. $46\frac{1}{2}$. 3. $153\frac{3}{5}$. 4. $11\frac{1}{7}$. 5. $191\frac{1}{3}$. 6. $373\frac{5}{7}$. 7. $492\frac{4}{5}$. 8. $185\frac{1}{3}$. 9. $37\frac{1}{5}$. 10. $26\frac{10}{11}$. 11. $374\frac{3}{8}$. 12. $113\frac{1}{4}$. 13. 48 rd. 14. 396 days. 15. 250 lb.

Page 92. — 16. 32 turns. 17. 8 waists. 18. 784 loaves. 19. $25\frac{1}{3}$ da.

Page 93. — 2. 102. 3. 135. 4. 114. 5. 320. 6. 660. 7. 1328. 8. 3592. 9. 5855. 10. 9681. 11. 285 mi. 12. \$1020. 13. 1060 lb. 14. 150 mi. 15. 96,000. 16. 156,000 lb.

Page 94. — 1. 162 cu. in. 2. First weighs 7 lb. more than second; first weighs $1\frac{1}{8}$ times second. 3. \$10.25. 4. 25¢. 5. \$9.50. 6. \$9.51. 7. \$84.01. 8. \$185.31.

Page 95. — 9. 8 lb. dates; 8 qt. peanuts; \$1.97. 10. 83¢. 11. 13¢ each. 12. \$4.00. 13. \$69.05.

Page 96. — 2. $\frac{1}{2}$. 3. $\frac{2}{5}$. 4. $1\frac{3}{5}$. 5. $7\frac{4}{5}$. 6. $8\frac{2}{5}$. 7. $\frac{3}{20}$. 8. $\frac{1}{4}$. 9. $\frac{3}{25}$. 10. $\frac{13}{20}$. 11. $\frac{19}{20}$. 12. $7\frac{1}{20}$. 13. $3\frac{3}{4}$. 14. $\frac{1}{8}$. 15. $\frac{3}{8}$. 16. $\frac{7}{8}$. 17. $7\frac{5}{8}$. 18. $4\frac{7}{8}$. 19. $9\frac{81}{250}$. 20. $6\frac{3}{40}$. 21. $5\frac{3}{250}$.

Page 97. — 2. .4. 3. .6. 4. .8. 5. .25. 6. .5. 7. .375. 8. .625. 9. .875. 10. $.333\frac{1}{3}$. 11. $.166\frac{2}{3}$. 12. $.833\frac{1}{3}$. 13. $.285\frac{5}{7}$. 14. $.444\frac{4}{9}$. 15. $.777\frac{7}{9}$. 16. $.272\frac{8}{11}$. 17. $.416\frac{2}{3}$. 18. .85. 19. .026. 20. .385. 21. $.008\frac{4}{7}$.

Page 98. — 1. 124.882. 2. 266.958. 3. 791.498. 4. 798.000. 5. 510.839. 6. 336.227. 7. 1034.580. 8. 469.540. 9. \$3306.694. 10. \$2285.825. 11. \$1883.370. 12. \$12,051.910. 13. 2024.112. 14. \$2356.48. 15. \$1315.58. 16. \$30,050.62.

Page 99. — 17. 290.906. 18. 172.598. 19. 199.557. 20. 274.067. 21. 937.128. 23. 1332.471. 24. 9943.092. 25. 152.594. 26. 361.798. 27. 304.32. 28. 225.36. 29. 103.52. 30. 652.25. 31. 349.372. 32. 4847.01. 33. 354.14. 34. 429.125. 35. 552.017. 36. 471.735. 37. 960.75. 38. 943.375. 39. 892.888. 40. 988.911.

Page 100. — 1. 67.772 cu. in. 2. Milk, .267 lb. 3. 246.50 ft. 4. 344.56 ft. 5. $359.16\frac{2}{3}$ ft. 6. .70¢; .43¢; .36¢; 1.02¢; .94¢; .34¢; .01¢. 7. 76.98 in. 8. 8924.28 mi.

Page 101. — 5. \$3071.90. 6. 3421.2. 7. \$4755. 8. 333.33. 9. 3371.06. 10. 348.255. 11. 325. 12. 557.858. 14. $316.84\frac{4}{5}$. 15. $32.906\frac{1}{4}$. 16. 92.631. 17. 601.56. 18. 48.22. 19. $33.638\frac{2}{7}$. 20. $3020.61\frac{1}{6}$. 21. $1120.940\frac{5}{8}$. 22. $1118.284\frac{4}{5}$. 23. 3468.01. 24. 1003.959. 25. $823.669\frac{3}{5}$.

Page 103. — **2.** 68.4; 102.6; 136.8; 171; 205.2. **3.** 11.34; 17.01; 22.68; 28.35; 34.02. **4.** 89.6; 134.4; 179.2; 224; 268.8. **5.** .14; .21; .28; .35; .42. **6.** \$47.30; \$70.95; \$94.60; \$118.25; \$141.90. **7.** \$496.40; \$744.60; \$992.80; \$1241.00; \$1489.20. **8.** 1742; 2613; 3484; 4355; 5226. **9.** 972; 1458; 1944; 2430; 2916. **10.** 267.4; 305.6; 343.8; 458.4. **11.** 557.2; 636.8; 716.4; 955.2. **12.** \$3003; \$3432; \$3861; \$5148. **13.** \$11,487; \$13,128; \$14,769; \$19,692. **14.** 17,535; 20,040; 22,545; 30,060. **15.** 85,960; 98,240; 110,520; 147,360. **16.** 55,720; 63,680; 71,640; 95,520. **17.** 28,630; 32,720; 36,810; 49,080. **18.** 13,524; 157,780; 24,794,000. **19.** 474; 5530; 869,000. **20.** \$25,080; \$292,600; \$45,980,000. **21.** 59,400; 693,000; 108,900,000.

Page 104. — **23.** \$3259.18. **24.** \$6207.30. **25.** 2781.6. **26.** 2904.486. **27.** 7731.122. **28.** 4959.375. **29.** 23,721.32. **30.** 53,065.8. **31.** 76,056.76. **32.** 566,274.16. **33.** 7400 lb. **34.** 15,000 cu. ft. **35.** 6000 lb.

4. 5.705$\frac{2}{3}$. **5.** 7.089. **6.** 13.600$\frac{3}{7}$. **7.** 9.380$\frac{5}{9}$. **8.** 7.258. **9.** 6.861$\frac{1}{9}$. **10.** .725. **11.** .690$\frac{10}{11}$. **12.** 38.388$\frac{8}{9}$. **13.** 6.624$\frac{4}{7}$. **14.** .087. **15.** .278.

Page 105. — **16.** .564; .775; \$1.83. **17.** \$1.44; 6.008; .933$\frac{1}{3}$.

Page 106. — **2.** .2; .1; .066$\frac{2}{3}$; .05; .033$\frac{1}{3}$. **3.** 3.55; 1.775; 1.183$\frac{1}{3}$; .887$\frac{1}{2}$; .591$\frac{2}{3}$. **4.** .025; .012$\frac{1}{2}$; .008$\frac{1}{3}$; .006$\frac{1}{4}$; .004$\frac{1}{6}$. **5.** .21; .105; .07; .052$\frac{1}{2}$; .035. **6.** 1.75; .875; .583$\frac{1}{3}$; .437$\frac{1}{2}$; .291$\frac{2}{3}$. **7.** .28; .14; .093$\frac{1}{3}$; .07; .046$\frac{2}{3}$. **8.** .048; .024; .016; .012; .008. **9.** .181$\frac{1}{2}$; .090$\frac{3}{4}$; .060$\frac{1}{2}$; .045$\frac{3}{8}$; .030$\frac{1}{4}$. **10.** .035$\frac{5}{7}$; .027$\frac{7}{9}$; .022$\frac{8}{11}$. **11.** .228$\frac{4}{7}$; .177$\frac{7}{9}$; .145$\frac{5}{11}$. **12.** 3.947$\frac{1}{7}$; 3.07; 2.511$\frac{9}{11}$. **13.** 6.914$\frac{2}{7}$; 5.377$\frac{7}{9}$; 4.4. **14.** .053$\frac{4}{7}$; .041$\frac{2}{3}$; .034$\frac{1}{11}$. **15.** .073$\frac{1}{7}$; .056$\frac{8}{9}$; .046$\frac{6}{11}$. **16.** .000$\frac{6}{7}$; .000$\frac{2}{3}$; .000$\frac{6}{11}$. **17.** .020$\frac{4}{7}$; .016; .013$\frac{1}{11}$. **18.** .005$\frac{1}{4}$; .003$\frac{9}{11}$; .003$\frac{1}{2}$. **19.** .016$\frac{1}{2}$; .012; .011. **20.** .21; .152$\frac{8}{11}$; .14. **21.** .132; .096; .088.

Page 107. — **23.** 3.44. **24.** 8.277. **25.** \$.64. **26.** \$6.85. **27.** \$4.888. **28.** 3.459. **29.** .432. **30.** .905. **31.** 60.5. **32.** 5.468. **33.** .312$\frac{1}{2}$. **34.** .458$\frac{1}{3}$. **35.** .093$\frac{3}{4}$. **36.** 3.187$\frac{1}{2}$. **37.** 1.562$\frac{1}{2}$. **38.** 3.75 lb. **39.** 7.4 tons; \$15.50; \$114.70. **40.** 17.5¢. **41.** 20.888$\frac{8}{9}$ lb. **42.** 19.68¢.

Page 110. — **1.** \$6. **2.** \$6.80. **3.** \$4.50. **4.** \$8. **5.** \$17.50. **6.** \$27. **7.** \$55. **8.** \$36. **9.** \$22. **10.** \$4.50. **11.** \$8.25. **12.** \$26.25. **13.** \$12.50. **14.** \$36. **15.** \$52.50. **16.** \$62.50.

Page 111. — **21.** \$84. **22.** \$45. **23.** \$112.50. **24.** \$18.75. **25.** \$42. **26.** \$74.25. **27.** \$157.50. **28.** \$65.25. **29.** \$52.50. **30.** \$56. **31.** \$24.50. **32.** \$48. **33.** \$9; \$27; \$36. **34.** \$144; \$146.88; \$141.12. **35.** \$54; \$56.16; \$51.84; \$55.08; \$52.92. **36.** \$80; \$79.20; \$80.40. **37.** \$81.25.

Page 112. — **2.** 52. **3.** 34. **4.** 8. **5.** 72. **6.** 147. **7.** 390. **8.** 240. **9.** 120.

Page 114. — **1.** \$4.53. **2.** \$10.65.

Page 115. — **3.** \$7.02. **4.** \$75. **5.** \$12.50. **6.** \$14.95. **7.** \$88.63. **8.** \$8.74. **9.** \$7.37. **10.** \$127.50. **11.** \$74.90. **12.** \$127.03.

Page 116. — **1.** 1 hr. 39 min. **2.** \$7.96. **3.** \$.36. **4.** \$3.05. **5.** \$9.42. **6.** \$1.86. **7.** \$2.79. **8.** \$17.12.

Page 117. — **9.** \$113.25. **10.** \$208. **11.** \$121.65. **12.** \$75.45. **13.** \$45.40. **14.** \$43.35. **15.** \$26.95. **16.** \$800.

Page 121. — **1.** \$1.20. **2.** \$.90. **3.** \$4. **4.** \$1.25. **5.** \$35. **6.** \$2.40. **7.** \$4.20. **8.** \$1.20. **9.** \$22.50. **10.** \$2.10. **11.** \$1. **12.** \$.90. **13.** \$2.50. **14.** \$.30. **15.** \$.50. **16.** \$.80. **17.** \$1. **18.** \$1.40.

Page 128. — **3.** $279\frac{1}{2}$ sq. ft. **4.** 246 sq. ft. **5.** 422 sq. ft. **6.** 8.5 in. **7.** 14 yd. **8.** 1611 sq. ft. **9.** 784 sq. yd. **10.** 2250 sq. ft. **11.** 80 rd. **12.** $21,235\frac{1}{2}$ sq. ft. **13.** 4 ft. **14.** Rectangle, 9 in. wide. **15.** 80 rd. **16.** 330 ft. **17.** $2\frac{3}{4}$ mi. **18.** 33 mi. **19.** 200 tablets.

Page 129. — **1.** 314.4 sq. in.; 288 sq. in.; 126 sq. in. **2.** 22 in. **3.** 77 sq. ft. **4.** 56 sq. ft. **5.** 15,012 sq. ft. **6.** 49 rd. **7.** 5808 sq. ft. **8.** 95 rd.

Page 130. — **1.** 225 sq. in. **2.** 336 sq. ft. **3.** 801.6 sq. ft. **4.** $3187\frac{1}{2}$ sq. ft. **5.** 21 acres. **6.** 27 acres. **7.** 5366.4 sq. mi. **8.** $157\frac{1}{2}$ sq. ft. **9.** $5\frac{1}{3}$ sq. yd.; $22\frac{2}{3}$ sq. yd.; $55\frac{1}{3}$ sq. yd. **10.** 240 sq. ft.

Page 131. — **1.** 154 cu. ft. **2.** 3360 cu. in. **3.** 3087 cu. ft. **4.** 150 cu. yd. **5.** 546 cu. ft. **6.** $1687\frac{1}{2}$ cu. ft. **7.** 20 cu. ft. **8.** $6\frac{17}{108}$ cu. ft.

Page 133. — **13.** 2 min. 4 sec. **14.** Chemical, 500 ft.; steamer, 1250 ft. **15.** 197,000 gal. **16.** $19.70. **17.** $10.80. **18.** $29.20. **19.** 4.8 tons.

Page 136. — **1.** $\frac{11}{16}$. **2.** $\frac{3}{8}$. **3.** $\frac{7}{11}$. **4.** $\frac{3}{10}$. **5.** $\frac{1}{8}$. **6.** $\frac{3}{7}$. **7.** $\frac{3}{8}$. **8.** $\frac{4}{9}$. **9.** $\frac{5}{12}$. **10.** $\frac{5}{8}$. **11.** $\frac{89}{12}$; $\frac{88}{10}$; $\frac{59}{4}$; $\frac{59}{8}$; $\frac{182}{5}$; $\frac{268}{8}$; $\frac{245}{6}$; $\frac{258}{5}$. **12.** 38; $21\frac{1}{3}$; $23\frac{3}{4}$; 23; $19\frac{1}{2}$; $24\frac{3}{5}$; $28\frac{3}{4}$; 29; $31\frac{2}{3}$. **13.** $1\frac{17}{24}$. **14.** $2\frac{11}{24}$. **15.** $\frac{11}{24}$. **16.** $\frac{11}{16}$. **17.** $4\frac{1}{24}$. **18.** $2\frac{1}{2}$. **19.** $71\frac{3}{8}$; $16\frac{1}{8}$. **20.** $118\frac{2}{15}$; $33\frac{7}{15}$. **21.** $133\frac{1}{24}$; $55\frac{7}{24}$. **22.** $172\frac{7}{10}$; $79\frac{9}{10}$. **23.** $437\frac{8}{15}$; $145\frac{13}{15}$.

Page 137. — **24.** $84\frac{23}{24}$. **25.** $154\frac{3}{4}$. **26.** $159\frac{3}{28}$. **27.** $1104\frac{17}{40}$. **28.** $1510\frac{7}{24}$. **29.** 80. **30.** $73\frac{1}{2}$. **31.** $\frac{1}{36}$. **32.** 60. **33.** $46\frac{1}{2}$. **34.** $44\frac{1}{3}$. **35.** $\frac{3}{32}$. **36.** $83\frac{5}{8}$. **37.** 3032. **38.** $13,531\frac{1}{5}$. **39.** $18\frac{11}{16}$. **40.** $23,381\frac{1}{3}$. **41.** $12\frac{13}{24}$. **42.** $1790\frac{5}{8}$. **43.** $48,592\frac{1}{2}$. **44.** \$$168\frac{3}{4}$. **45.** \$$268\frac{1}{5}$. **46.** \$$2425\frac{4}{5}$. **47.** \$$214\frac{1}{2}$. **48.** \$$382\frac{1}{2}$. **49.** \$$2392\frac{1}{2}$. **50.** $4950. **51.** 490 bbl. **52.** 15 mi. **53.** $57\frac{5}{11}$ bu. **54.** 380 mi.

Page 140. — **2.** $\frac{1}{2}$. **3.** $\frac{5}{9}$. **4.** $\frac{15}{16}$. **5.** $\frac{4}{5}$. **6.** $\frac{3}{4}$. **7.** $\frac{4}{5}$. **8.** $\frac{3}{4}$. **9.** $\frac{3}{8}$. **10.** $\frac{3}{5}$. **11.** $\frac{3}{10}$. **12.** $\frac{6}{7}$. **13.** $\frac{1}{10}$. **14.** $2\frac{1}{2}$. **15.** $6\frac{3}{4}$. **16.** $10\frac{1}{5}$. **17.** $9\frac{1}{2}$. **18.** $4\frac{1}{2}$. **19.** $44\frac{1}{5}$. **20.** $29\frac{9}{16}$. **22.** $16\frac{1}{2}$. **23.** 210. **24.** $6\frac{1}{5}$. **25.** $11\frac{1}{4}$. **26.** $18\frac{7}{9}$. **27.** 9. **28.** $13\frac{3}{4}$ ft.; $45\frac{3}{8}$ ft. **29.** \$$1\frac{2}{5}$. **30.** \$$37\frac{1}{3}$; \$$78\frac{3}{10}$. **31.** $28\frac{1}{2}$ bu. **32.** $86\frac{1}{3}$ sq. ft.

Page 141. — **33.** $1.17. **34.** $687\frac{1}{2}$ cu. ft. **35.** 35 lb.

Page 142. — **2.** $2\frac{2}{3}$. **3.** $1\frac{5}{16}$. **4.** $\frac{11}{12}$. **5.** $5\frac{2}{3}$. **6.** $1\frac{1}{15}$. **7.** $1\frac{1}{3}$. **8.** $3\frac{8}{9}$. **9.** $4\frac{1}{5}$. **10.** $7\frac{7}{9}$. **11.** $2\frac{1}{25}$. **12.** $3\frac{5}{6}$. **13.** $2\frac{1}{2}$.

Page 143. — **14.** $7\frac{3}{5}$. **15.** $7\frac{1}{3}$. **16.** $9\frac{1}{5}$. **17.** $11\frac{1}{5}$. **18.** $9\frac{9}{16}$. **19.** $8\frac{3}{4}$. **20.** $2\frac{2}{3}$. **21.** $2\frac{2}{9}$. **22.** $5\frac{4}{9}$. **23.** $2\frac{3}{10}$. **24.** $3\frac{1}{5}$. **25.** $8\frac{2}{3}$. **26.** $7\frac{1}{7}$. **28.** $\frac{5}{8}$. **29.** $1\frac{7}{8}$. **30.** $1\frac{1}{6}$. **31.** $\frac{7}{32}$. **32.** $\frac{15}{256}$. **33.** $\frac{2}{3}$. **34.** $1\frac{1}{2}$. **35.** $8\frac{1}{3}$. **36.** $\frac{25}{96}$. **37.** 10. **38.** $\frac{27}{50}$. **39.** 28. **40.** $3\frac{3}{4}$ lb. **41.** 22 times.

Page 144. — **42.** 6 times. **43.** 382 bu. **44.** $\frac{1}{12}$ lb. **45.** 8 tents. **46.** 28 loads. **47.** 6 bu. **48.** $12\frac{3}{4}$ bu.

Page 145. — **2.** $\frac{5}{6}$. **3.** $2\frac{1}{7}$. **4.** $1\frac{1}{2}$. **5.** $\frac{8}{9}$. **6.** 10. **7.** $\frac{1}{15}$. **8.** $1\frac{4}{5}$. **9.** $1\frac{3}{8}$. **10.** $1\frac{1}{6}$. **11.** $\frac{3}{4}$. **12.** $\frac{9}{16}$. **13.** $\frac{14}{15}$. **14.** $\frac{1}{4}$. **15.** $\frac{1}{4}$. **16.** $\frac{1}{6}$. **17.** $\frac{15}{16}$. **18.** $\frac{2}{3}$. **19.** $\frac{4}{5}$. **20.** $7\frac{1}{4}$. **21.** 1.

2. $\frac{5}{9}$; $\frac{5}{8}$; $\frac{1}{32}$.

Page 146. — **3.** $\frac{1}{3}$. **4.** $\frac{5}{16}$. **5.** $\frac{7}{8}$. **6.** $\frac{1}{3}$. **7.** $\frac{3}{5}$. **8.** $\frac{9}{20}$. **9.** $\frac{9}{10}$. **10.** $\frac{2}{3}$. **11.** $\frac{1}{10}$. **12.** $\frac{1}{12}$. **13.** $\frac{4}{5}$. **14.** $\frac{2}{3}$. **15.** $\frac{8}{15}$. **16.** $\frac{3}{10}$. **17.** $\frac{9}{16}$. **18.** $\frac{2}{3}$ acre. **19.** $\frac{1}{4}$. **20.** $\frac{2}{5}$. **21.** $\frac{1}{15}$. **22.** $\frac{1}{3}$.

Page 147. — 1. $\frac{9}{10}$ is $1\frac{1}{5}$ times $\frac{3}{4}$. 2. $\frac{5}{12}$ is $\frac{5}{8}$ of $\frac{2}{3}$. 3. $\frac{15}{16}$ is $1\frac{1}{2}$ times $\frac{5}{8}$. 4. $\frac{11}{16}$ is $\frac{11}{12}$ of $\frac{3}{4}$. 5. 24 is $3\frac{3}{5}$ times $6\frac{2}{3}$. 6. 88 is $15\frac{5}{7}$ times $5\frac{3}{5}$. 7. $8\frac{1}{4}$ is $\frac{1}{8}$ of 66. 8. $6\frac{1}{6}$ is $\frac{1}{12}$ of 74. 9. $87\frac{1}{2}$ is $10\frac{1}{2}$ times $8\frac{1}{3}$. 10. $15\frac{3}{4}$ is $\frac{1}{6}$ of $94\frac{1}{2}$. 11. $46\frac{2}{3}$ is $7\frac{3}{11}$ times $6\frac{5}{12}$. 12. $91\frac{1}{3}$ is 4 times $22\frac{5}{6}$. 13. 168 loaves. 14. 6 barrels.

Page 148. — 15. 250 qt. 16. $51\frac{3}{4}$ acres. 17. $2\frac{2}{3}$ mi. 18. 8 cans. 19. 24 bu. 20. 135 trees.

1. $\$1\frac{3}{4}$. 2. $102\frac{1}{2}$ T.; $\$92\frac{1}{4}$. 3. $52\frac{1}{2}$ mi.; 56 min. 4. 260,268 T. 5. $6\frac{2}{3}$ ¢.

Page 149. — 6. $\$3\frac{1}{2}$. 7. $\$\frac{3}{4}$. 8. $\$\frac{1}{2}$. 9. $\$\frac{2}{5}$. 10. 224 lb. 11. $\frac{2}{3}$. 12. $\frac{3}{16}$; 120 acres. 13. 24,255 ties; $19,404. 14. S.C., $2\frac{1}{3}$ bu.; Japan, $1\frac{1}{4}$ bu.; $1\frac{1}{12}$ bu. more in S.C. 15. 3249 bu. 16. $4\frac{1}{2}$ sq. yd.; $6\frac{1}{4}$ sq. yd.

Page 152. — 2. $\frac{5}{8}$. 3. $\frac{3}{16}$. 4. $\frac{5}{16}$. 5. $\frac{7}{16}$. 6. $\frac{9}{16}$. 7. $\frac{11}{16}$. 8. $\frac{13}{16}$. 9. $\frac{1}{32}$. 10. $\frac{3}{32}$. 11. $\frac{5}{32}$. 12. $\frac{7}{32}$. 13. $\frac{9}{32}$. 14. $\frac{11}{32}$. 15. $\frac{13}{32}$. 16. $\frac{17}{32}$. 17. $\frac{21}{32}$. 18. $\frac{25}{32}$. 19. $\frac{31}{32}$. 20. $\frac{57}{64}$. 21. $\frac{39}{64}$. 22. $\frac{51}{64}$.

Page 154. — 3. .25. 4. .125. 5. .375. 6. .166667−. 7. .75. 8. .875. 9. .1875. 10. .833333+. 11. .625. 12. .428571+. 13. .8. 14. .555556−. 15. .714286−. 16. .181818+. 17. .416667−. 18. .3125. 19. .4375. 20. .5625. 21. .6875. 22. .8125. 23. .15625. 24. .28125. 25. .40625. 26. .78125. 27. .916667−. 28. .266667−. 29. .140625. 30. .515625.

1. 299.45. 2. 133.7145. 3. 17.7755. 4. 74.3875. 5. 40. 6. .35854. 7. 1.49479. 8. 221.0295.

Page 155. — 9. .4316. 10. 4.1764. 11. 4.333. 12. 18.4401. 13. 81.8573. 14. 2.5575. 15. .51844. 16. 6.858407. 17. 4.993005. 18. .9375. 19. .6901. 20. .5625. 21. .8173. 22. .59375. 23. .46333. 24. .928125. 25. .777586. 26. 290.72 mi. 27. $.0504; $.0049; $.0115; $.057; $.0066. 28. .875 in. 29. .9364 lb. 30. Southern, 7198.68 mi.; B. & O., 4442.35 mi.

Page 156. — 31. 27.7 hr.; 11.167 hr.; 39.583 hr.; 11.917 hr. 32. 3.61 lb.; 7.66 lb.; 5.77 lb.; 9.09 lb.; 15.88 lb. 33. $101.99.

Page 157. — 2. .2744. 3. .3024. 4. .75738. 5. .30312. 6. .180435. 7. .359723. 8. .329043. 9. .397761.

Page 158. — 13. .0994. 14. .0812. 15. 1.6284. 16. 6.8474. 17. 3.55745. 18. 1.66225. 19. 1.65. 20. 7.49. 21. 9.32249. 22. 36.25792. 23. 93.07701. 24. .378. 25. .093886. 26. .008008. 27. 65.02518. 28. 806.932276. 29. $116. 30. 84.9618 mi. 31. $519.75. 32. $1187.01. 33. $51.45.

Page 161. — 2. 2.5. 3. 3.3. 4. 7.2. 5. 6.72. 6. 4.5. 7. 8.8. 8. 4.3. 9. 8.4. 10. 1.26. 11. 3.36. 12. 2.44. 13. 4.76. 14. 7.5. 15. 2.564. 19. .0075. 20. .0055. 21. .0036. 22. .062. 23. .846. 24. .084. 25. .0364. 26. .00035. 27. .0345. 28. 8. 29. 1680. 30. 32. 31. 64. 32. 224. 33. 28.8. 34. 24.64. 35. 4.985. 36. 6400. 37. 1.715. 38. 64.175. 39. 625. 40. 407.5.

Page 162. — 2. 12. 3. 32.5. 4. .7. 5. 420. 6. 1.5. 7. .4. 8. 1.25. 9. .5. 10. 1.2. 11. 4.2. 12. 18.75. 13. 420. 14. .011. 15. .36.

Page 163. — 1. 1762 bu. 2. 109.4 acres. 3. 17 mo. 4. 74.5 lb. 5. 132 lb. 6. 872 boxes. 7. 135 barrels. 8. 2340 barrels. 9. 1240 rubles. 10. 25.75 rd. 11. 27.8 in. 12. 1524 ft. 13. 72.8 mi.

Page 164. — 1. $528.30. 2. 2669 qt. 3. $321.80. 4. $3071.25. 5. $.52. 6. 55 acres. 7. 214,000 cu. ft. 8. $89.995. 9. $24.843. 10. $2193.50.

Page 165.—**1.** \$13.26. **2.** \$12.35. **3.** \$7.15. **4.** \$30.68. **5.** \$107.25. **6.** \$170.69; \$26.26. **7.** $54\frac{1}{2}$ bu. **8.** \$340.08. **9.** \$169.39. **10.** \$48.49. **11.** \$59.71. **12.** Va. produced: 252,908 bu. more than N.C.; 2,277,572 bu. more than Ga.; 2,691,639 bu. more than Ala.; 2,745,420 bu. more than Fla.; and 2,965,679 bu. more than Tenn.

Page 166.—**13.** \$1.35. **14.** \$1.20. **15.** \$2.20. **16.** \$5.40. **17.** \$3.84. **18.** \$4.73. **19.** \$2.52. **20.** \$9.10. **21.** \$8.40.

Page 167.—**22.** \$4.84. **23.** \$1.65. **24.** \$43.88. **25.** \$5.00. **26.** 3060 stems. **27.** \$15.30. **28.** \$11.20. **29.** \$31.50; \$75.38. **30.** \$16.42 **31.** \$58.96; \$14.74.

Page 168.—**32.** \$15; \$15; \$78.75. **33.** \$22.50; \$14.40. **34.** \$72; \$81. **35.** \$189.90. **36.** \$268.65. **37.** \$432. **38.** \$163.35. **39.** \$2.88; \$.09; \$.16; \$.06$\frac{1}{16}$. **40.** 6400 qt. per acre.

Page 169.—**1.** Sept., 50,256; Oct., 54,512; Nov., 52,640; Dec., 47,872; Jan., 57,360; Feb., 46,016; Mar., 54,728; Apr., 50,264; May, 52,120; June, 43,072. Total for the year, 508,840. Lincoln, 49,690; Greeley, 91,360; Park, 40,820; Douglass, 50,830; Irving, 61,080; White, 81,630; Whittier, 51,350; Longfellow, 82,080. Total for the year, 508,840.

2. Lincoln, 4969; Greeley, 9136; Park, 4082; Douglass, 5083; Irving, 6108; White, 8163; Whittier, 5135; Longfellow, 8208. Average for Greeley 928 greater than for Longfellow.

3. Sept., 6282; Oct., 6814; Nov., 6580; Dec., 5984; Jan., 7170; Feb., 5752; Mar., 6841; Apr., 6283; May, 6515; June, 5384. Average for Jan. 1186 greater than for Dec. **4.** 2544.2.

Page 170.—**5.** 1085.367. **6.** 1841.65. **7.** 972.46. **8.** 1590.4658. **9.** 29.389. **10.** 44.752. **11.** 94.728. **12.** 475.62. **13.** 71.5858. **14.** 3.6183. **15.** 8.5735. **16.** 18.5127. **17.** 2730. **18.** 1.548. **19.** 1.356. **20.** 23.994. **21.** 7.07875. **22.** .27302. **23.** 1.349. **24.** 86.40784. **25.** 2990.1494. **26.** 6.15804. **27.** 2, 3, 17. **28.** 3, 67. **29.** 3^4, 5. **30.** 2^3, 3^2, 7. **31.** 2^3, 31. **32.** 3^6. **33.** 2^6, 3^3. **34.** 2^4, 3^3, 5. **35.** 2^2, 3, 11^2. **36.** 2^4, 3^4. **37.** 2, 3, 5^2, 7. **38.** 2, 5^2, 73. **39.** 2, 3^5, 5^2, 7. **40.** 2^2, 3^4, 7^2. **41.** 2^3, 3^4. **42.** 2^7, 3, 107.

Page 171.—**44.** $\frac{3}{4}$. **45.** $\frac{2}{3}$. **46.** $\frac{3}{5}$. **47.** $\frac{5}{6}$. **48.** $\frac{3}{5}$. **49.** $\frac{2}{3}$. **50.** $\frac{1}{3}$. **51.** $\frac{7}{8}$. **52.** $\frac{3}{7}$. **53.** $\frac{1}{4}$. **54.** $\frac{35}{48}$. **55.** $\frac{1}{40}$. **56.** $\frac{11}{16}$; $\frac{1}{16}$. **57.** $1\frac{9}{16}$; $\frac{3}{16}$. **58.** $1\frac{5}{12}$; $\frac{1}{12}$. **59.** $2\frac{1}{72}$; $\frac{17}{72}$. **60.** $2\frac{5}{24}$; $\frac{11}{24}$. **61.** $1\frac{67}{80}$; $\frac{3}{80}$. **62.** $2\frac{3}{4}$; $\frac{11}{12}$. **63.** $4\frac{1}{12}$; $\frac{11}{12}$. **64.** $6\frac{3}{8}$; $\frac{7}{8}$. **65.** $15\frac{8}{21}$; $3\frac{20}{21}$. **66.** $12\frac{49}{100}$; $2\frac{79}{100}$. **67.** $7\frac{97}{150}$; $1\frac{11}{30}$. **71.** $4\frac{1}{16}$. **72.** $18\frac{3}{5}$. **73.** $2\frac{3}{4}$. **74.** $6\frac{1}{12}$. **75.** $1\frac{29}{60}$. **76.** $1\frac{73}{120}$. **77.** $\frac{37}{10}$. **78.** $\frac{55}{8}$. **79.** $\frac{59}{12}$. **80.** $\frac{379}{25}$. **81.** $\frac{100}{3}$. **82.** $\frac{200}{3}$. **83.** $\frac{175}{2}$. **84.** $\frac{250}{3}$. **85.** $7\frac{1}{2}$. **86.** $14\frac{1}{4}$. **87.** 9. **88.** $1\frac{17}{32}$. **89.** $4\frac{1}{4}$. **90.** 210. **91.** $97\frac{1}{2}$. **92.** $3\frac{17}{20}$. **93.** 150. **94.** $3\frac{21}{25}$. **95.** 8. **96.** 30. **97.** 4. **98.** $16\frac{2}{3}$. **99.** 16.

Page 172.—**101.** $\frac{7}{8}$ is $\frac{7}{40}$ greater than $\frac{7}{10}$; $\frac{7}{8}$ is $1\frac{1}{4}$ times $\frac{7}{10}$. **102.** $\frac{7}{12}$ is $\frac{7}{60}$ less than $\frac{7}{10}$; $\frac{7}{12}$ is $\frac{5}{6}$ of $\frac{7}{10}$. **103.** $74\frac{1}{4}$ cu. in. **104.** Milwaukee, $9\frac{3}{128}$ cu. in. greater; Maine, $14\frac{17}{128}$ cu. in. less; North River, $11\frac{1}{4}$ cu. in. less. **105.** $1121\frac{19}{33}$; $848\frac{16}{33}$. **106.** .1848. **107.** 2.128. **108.** .018. **109.** 2.7648. **110.** .055. **111.** .00468. **112.** .002943. **113.** 12.35. **114.** 3360. **115.** 28.74. **116.** 13.20025. **117.** 135.0165. **118.** .05. **119.** 1.034. **120.** 4.775. **121.** 25.08. **122.** .0486. **123.** .007. **124.** .5163. **125.** 3.7009. **126.** 5.85. **127.** 8.2. **128.** $\frac{5}{16}$. **129.** $\frac{7}{800}$. **130.** $\frac{41}{4000}$. **131.** $\frac{9}{40000}$. **132.** .444+. **133.** .778–. **134.** .917–. **135.** 1.706–. **136.** 1.830–.

Page 173. — 1. 195,000 barrels. **2.** \$6.48. **3.** \$98.81. **4.** \$215.62½; \$67,490.63. **5.** $70\frac{4}{19}$ mi. **6.** 109.35 mi. **7.** \$12.67. **8.** \$119.60. **9.** \$28.40.

Page 174. — 10. 62,390 bunches. **11.** 72 boxes. **12.** \$2.25. **13.** 30 pineapples; \$$2\frac{1}{7}$; $233\frac{1}{3}$ crates. **14.** \$712.50. **15.** 10,000 qt. **16.** 25,000 lb. **17.** \$26.88. **18.** \$3500. **19.** \$7186.73.

Page 175. — **20.** \$903. **21.** Gained \$43. **22.** \$.19. **23.** \$550. **24.** \$5250; \$8820; \$18,900. **25.** 30¢. **26.** First offer; \$2.50. **27.** \$407.16. **28.** \$4500.

Page 176. — **29.** \$5.28. **30.** 220 poles; \$275. **31.** \$41.80. **32.** \$28.82. **33.** \$37.40. **34.** \$86.21. **35.** \$33; \$507.51. **36.** 414 subscribers. **37.** \$5899.50. **38.** \$12,420. **39.** 1.6¢. **40.** 2.47 sec.

Page 178. — **3.** 31 da. **4.** 117 oz. **5.** 451 pk. **6.** 2650 in. **7.** 1875 rd. **8.** 175 mo. **9.** 132 in. **10.** 51,200 lb. **11.** 900 sec. **12.** 364 pt. **13.** 14 ft. 6 in. **14.** 5 gal. 3 qt. 1 pt. **15.** 2 bu. 2 pk. 5 qt. **16.** 6 hr. 5 min. 29 sec. **17.** 1 cwt. 20 lb. 4 oz. **18.** 58 mi. 85 rd. **19.** 3 T. 12 cwt. 32 lb. **20.** 8 bu. 1 pk. 4 qt. **21.** 17 gal. 1 qt. 1 pt. **22.** 1 hr. 25 min. 15 sec. **23.** 6 sq. ft. 136 sq. in. **24.** 169 sq. yd. 4 sq. ft. **25.** 14 cu. ft. 808 cu. in. **26.** 455 cu. yd. 15 cu. ft. **27.** 60 cu. ft. 1285 cu. in.

Page 179. — **28.** 5 mi. 2602 ft. **29.** 16¢. **30.** 114 persons. **31.** \$2.45. **32.** 66 ft. **33.** 30 mi. **34.** 12½ mi. **35.** 10 ft. $7\frac{25}{29}$ in.

Page 180. — **4.** $\frac{3}{16}$ da. **5.** $\frac{3}{8}$ yd. **6.** $\frac{11}{16}$ gal. **7.** $\frac{19}{32}$ bu. **8.** $\frac{25}{28}$ L. T. **9.** $\frac{3}{4}$ cu. yd. **10.** .245 mi. **11.** .382 T. **12.** .03 hr. **13.** .35 wk. **14.** .05 right angle. **15.** .36 sq. mi. **16.** 1026 in. **17.** 102 qt. **18.** 4856 lb. **19.** 5115 ft. **20.** 1 qt. ½ pt. **21.** $\frac{21}{32}$°. **22.** 78° 45′. **23.** 16 cu. ft. 1512 cu. in. **24.** 21 hr. 36 min.

Page 181. — **2.** 18 ft. 2 in. **3.** 10 yd. 9 in. **4.** 72 gal. 2 qt. **5.** 33 mi. 80 rd. **6.** 9 lb. 1 oz. **7.** 15 hr. 30 min. **8.** 30 wk. **9.** 7 mo. 26 da. **10.** 2 hr. **11.** 35 ft. **12.** 9 yr. 9 mo. **13.** 5 T. 16 cwt. 70 lb. **14.** 711 cu. yd. **15.** 193 bu. **16.** 90° 0′ 40″.

Page 182. — **18.** 12 bu. 3 pk. **19.** 49 ft. $10\frac{2}{3}$ in. **20.** 2 yd. 27 in. **21.** 7 hr. 10 min. **22.** 135° 15′. **23.** 3 T. 3 cwt. 80 lb. **24.** 280 rd. **25.** 3 mi. 1780 ft. **26.** 2 hr. 14 min. 22 sec. **28.** 2 yr. 2 mo. 7 da. **29.** 5 yr. 4 mo. 28 da. **30.** 4 yr. 6 mo. 26 da.

Page 183. — **32.** 57 yr. 2 mo. 8 da.; 52 yr. 22 da.; 46 yr. 10 mo. 7 da.; 42 yr. 10 mo. 17 da. **33.** 185 ft. 8 in. **34.** 54 hr. 25 min. **35.** 9 bu. 2½ pk. **36.** \$3.875. **37.** 313 A. 114 sq. rd. **38.** 197 rd. 16 ft. **39.** 6 da. 8 hr. 15 min. **40.** 8 bu. 1 pk. **41.** 6° 36′, or 455.4 mi.; 3° 46′, or 259.9 mi.; 3° 26′, or 236.9 mi.; 2° 54′, or 200.1 mi.

Page 184. — **2.** 58 ft. **3.** 38 gal. 3 qt. **4.** 17 hr. 12 min. **5.** 27 bu. **6.** 66° 21′ 20″. **7.** 165° 4′. **8.** 491 lb. 4 oz. **9.** 54 yr. 1 mo. **11.** 28.646− sq. ft. **12.** 2434.87− sq. ft. **13.** 227.764− sq. ft. **14.** 1024.566− sq. ft. **15.** 342.25 sq. ft. **16.** 144,474 sq. ft.

Page 185. — **17.** 1693.519− cu. ft. **18.** 1500.213− cu. ft. **19.** 2876.32− cu. ft. **20.** 10,283.802+ cu. ft.

Page 186. — **4.** 5 gal. 3 qt. **5.** 16 lb. 5 oz. **6.** 8 ft. 11 in. **7.** 10 yd. 11 in. **8.** 18° 22′. **9.** 650 lb. **10.** 7 mi. 100 rd. **11.** 9 sq. ft. 52 sq. in. **12.** 9. **13.** 25. **14.** 7. **15.** 640. **16.** 300 rd.; 562½ acres. **17.** 12 wk. **18.** 35 wk. 5 da. **20.** 4.5 ft. **21.** 15 rd. **22.** 60 rd.

Page 187. — 1. 79 T. 750 lb. 2. 28 ft. 9 in. 3. $12.15. 4. $78. 5. 185 lb. 6. 18.9 tons. 7. $1\frac{7}{8}$ gal. 8. $9\frac{3}{4}$ lb. 9. 3 mi. 10. $65.75. 11. $1.62. 12. 291 lb.

Page 188. — 1. $\frac{1}{100}$. 2. $\frac{7}{100}$. 3. $\frac{9}{100}$. 4. $\frac{11}{100}$. 5. $\frac{19}{100}$. 6. $\frac{17}{100}$. 7. $\frac{27}{100}$. 8. $\frac{33}{100}$. 9. $\frac{51}{100}$. 10. $\frac{63}{100}$. 11. $\frac{81}{100}$. 12. $\frac{77}{100}$. 13. .15. 14. .24. 15. .36. 16. .45. 17. .04. 18. .06. 19. .10. 20. .30. 21. 9%. 22. 13%. 23. 41%. 24. 48%. 25. 75%. 26. 55%. 27. 8%. 28. 20%. 29. 2%. 30. 60%. 31. 9%. 32. 90%.

Page 190. — 2. $52.50. 3. $146.12. 4. $4.17. 5. $4.26. 6. $13.50. 7. $71.62. 8. $15.87. 9. $32.74. 10. $540. 11. $3116.96. 12. 256.75\frac{1}{2}$. 13. $571.06. 14. 55.8 lb. 15. $13.20. 16. 552 persons. 17. 455.7 mi. 18. $357. 19. $3.23. 20. $106.26.

Page 191. — 2. .22$\frac{1}{2}$. 3. .35$\frac{3}{4}$. 4. .46$\frac{2}{3}$. 5. .03$\frac{1}{8}$. 6. .06$\frac{1}{4}$. 7. .08$\frac{1}{3}$. 8. .00$\frac{1}{5}$. 9. .00$\frac{3}{4}$. 10. .00$\frac{5}{8}$. 11. 1.06. 12. 1.20. 13. 1.50. 14. $\frac{1}{5}$. 15. $\frac{2}{5}$. 16. $\frac{1}{2}$. 17. $\frac{3}{5}$. 18. $\frac{3}{4}$. 19. $\frac{1}{8}$. 20. $\frac{3}{8}$. 21. $\frac{5}{8}$. 22. $\frac{7}{8}$. 23. $\frac{2}{3}$. 24. $\frac{1}{30}$. 25. $\frac{1}{15}$. 26. $\frac{1}{16}$. 27. $\frac{1}{12}$. 28. $\frac{1}{6}$. 29. $\frac{5}{6}$. 30. $\frac{9}{8}$. 31. $\frac{4}{3}$. 32. $\frac{11}{8}$. 33. $\frac{5}{3}$. 35. 50%. 36. 25%. 37. 80%. 38. $33\frac{1}{3}$%. 39. $37\frac{1}{2}$%. 40. $16\frac{2}{3}$%. 41. $66\frac{2}{3}$%. 42. $83\frac{1}{3}$%. 43. $87\frac{1}{2}$%. 44. 175%. 45. $162\frac{1}{2}$%. 46. $233\frac{1}{3}$%. 47. $41\frac{2}{3}$%. 48. $68\frac{3}{4}$%. 49. $25\frac{3}{5}$%.

Page 193. — 2. $23.46. 3. $15.36. 4. $34.26. 5. $13.29. 6. $56.28. 7. $92.32. 8. $292.97. 9. $150. 10. $66.27. 11. $583.54. 12. $749.40. 13. $791.20. 14. $956.97. 15. $120.64. 16. $163.15. 17. 600 fish. 18. State, $3790.20; county, $947.55; town, $947.55. 19. $4.20.

Page 194. — 20. 1337. 21. $5463; $4856. 22. 3126 lb.; $2344\frac{1}{2}$ lb. 1. 14.58. 2. 13.8. 3. 70.65. 4. 5.65. 5. 42.625. 6. 190. 7. 132.84. 8. 370.04. 9. 91.5. 10. 847. 11. $224.50+. 12. $545.91−. 13. $853.60. 14. $879.84. 15. $2606.56−. 16. 80 lb. 17. 8 bunches; 18 lb. 18. $58.95; $556.75.

Page 195. — 19. 1875 lb. 20. 11,520 lb.

Page 196. — 3. 48%. 4. 25%. 5. $62\frac{1}{2}$%. 6. $33\frac{1}{3}$%. 7. 162%. 8. $2\frac{1}{2}$%. 9. 8%. 10. $6\frac{1}{4}$%. 11. $166\frac{2}{3}$%. 12. $12\frac{1}{2}$%. 13. 75%. 14. 96%. 15. $18\frac{3}{4}$%. 16. $16\frac{2}{3}$%. 17. $31\frac{1}{4}$%. 18. $16\frac{1}{4}$%. 19. $15\frac{5}{8}$%. 20. $45\frac{1}{2}$%. 21. $87\frac{1}{2}$%. 22. 40%.

Page 197. — 23. $22\frac{1}{2}$%. 24. $37\frac{1}{2}$%. 25. 75%. 26. $24\frac{1}{2}$%. 27. 44%. 28. $87\frac{1}{2}$%. 29. $19\frac{1}{3}$%. 30. $\frac{1}{2}$%. 31. $22\frac{2}{3}$%. 32. $37\frac{1}{2}$%. 33. Hale, $62\frac{1}{2}$%; Chesley, 25%; Hoit, $12\frac{1}{2}$%.

Page 199. — 3. 1500. 4. 880. 5. 1536. 6. 1800. 7. 1600. 8. $414. 9. $923.75. 10. $103.50. 11. $146.25. 12. $90.96. 13. $152. 14. $672. 15. $212.32. 16. $52.77. 17. $49.62. 18. 6250 lb.

Page 200. — 19. 10,000 tons. 20. 7500 tons. 21. 23,760 lb. 22. 11,000 tons. 23. 38,750 lb. 24. 125,550. 25. 225 lb. 26. $10.40. 27. $4.90. 28. 1500 ft. 29. 280,400 sq. mi.

Page 201. — 1. 19.6. 2. 25%. 3. $1875. 4. $41.56. 5. $529. 6. $37\frac{1}{2}$%. 7. 899.14\frac{1}{2}$.

Page 202. — 8. $500. 9. 12,880 bu. 10. 25%. 11. 7000 shirts. 12. $4755.25. 13. $44\frac{3}{4}$%. 14. N.Y., $9\frac{3}{4}$%; N.H., $9\frac{1}{4}$%; O., $6\frac{3}{4}$%; Pa., $5\frac{1}{4}$%; Me., $4\frac{3}{4}$%; Ill., 4%. 15. $15\frac{1}{2}$%. 16. 3,671,904 farms. 17. 1,509,200 cords.

Page 204. — **1.** \$.18. **2.** \$.19. **3.** \$.33. **4.** \$2.95. **5.** \$8.87. **6.** \$18.25. **7.** \$27.16; \$325.92; \$10.08. **8.** \$31.50; \$472.50; \$52.50. **9.** \$20; \$400; \$240. **10.** \$2.50; \$45; \$9. **11.** \$5; \$120; \$60. **12.** \$1.40; \$50.40; \$7.20. **13.** \$1.40; \$42; \$10.50.

Page 205. — **14.** \$26. **15.** \$20.75. **16.** \$15. **17.** \$9.75. **18.** \$12.95. **19.** \$191.25. **20.** \$20.25. **21.** \$14.85. **22.** \$12. **23.** \$13.50. **24.** \$235.71. **25.** \$33; \$31.35. **26.** \$189.

Page 206. — **1.** 2 qt. **2.** 225 lb. **3.** \$16.65. **4.** \$180. **5.** 29 shelves. **6.** 345 lb. **7.** 575 lb. **8.** $1\frac{2}{3}$ bales; 60%. **9.** \$724.50. **10.** \$1242.

Page 207. — **11.** \$5 less per ton. **12.** 2070 doz. brooms; 207 brooms. **13.** 45,480 tons. **14.** 720 pieces. **15.** 10,080 pieces. **16.** 840 bricks.

Page 208. — **17.** 1050 lb.; 15¢. **18.** 280 more bricks. **19.** \$52.50. **20.** 1890 lb. **21.** 63 10-lb. baskets. **22.** 315 3-lb. baskets. **23.** 63 5-lb. baskets. **24.** \$409.50. **25.** 40¢; \$1.20. **26.** \$141.75. **27.** \$929.25. **28.** \$612.50.

Page 209. — **29.** 8750 barrels. **30.** \$10,937.50. **31.** 525,000; 131,250,000. **32.** 49,875,000. **33.** 68,355,000. **34.** \$68,125. **35.** \$78,750. **36.** \$97,762.50. **37.** \$244,637.50.

Page 210. — **38.** \$4.56. **39.** 90 acres. **40.** 12.39 in. **41.** 4 days. **42.** 10 sacks. **43.** 36 bu. **44.** 3240 bu. **45.** 20%; 80%. **46.** 2592 bu.

Page 211. — **47.** \$2203.20. **48.** \$1609.20. **49.** \$594. **50.** 116,640 lb. **51.** 72,000 lb. **52.** $13\frac{1}{3}$ hr. **53.** 44,640 lb. **54.** 20%. **55.** \$159.84. **56.** 16,560 lb. broken rice; 55,440 lb. whole rice. **57.** \$3187.80. **58.** $2\frac{1}{2}$¢. **59.** 90%. **60.** \$1092.

Page 212. — **61.** 4850. **62.** Oct., $8\frac{1}{3}$%; Nov., $10\frac{1}{2}$%; Dec., 15%; Jan., 18%; Feb. $16\frac{2}{3}$%; Mar., 15%; Apr., $12\frac{1}{2}$%; May, 4%. **63.** Oct., 18,750; Nov., 23,625; Dec., 16,875; Jan., 27,000; Feb., 37,500; Mar., 45,000; Apr., 37,500; May, 12,000; total, 218,250. **64.** 45 violets. **65.** \$174.60. **66.** \$523.80. **67.** \$1726.20.

Page 214. — **1.** Cotton, 1240 gr. **2.** $437\frac{1}{2}$ gr. **3.** .911+. **4.** 26 lb. 4 oz. **5.** 8 lb.; \$33.92. **6.** \$5143.94. **7.** \$227.70. **8.** \$116.99. **9.** \$248.06196. **10.** \$44.88. **11.** \$22.44. **12.** $\$2.80\frac{1}{2}$. **13.** \$7.39. **14.** \$22.54. **15.** \$8.47. **16.** $49\frac{1}{2}$¢. **17.** 34 spoons.

Page 215. — **18.** 464.4 gr. **19.** \$1.0336. **20.** .9675 pwt. **21.** \$288.40. **22.** 22 lb. $4\frac{3}{4}$ oz. (troy). **23.** 67 lb. $2\frac{1}{4}$ oz. (troy). **24.** 21.3925 pwt. **25.** \$19.85. **26.** $4479\frac{1}{6}$ lb. (troy). **27.** \$175; $.01\frac{3}{4}$%. **28.** \$1.52+. **29.** \$19.298−.

Page 216. — **1.** 15 cords. **2.** $14\frac{1}{4}$ cords. **3.** 330 cords. **4.** $1\frac{1}{3}$ cords. **5.** $57\frac{3}{4}$ cu. in.; $67\frac{1}{5}$+ cu. in. **6.** 3136 bu. **7.** 1152 gal.

Page 217. — **8.** 2400 cu. ft. **9.** 61,440 lb. **10.** 18,560 lb. **11.** 35,200 lb. **12.** 240 bu. **13.** $5\frac{5}{9}$ ft. **14.** 102,200 lb. **15.** 88,500 lb. **16.** \$54.40.

Page 218. — **17.** 3136 lb.; \$76. **18.** 5600 lb.; \$23. **19.** 24,000 lb.; \$1536. **20.** 555 lb.; \$74.93. **21.** 800 lb.; \$60.40. **22.** \$5.88. **23.** \$1.53. **24.** \$11.25. **25.** \$7.25. **26.** \$7.99. **27.** \$3.25. **28.** \$.60. **29.** \$3.20. **30.** \$3.00. **31.** \$9.45. **32.** \$21.60. **33.** \$7.50. **34.** \$72.00. **35.** \$36.00.

Page 219. — **1.** 90 bbl.; 250 bbl. **2.** 1000 cu. ft.; $238\frac{2}{21}$ bbl. **3.** $2\frac{1}{2}$ million cu. ft. **4.** 16,500,000 gal. **5.** 5,742,950 million gal. **6.** $10\frac{1}{3}$ lb. **7.** 5150 lb. **8.** $287\frac{1}{2}$ lb.

Page 220. — **9.** First, 2.62 times; second, 2.95 times. **10.** 15 lb. **11.** 6800 lb. **12.** $535\frac{15}{16}$ lb. **13.** $254\frac{3}{8}$ lb.

Page 221. — **1.** 400°. **2.** 410°; 230°. **3.** 163°. **4.** 122°. **5.** 140°. **6.** 103°. **7.** 59°. **8.** 114°. **9.** 104°. **10.** 115°. **11.** 69°. **12.** 110°. **13.** 81°.

Page 223. — **1.** 576 ft. **2.** 210 ft. **3.** $1666\frac{2}{3}$ ft. **4.** 3584 ft. **5.** $1194\frac{2}{3}$ ft. **6.** 1920 ft. **7.** 4500 ft. **8.** 5400 ft. **9.** 1536 ft. **10.** 4851 ft. **11.** 10,976 ft. **12.** 20,480 ft. **13.** $1365.47. **14.** $30.41. **15.** $75.27. **16.** $66.32. **17.** $86.52.

Page 224. — **18.** $70.40. **19.** $211.20. **20.** $50.18. **21.** $108.80. **22.** $16. **23.** $225. **24.** $17.28. **25.** $52.80. **26.** 72 pieces. **27.** $15.56. **28.** $137.16. **29.** 840 ft. **30.** $29\frac{2}{7}$ lb.

Page 225. — **1.** Cost of 100 sq. yd., $33.20; 33.2¢. **2.** $26.45. **3.** $17.69.

Page 226. — **4.** $87.84. **5.** $63.36. **6.** $31.49. **7.** $25.09. **8.** $21.54. **9.** $40. **10.** $14.60. **11.** $11.20. **12.** $16.52. **13.** $12.61. **14.** 19 bundles. **15.** $23.89.

Page 227. — **1.** 143 bunches. **2.** $78.65. **3.** $125.90. **4.** $152. **5.** 24 strips. **6.** $4.80. **7.** $28. **8.** $56.56. **9.** $1.77.

Page 229. — **1.** $8.40. **2.** $22.40. **3.** $12.84. **4.** $10.50.

Page 230. — **1.** $14.75. **2.** $13.57. **3.** $19.77. **4.** $33.06. **5.** $63. **6.** $212.50. **7.** $17,465. **8.** $2.33. **9.** $.33.

Page 231. — **10.** 432 tons. **11.** 32,423,880 yd. **12.** 8100 lb. **13.** 135 strips. **14.** 24,000 strips. **15.** $\frac{3}{4}$ gal. **16.** 16 tons. **17.** 216 tons. **18.** 86¢. **19.** $36.55.

Page 232. — **20.** 6000 bricks. **21.** $81.25. **22.** 130 lb. **23.** $596.70. **24.** $76\frac{1}{2}$ cu. yd. **25.** 1350 cu. yd. **26.** 22 days. **27.** $13,625. **28.** $57,187\frac{1}{2}$ lb. **29.** $32,340. **30.** $18,480. **31.** 1440 cu. ft.; 10,800 gal.

Page 236. — **1.** .348. **2.** .226. **3.** .6723. **4.** .4091. **5.** .076. **6.** .054. **7.** .0827. **8.** .0999. **9.** 25.7%. **10.** 42.1%. **11.** 87.24%. **12.** 50.63%. **13.** 2.6%. **14.** 8.2%. **15.** 8.44%. **16.** 6.19%. **17.** 6.2%; 6.15%. **18.** 4.9%; 4.90%. **19.** 12.6%; 12.59%. **20.** 45.6%; 45.56%. **21.** 86.5%; 86.47%. **22.** 24.4%; 24.35%. **23.** 36.4%; 36.40%. **24.** 91.0%; 90.98%.

1. $1.79. **2.** $3.44. **3.** $7.10. **4.** $$42.70\frac{1}{2}$. **5.** $4.36. **6.** $6.76. **7.** $62.72. **8.** $318.65. **9.** $331.59. **10.** $180.26. **11.** $5669.62. **12.** $7718.85.

Page 237. — **13.** $33\frac{1}{3}$%. **14.** 8%. **15.** $\frac{5}{8}$%. **16.** $12\frac{1}{2}$%. **17.** 35%. **18.** $16\frac{2}{3}$%. **19.** 4.8%. **20.** 16.5%. **21.** 11.2%. **22.** $8520. **23.** $620.80. **24.** $2205. **25.** $17,280. **26.** $3083.76. **27.** $17,250. **28.** $7661.88. **29.** $27,400. **30.** 34.18%. **31.** 164.29%. **32.** 17.05%. **33.** 273.97%. **34.** 61.52%. **35.** 63.08%. **36.** 1639.456 lb. **37.** 9000 lb. **38.** 69.3%. **39.** 34.9%. **40.** 98.3%. **41.** 204 horse vehicles; 450 street cars; 594 motor vehicles; 4752 bicycles.

Page 238. — **42.** 27.6%. **43.** $28,100. **44.** 18.5%. **45.** 980 bu. **46.** Sand, 35%; limestone, 34%; soda, 10%; broken glass, $8\frac{1}{2}$%; other materials, $12\frac{1}{2}$%. **47.** 15%. **48.** Great Britain, $2,209,343.90+; United States, $319,754.10+; Spain, $759,170.78−. **49.** Otter, 43.4%; beaver, 12.1%; seal, 25.1%; mink, 79.4%; muskrat, 76.3%.

Page 240. — **1.** $7630. **2.** $4259.50. **3.** 20%. **4.** $2 per day. **5.** 8.8%. **6.** $4,613,133.65−. **7.** Males, 64.3%; females, 111.4%. **8.** 5% increase in yield; 16% decrease in price. **9.** 519.7%; 46.5%. **10.** 26.5%; 7.5%. **11.** .37%.

Page 241. — **2.** 384. **3.** $248. **4.** $530. **5.** $594.72. **6.** $282.56.

Page 242. — **7.** $1400. **8.** 40 words a minute. **9.** 450 children. **10.** $781.25. **11.** $425. **12.** 140,500. **13.** 35,000 mi. **14.** 65,000 yd. **15.** 64,000 cu. ft. **16.** $18,322,500.

Page 243. — **2.** 582.

Page 244. — **3.** $245. **4.** $950. **5.** $726. **6.** $4454.16. **7.** 120 lb. **8.** $17\frac{1}{2}$ lb. **9.** $32,000. **10.** $39,000. **11.** 40 yd. **12.** $6 per ton. **13.** 42,000. **14.** 800 bbl. **15.** 10¢. **16.** 1,187,500. **17.** 576 hats. **18.** 12,500.

Page 247. — **1.** $5.40. **2.** $8.50. **3.** $4.48. **4.** $37\frac{1}{2}$%. **5.** 26¢. **6.** $1854. **7.** $66\frac{2}{3}$%. **8.** $15. **9.** $16\frac{2}{3}$%. **10.** $$1.87\frac{1}{2}$. **11.** $7095. **12.** $20. **13.** $11,435.58. **14.** $2\frac{1}{2}$¢.

Page 248. — **15.** 20%. **16.** 5¢. **17.** 20%. **18.** 127.3%. **19.** $10. **20.** 60%. **21.** 29.9%. **22.** $272. **23.** 30%. **24.** $6206.25. **25.** Gained $68.

Page 250. — **2.** Com., $2.70; net proceeds, $51.30. **3.** $4396.40. **4.** Com., $52.15; net proceeds, $692.85. **5.** $99.42.

Page 251. — **6.** $1950.93. **7.** $4689.30. **8.** $18. **9.** Com., $56; net proceeds, $75. **10.** Com., $371.25; net proceeds, $11,878.75. **11.** $18.75; $356.25. **12.** $12.75; $199.75. **13.** $63; $567. **14.** $12; $138. **15.** $8.64; $106.56. **16.** $27; $513. **17.** $5.28; $100.32. **18.** $24.75; $470.25. **19.** $22.88; $205.92. **20.** $19.60; $372.40. **21.** $1800.

Page 252. — **22.** $2904.55. **23.** $5000. **24.** $2\frac{1}{2}$%. **25.** 5%. **26.** $81.60. **27.** $672. **28.** 4%. **29.** $1200. **30.** $2\frac{1}{2}$%. **31.** 18,000 bu. **32.** $50.74; $1696.26.

Page 253. — **2.** $18. **3.** $32.30. **4.** $4.90. **5.** $2.85. **6.** $15.12; $45.36; $26.64. **7.** $5.70; $25.65; $14.85. **8.** $3.51; $21.06; $17.94. **9.** $4.16; $22.88; $21.12. **10.** $3.42; $51.30; $98.70.

Page 254. — **11.** $162. **12.** $28. **13.** $16.20. **14.** $17.55. **15.** $48.45. **16.** $216. **17.** $64.98. **18.** $17.10. **19.** $11.88. **20.** $4.86. **21.** $31.59. **22.** $138.89. **23.** $7.02.

Page 255. — **1.** $152. **2.** $270. **3.** $486. **4.** $204. **5.** $324. **6.** $613.36. **7.** $214.13. **8.** $398.40. **9.** $297.50. **10.** $434. **11.** $769.50. **12.** $1324.80. **13.** Single discount, $9.60 more. **14.** $161. **15.** $301.15. **16.** $75.60. **17.** $496. **18.** $106.02.

Page 256. — **20.** $37.80. **21.** $96.39. **22.** $62.10. **23.** $290.70.

Page 257. — **1.** 20%. **2.** $33\frac{1}{3}$%. **3.** $12\frac{1}{2}$%. **4.** 60%. **5.** $37\frac{1}{2}$%. **6.** $16\frac{2}{3}$%. **7.** 28%. **8.** 25%. **9.** $15\frac{1}{3}$%. **10.** 24%.
11. $\frac{ht}{ih}$. **12.** $\frac{sb}{bh}$. **13.** $\frac{tih}{eas}$. **14.** $\frac{sbo}{bho}$. **15.** $\frac{wtk}{wbh}$. **16.** $\frac{amb}{woeh}$. **17.** $\frac{bs}{whm}$.
18. $\frac{wih}{was}$. **19.** $\frac{hmks}{imtm}$. **20.** $\frac{tsie}{swmo}$.

Page 258. — **21.** $\frac{to}{tm}$. **22.** $\frac{ek}{sk}$. **23.** $\frac{heo}{iok}$. **24.** $\frac{she}{aeo}$. **25.** $\frac{ao}{mt}$.
26. $\frac{mo}{bs}$. **27.** $\frac{hme}{ith}$. **28.** $\frac{twe}{tbm}$. **29.** $\frac{io}{is}$. **30.** $\frac{eaoe}{smts}$. **31.** $\frac{ht}{ik}$. **32.** $\frac{to}{ek}$.
33. $\frac{mk}{whw}$. **34.** $\frac{bs}{wih}$. **35.** $\frac{hmo}{ime}$. **36.** $\frac{aho}{bko}$. **37.** $\frac{tkm}{sws}$. **38.** $\frac{smo}{bie}$. **39.** $\frac{was}{hth}$.

40. $\frac{hkt}{iom}$. 42. $\frac{ih}{eo}$. 43. $\frac{ah}{wok}$. 44. $\frac{tm}{mo}$. 45. $\frac{ito}{eok}$. 46. $\frac{sah}{bso}$. 47. $\frac{mto}{whso}$.
48. $\frac{blh}{hte}$. 49. $\frac{bre}{wmh}$. 50. $\frac{ien}{hmo}$. 51. $\frac{pe}{mt}$. 52. $\frac{bhe}{hwo}$. 53. $\frac{bte}{wbs}$. 54. $\frac{ihe}{ieo}$.
55. $\frac{le}{bm}$.

Page 259.—**1.** \$17.60. **2.** 8 hr. 15 min.; 9 hr. **3.** $53\frac{1}{3}$ mi. per hr. going; $48\frac{8}{9}$ mi. per hr. returning. **4.** 302 passengers. **5.** \$2845.76. **6.** \$140.90. **7.** \$15.40.

Page 260.—**8.** 15 days. **9.** 32,916 mi. **10.** 1125 tons. **11.** \$63,375. **12.** \$2625; \$34,125. **13.** \$42,832.50. **14.** \$20,542.50; \$1580.19+. **15.** 77.042 mi. **16.** 12.10185+ ft.

Page 261.—**17.** \$180. **18.** \$339. **19.** \$26.25. **20.** \$41; \$6. **21.** 1080 bu. oats; 1410 bu. wheat; 2568 bu. corn. **22.** \$577.50. **23.** \$54.96. **24.** \$1065.

Page 262.—**25.** 42,900 trees. **26.** 68,640 lb. **27.** $6\frac{7}{8}$ ¢. **28.** \$1767.48. **29.** $4\frac{3}{10}$ ¢; $2\frac{23}{40}$ ¢. **30.** \$20.60. **31.** \$154.44. **32.** 520 bags. **33.** \$193.44. **34.** 58,344 lb.

Page 263.—**35.** 831,000 lb. **36.** 2136 plants. **37.** $1\frac{2}{3}$ ft. **38.** 70 ¢. **39.** 2403 lb. **40.** $\frac{1}{4}$ lb. **41.** 534 lb.; 128,160 lb.

Page 264.—**42.** 3150 trays. **43.** 2925 lb. **44.** 3204 chests. **45.** 24 chests. **46.** 14 ¢. **47.** 112,906,000 lb. **48.** 147,517,000 lb. more from India; 42,109,000 lb. less from China. **49.** Great Britain, 6.03 lb.; Russia, .95 lb.; Germany, .12 lb.; France, .06 lb.; United States, 1.3 lb.

Page 265.—**50.** 54 gal. **51.** $121\frac{1}{2}$ lb. **52.** 1944 lb. **53.** $13\frac{1}{2}$ lb. **54.** $4\frac{1}{2}$ lb. **55.** 162 loaves. **56.** 6 chests and 12 loaves. **57.** \$1.24. **58.** \$16.74.

Page 266.—**59.** 60,000 tons. **60.** 8883 tons. **61.** 49,979,739 pairs. **62.** \$22.75. **63.** 450,000. **64.** 1st da., 22,500; 2d da., 168,750; 3d da., 180,000; 4th da., 56,250; 5th da., 22,500. **65.** 180 trees. **66.** 337,500.

Page 267.—**67.** 1350 lb. **68.** 432 lb. **69.** \$378. **70.** 108 lb.; 34.56 lb.; \$30.24. **71.** 118.8 lb. **72.** $\frac{3}{4}$ lb. **73.** \$493.02. **74.** 150 oz. **75.** \$337.50. **76.** 42,367,115 lb. **77.** In order: \$1056; \$948; \$1128; \$780.

Page 268.—**78.** \$99.75. **79.** \$266. **80.** \$704. **81.** \$153.60. **82.** \$112.50. **83.** \$198.40. **84.** \$70.40. **85.** \$81.60.

Page 269.—**86.** \$1610.75. **87.** 112 bbl.; $4\frac{2}{3}$ bbl. **88.** 4704 gal. **89.** 98 bbl. **90.** \$4101.30. **91.** \$4687.20. **92.** 234.95 mi. **93.** 39 wk. 2 da. **94.** Gasoline, etc., $12\frac{1}{2}$%; illuminating oil, 75%; lubricating oil, 3%; residuum, etc., $9\frac{1}{2}$%. **95.** \$12,127.50. **96.** 19,404 bbl.

Page 270.—**97.** $8\frac{3}{4}$ ¢. **98.** 7500 gal. **99.** 160 cars. **100.** 3900 tons. **101.** 864 cases. **102.** 8640 gal. **103.** 43,200 gal. **104.** 650 gal. **105.** $45\frac{2}{3}$%.

Page 271.—**106.** 34,100 lb. **107.** \$6.48. **108.** \$3.48. **109.** \$55,000. **110.** 9.9 lb.; 213,600 lb. **111.** \$2361.15. **112.** $26\frac{1}{2}$ ¢. **113.** \$81.62; $3\frac{2}{3}$%.

Page 272.—**114.** \$4.10. **115.** 950 lb.; \$38.95. **116.** Nov., 11,600 lb.; Dec., 12,000 lb.; Jan., 12,800 lb.; Feb., 14,400 lb.; Mar., 15,600 lb. **117.** Nov., 58 lb.; Dec., 60 lb.; Jan., 64 lb.; Feb., 72 lb.; Mar., 78 lb. **118.** Nov., \$1.14; Dec., \$2.04+. **119.** Jan., \$741.30; Feb., \$452.20. **120.** Jan., \$3.71−; Feb., \$2.26+. **121.** \$1020.20.

Page 273. — **122.** \$4.29+. **123.** \$1.44−. **124.** \$3.14−. **125.** \$357.80; \$275. **126.** \$94.90; \$232.40. **127.** \$468. **128.** \$53.20. **129.** \$256.40. **130.** \$120. **131.** \$12,244. **132.** 28 lb. **133.** \$1604.

Page 274. — **134.** \$275.75. **135.** \$394.40. **136.** \$1879.15. **137.** 58%. **138.** \$15.39. **139.** \$84.20.

Page 275. — **3.** \$535.45. **4.** \$1923.94. **5.** \$30,592.94. **6.** \$151,189.40. **7.** \$2,974,622.52.

Page 276. — **8.** 33,418. **9.** 56,535. **10.** \$138.85. **11.** 3.4614. **12.** 3.3585. **13.** \$62,517.12. **22.** 335.8184. **23.** 47.3008. **24.** 684.74. **25.** 150.3255. **26.** 366.29104. **27.** 47.37555. **28.** 301.176. **29.** .329765. **30.** 2.97242. **31.** 2,500,000. **32.** 8,133,000. **33.** 271,875. **34.** 101.5625. **35.** 16. **36.** 1024. **37.** .90625. **38.** 2.25. **39.** 5.76. **40.** 56.8. **41.** 4260. **42.** 2109.375. **43.** 1728. **44.** 715. **45.** .206. **46.** 2, 3^3, 5. **47.** 2^3, 3^3. **48.** 3^2, 5^2. **49.** 2^2, 3^2, 5. **50.** 2^2, 3^4. **51.** 3^2, 89. **52.** 3^2, 5, 11. **53.** 3, 11^2. **54.** 3^4, 7. **55.** 2^3, 3^4. **56.** 2^2, 3, 5, 13. **57.** 3^4, 11. **58.** 2^{10}. **59.** 2^6, 3^3. **60.** 2^2, 3^3, 11^2. **61.** 2^7, 5^4. **62.** $\frac{32}{48}$ and $\frac{20}{48}$; $\frac{64}{96}$ and $\frac{40}{96}$; $\frac{96}{144}$ and $\frac{60}{144}$; $\frac{160}{240}$ and $\frac{100}{240}$; $\frac{256}{384}$ and $\frac{160}{384}$; $\frac{352}{528}$ and $\frac{220}{528}$.

Page 277. — **63.** $\frac{7}{12}$. **64.** $\frac{4}{5}$. **65.** $\frac{3}{4}$. **66.** $\frac{5}{6}$. **67.** $\frac{1}{3}$. **68.** $\frac{7}{8}$. **69.** $\frac{2}{3}$. **70.** $\frac{5}{12}$. **71.** $\frac{3}{8}$. **72.** $\frac{1}{8}$. **73.** $\frac{113}{4}$. **74.** $\frac{261}{8}$. **75.** $\frac{650}{3}$. **76.** $\frac{1299}{4}$. **77.** $\frac{427}{10}$. **78.** $\frac{821}{12}$. **79.** $\frac{539}{18}$. **80.** $\frac{1363}{24}$. **81.** 29. **82.** 23. **83.** 49. **84.** $34\frac{5}{8}$. **85.** $29\frac{1}{2}$. **86.** $12\frac{1}{3}$. **87.** $20\frac{3}{5}$. **88.** $68\frac{5}{12}$. **89.** 76. **90.** $56\frac{2}{3}$. **91.** $\frac{11}{25}$. **92.** $\frac{17}{20}$. **93.** $\frac{1}{40}$. **94.** $\frac{1}{16}$. **95.** $\frac{5}{16}$. **96.** $20\frac{3}{4}$. **97.** $\frac{5}{32}$. **98.** $\frac{7}{32}$. **99.** $\frac{9}{32}$. **100.** $53\frac{1}{8}$. **101.** $65\frac{5}{8}$. **102.** $7\frac{13}{16}$. **103.** $.55\frac{5}{9}$. **104.** $.41\frac{2}{3}$. **105.** $.53\frac{1}{3}$. **106.** 1.75. **107.** .9375. **108.** .84. **109.** .078125. **110.** .425. **111.** $.69\frac{4}{9}$. **112.** 3.45. **113.** .856364−. **114.** .003125. **115.** $\frac{25}{60}$, $\frac{28}{60}$. **116.** $\frac{72}{80}$, $\frac{45}{80}$. **117.** $\frac{52}{72}$, $\frac{51}{72}$. **118.** $\frac{32}{72}$, $\frac{39}{72}$, $\frac{46}{72}$. **119.** $\frac{195}{225}$, $\frac{162}{225}$, $\frac{155}{225}$. **120.** $\frac{135}{240}$, $\frac{128}{240}$, $\frac{408}{240}$. **121.** $\frac{56}{60}$, $\frac{27}{60}$, $\frac{58}{60}$. **122.** $\frac{42}{72}$, $\frac{22}{72}$, $\frac{69}{72}$. **123.** $\frac{35}{42}$, $\frac{6}{42}$, $\frac{13}{42}$. **124.** $2\frac{5}{8}$. **125.** $2\frac{7}{10}$. **126.** $\frac{3}{16}$. **127.** $\frac{14}{25}$. **128.** $3\frac{2}{15}$. **129.** $\frac{7}{20}$.

Page 278. — **130.** $167\frac{1}{4}$. **131.** $221\frac{7}{8}$. **132.** $20.8\frac{3}{5}$. **133.** $26.71\frac{1}{3}$. **134.** $201\frac{9}{10}$. **135.** $23.66\frac{1}{4}$. **136.** $\frac{1}{16}$. **137.** $\frac{5}{12}$. **138.** $\frac{4}{9}$. **139.** $\frac{9}{20}$. **140.** 418.5. **141.** 704.7. **142.** 179.58. **143.** 333.25. **144.** \$9.75. **145.** \$$13.12\frac{1}{2}$. **146.** \$$11.60\frac{1}{2}$. **147.** \$18. **148.** $\frac{15}{16}$. **149.** $2\frac{1}{2}$. **150.** $1\frac{2}{3}$. **151.** $\frac{7}{12}$. **152.** $42\frac{5}{6}$. **153.** $17\frac{2}{15}$. **154.** $26\frac{3}{10}$. **155.** $18\frac{4}{25}$. **156.** 112. **157.** 32. **158.** 96. **159.** 84. **160.** $\frac{2}{3}$; $42\frac{2}{3}$. **161.** $22\frac{2}{5}$. **162.** 18. **163.** $\frac{5}{18}$. **164.** 1. **165.** $7\frac{7}{8}$. **166.** $5\frac{5}{6}$. **167.** $\frac{2}{3}$. **168.** $\frac{2}{5}$. **169.** $\frac{1}{2}$. **170.** $\frac{15}{16}$. **171.** $\frac{5}{6}$. **172.** $\frac{3}{4}$. **173.** $\frac{1}{3}$. **174.** $\frac{13}{24}$.

Page 279. — **175.** 126. **176.** $121\frac{1}{3}$. **177.** 280. **178.** 450. **179.** 12. **180.** $16\frac{2}{3}$. **181.** 2. **182.** $\frac{5}{8}$.

1. 969,103.272. **2.** 409,500 lb. **3.** \$432,000. **4.** 321 mi. **5.** $4\frac{7}{8}$ tons. **6.** British, $52\frac{1}{3}$ lb.; German, $58\frac{3}{4}$ lb. **7.** \$6,258,845.376. **8.** 6939 mi.

Page 280. — **9.** 2250 mi. **10.** 83,200. **11.** 4375 sacks; 1,225,000 lb.; 6250 bbl. **12.** \$4,706,100. **13.** \$15,300,000. **14.** \$2.10. **15.** 423.6 lb. **16.** \$6900. **17.** 177,085 visitors. **18.** 6,250,000 lb. to Great Britain; 1,700,000 lb. to U. S.

Page 281. — **19.** 330 trees. **20.** \$2862. **21.** 14,688 acres. **22.** 69 weeks. **23.** $\frac{9}{16}$ oz. **24.** $1\frac{1}{3}$ gal. **25.** \$102. **26.** \$4566.20. **27.** 24,500 tons. **28.** $12\frac{7}{8}$ ¢.

Page 282. — **29.** $17\frac{1}{2}$ tons. **30.** $234. **31.** 2400 pictures. **32.** $1191\frac{2}{3}$ mi. **33.** 12,600 lb. **34.** 150,000 locomotives; 3,000,000 freight cars. **35.** $837,500. **36.** .2872404 in. **37.** $\frac{3}{50}$. **38.** Elephant, 8750 lb.; camel, 1375 lb.

Page 283. — **2.** $17. **3.** $76.50. **4.** $14.17. **5.** $31.88. **6.** $106.25. **7.** $148.75.

Page 284. — **8.** $850. **9.** $2550. **10.** $8500. **11.** $7650. **12.** $425. **13.** $1275. **14.** 3.375. **15.** 600. **16.** 53,250. **17.** 2325. **18.** 58.75. **19.** 16,750. **20.** 49,500. **21.** 42,000. **22.** 506.25. **23.** 487.5. **24.** 20,900. **25.** 92,400. **26.** 52,800. **27.** 383,400. **28.** 1320. **30.** 180 yd. **31.** 160 yd. **32.** 144 yd. **33.** 96 yd. **34.** 80 yd. **35.** 48 yd. **36.** 188. **37.** 25.6. **38.** 6.4. **39.** 2.96. **40.** .392. **41.** .632. **42.** 1.712. **43.** 1.144. **44.** 19.2. **45.** 3.4. **46.** $144; $146.88; $141.12. **47.** $144; $142.56; $144.72. **48.** $138.24.

Page 287. — **1.** Bal. against Geo. Griffin, $1.19. **2.** Bal. in favor of Thomas Hinds & Co., $407.46. **3.** Bal. against Johnson & Mason, $275.90.

Page 294. — **2.** $405,312.50. **3.** $720. **4.** $21.60. **5.** $61.75. **6.** $550.20. **7.** $397.70. **8.** $503.70. **9.** $1158.30. **10.** $16,983. **11.** $378; $\frac{3}{4}$¢.

Page 295. — **12.** $160.88. **13.** $3703.22. **14.** $1095. **15.** $2531.25. **16.** $291. **17.** $831.79. **18.** $7020. **19.** $410; $1620. **20.** $\frac{3}{10}$. **21.** $56,700. **22.** $21.83. **23.** Neither gained nor lost. **24.** $28.67.

Page 298. — **2.** 36. **3.** 92. **4.** 26. **5.** 496. **6.** 12.5. **7.** $1\frac{1}{2}$. **8.** 108 books. **9.** 13. **10.** 26; 52.

Page 299. — **2.** $x=6$. **3.** $x=6$. **4.** $x=4$. **5.** $x=21$. **6.** $x=35$. **7.** $x=45$. **8.** $x=35$. **9.** $x=24$. **10.** $x=24$. **11.** $x=9$. **12.** $x=72$. **13.** $x=56$. **14.** 24. **15.** 400 pupils. **16.** 24 games. **17.** 640 persons. **18.** 80 ft. **19.** 36 min.

Page 302. — **3.** $x=2$. **4.** $x=11$. **5.** $x=5$. **6.** $x=9$. **7.** $x=7$. **8.** $x=2$. **9.** $x=6$. **10.** $x=15$. **11.** $x=2$. **12.** $x=7$. **13.** $x=1\frac{1}{2}$. **14.** $x=8$. **15.** $x=5$. **16.** $x=1$. **17.** $x=16$. **18.** $x=4$. **19.** $x=2$. **20.** $x=8$. **21.** $x=4\frac{4}{5}$. **22.** $x=9$. **23.** $x=3$. **24.** $x=11$. **25.** $x=12$. **26.** $x=13$. **27.** $x=4$.

Page 303. — **29.** $x=24$. **30.** $x=36$. **31.** $x=20$. **32.** $x=320$. **33.** $x=18$. **34.** $x=40$. **35.** $x=28$. **36.** $x=60$. **2.** 29, 87.

Page 304. — **3.** 40 clams, 120 clams. **4.** Steam, 40 cu. ft.; water, 80 cu. ft. **5.** 2. **6.** $9, $36. **7.** $10.50, $42. **8.** 18, 54. **9.** 13, 65. **10.** 45. **11.** 80. **12.** Jones, $25,000; Hollis, $75,000; Frye, $125,000. **13.** Jones, $260; Hollis, $780; Frye, $1300. **14.** $ .90 per ton. **15.** 1200 lb.; 3000 lb.

Page 305. — **16.** 162 ft. **17.** 400 gal.; 1600 gal. **18.** 250 violins; 500 violins. **19.** 4 16-candle power; 8 20-candle power. **21.** 20 words. **22.** 17 words. **23.** 28 words. **24.** 28 words. **25.** 37 words. **26.** 15 words. **27.** 21 words. **28.** 19 words. **29.** 14 words.

Page 306. — **30.** 8 min. **31.** 140 factories. **32.** 12,000 sq. yd., inside; 52,000 sq. yd., outside. **33.** 36. **34.** 128. **35.** 105. **36.** 133,000 miners. **37.** 320 lb. **38.** $2.80. **39.** 3300 ft. **40.** 42 mi. **41.** 78 vessels. **42.** 165 ft.

Page 307. — 44. \$16. 45. \$7250. 46. \$150. 47. 42. 48. 1725 passengers. 49. 800 steamers ; 225 sailing vessels. 50. 240 boys.

Page 308. — 52. $52\frac{1}{2}$, $31\frac{1}{2}$. 53. 60, 40. 54. 105 days. 55. 160 tons ; 40 tons ; 43.2 tons. 56. Length, 20 ft. ; width, 15 ft. 57. Ore, 200 car loads ; coke, 175 car loads ; limestone, 75 car loads.

Page 310. — 2. $x=4, y=1$. 3. $x=5, y=2$. 4. $x=8, y=2$. 5. $x=10, y=2$.

Page 311. — 6. $x=3, y=4$. 7. $x=4, y=3$. 8. $x=1, y=1$. 9. $x=5, y=2$. 10. $x=4, y=3$. 11. $x=1, y=2$. 12. $x=6, y=1$. 13. $x=4, y=5$.

Page 312. — 15. $x=2, y=1$. 16. $x=3, y=2$. 17. $x=4, y=3$. 18. $x=5, y=4$. 19. $x=3, y=\frac{1}{2}$. 20. $x=\frac{1}{2}, y=\frac{1}{3}$. 21. $x=6, y=4$. 22. $x=9, y=2$. 23. $x=8, y=5$. 24. $x=4, y=10$. 25. $x=7, y=13$. 26. $x=10, y=4$. 27. $x=4, y=15$.

Page 313. — 28. $x=12, y=8$. 29. $x=20, y=16$. 30. $x=12, y=18$. 31. $x=36, y=24$. 32. 11, 3. 33. 5, 8. 34. 16, 2. 35. Raspberries, 12 ¢ ; cherries, 10 ¢. 36. 60 3-grain capsules ; 160 2-grain capsules. 37. 600 @ 5 ¢ ; 250 @ 10 ¢. 38. 20 @ 12 ¢ ; 16 @ 10 ¢. 39. 26 1-dollar bills ; 12 2-dollar bills.

Page 314. — 40. Rate, 30 − 2. 41. Delaware, 75 ¢ ; Whitestone, 45 ¢. 42. 840 men ; 160 women. 43. 100 for adults ; 200 for children. 44. 18 in. ; 12 in. 45. 4 mo. ; 8 mo. 46. 84 tons ; 10 tons. 47. 1200 to grounds ; 400 to grand stand.

Page 315. — 1. 7920 in. 2. 369,500 lb. 3. 51 pk. 4. 1440 min. 5. 69.6 in. 6. 8658″. 7. 7 T. 420 lb. 8. 8 hr. 20 min. 9. $\frac{3}{16}$; $\frac{6}{25}$; $\frac{1}{50}$. 10. $\frac{6}{25}$; $\frac{13}{16}$.

Page 316. — 1. 301 pt. 12. $33\frac{1}{3}$ hr. 13. $7\frac{71}{125}$ T. 14. 3310 in. 15. 52 wk. 1 da. 6 hr. 16. 11° 15′. 17. 2 mi. 665 ft. 18. 12 T. 19. 6 T. 3 cwt. ; 2 T. 6 cwt. ; 24 lb. 5 oz. 20. 76 ft. ; 11 yd. 21. 21 ft. 5 in. ; 25 ft. $11\frac{1}{2}$ in. 22. 12° 18′. 23. 25° 31′ 27″. 24. 4° 8′ $7\frac{1}{2}$″ ; 12° 24′ $22\frac{1}{2}$″ ; 78 ft. 4 in. 25. 58 lb. 2 oz. ; 9 lb. 11 oz. 32. 62 gal. 1 qt. 1 pt. 33. 219 gross. 34. 7 bu. $3\frac{1}{2}$ pk. 35. 20 times. 36. 18,000. 37. 8 min. 24 sec.

Page 317. — 38. 67 A. 100 sq. rd. 39. 8223 lb. 7 oz. 40. 14 lb. 41. 24,000 jars. 42. \$750,000. 43. 20 bu. 44. \$912,109.25. 45. 2712 lb. 8 oz. 46. 14 oz. ; $2\frac{1}{4}$ oz. 47. 185 stamps. 48. 5.7 mi. 49. $2\frac{1}{8}$ min.

Page 318. — 50. 90 gal.

Page 319. — 1. \$2043.93. 2. \$120.45. 3. \$273.13. 4. \$347.40. 5. \$27.89. 6. \$202.48. 7. \$7.00. 8. \$9.61. 9. \$386. 10. \$2433.25 ; \$193.

Page 320. — 12. \$39.74. 13. \$24.94. 14. \$35.71. 15. \$60.24. 16. \$122.70. 17. \$323.24. 18. \$488.63. 19. \$609.98. 20. \$2145.83. 22. £205 9*s*. 9*d*. 23. £657 11*s*. 2*d*. 24. £1130 3*s*. 6*d*. 25. £9 18*s*. 4*d*. 26. £12 17*s*. 11*d*. 27. £18 10*s*. 3*d*. 28. £15 17*s*. 29. £17 6*s*. 10*d*. 30. £10 5*s*. 9*d*. 31. £25 3*s*. 2*d*. 32. £156 5*s*. 5*d*. 33. £66 19*s*. 2*d*. 34. £996 13*s*. 35. £742 18*s*. $11\frac{1}{4}$*d*. 36. £223 1*s*. 6*d*. ; £16 9*s*. 4*d*. ; £14 3*s*. $2\frac{1}{4}$*d*.

Page 321. — **37.** 7s. 11d. **38.** £15 12s.; $6.33. **39.** £1 2s. 1d. **40.** £4 17s.; $7.87–. **41.** $2.86–. **42.** 8 M. 32 pf. **43.** 33,694 fr. **44.** $19,902,211.72+. **45.** $1190. **46.** $2219.12. **47.** £17,207,148; $83,738,585.74+.

Page 324. — **2.** 23 min. 57 sec. after noon. **3.** 12 min. 11 sec. after noon. **4.** 17 min. 45 sec. after 11 A.M. **5.** 1 min. 47 sec. after 6 P.M. **6.** 17 min. 33 sec. after 5 P.M. **7.** 8 min. 12 sec. after 5 P.M. **8.** 48 min. 6 sec. after 5 P.M. **9.** 27 min. 45 sec. after 5 P.M. **10.** 22 min. 7 sec. after 6 P.M.

Page 325. — **12.** 41° 48′ 6″ W. **13.** 8° 3′ 6″ W. **14.** 21° 33′ 6″ W. **15.** 102° 33′ 6″ W. **16.** 108° 33′ 6″ W. **17.** 164° 48′ 6″ W. **18.** 143° 55′ 36″ W. **19.** 60° 36′ 51″ W. **20.** 137° 6′ 51″ W. **21.** 4 hr. 43 min. 4 sec. **22.** Chicago, 6 min. 48 sec. after midnight; Denver, 57 min. 23 sec. after 10 P.M. day before; San Fran., 47 min. 37 sec. after 9 P.M. day before. **23.** 4 hr. 53 min. 36 sec.; 1 hr. 34 min. 54 sec.

Page 326. — **24.** 16° 15′ W. **25.** Sat., 8° 45′ E.; Tues., 3° 15′ W.; 12° west. **27.** May 2, 59 min. 54 sec. after midnight. **28.** May 2, 3 min. 16 sec. after midnight. **29.** May 2, 29 min. 8 sec. after midnight. **30.** May 2, 35 min. 56 sec. after 2 A.M. **31.** May 1, 47 min. 17 sec. after 9 P.M. **32.** May 1, 9 min. 56 sec. after 6 P.M. **33.** 15 da. 18 hr. 8 min. 6 sec.

Page 328. — **1.** New York, 11:30 A.M.; New Orleans, 10:30 A.M.; San Antonio, 10:30 A.M.; Portland, 8:30 A.M. **2.** Salt Lake City, 11:15 A.M.; Portland, 1:15 P.M.; Manila, 2:15 A.M. (next day); Bombay, 11:15 P.M.; Berlin, 7:15 P.M.; London, 6:15 P.M.; Tokyo, 3:15 A.M. (next day). **3.** 7 A.M. **4.** Between 4:41 P.M. and 6:40 P.M., Apr. 30. **5.** 20 hr. 39 min.

Page 329. — **6.** 6 P.M. May 2. **7.** 5:15 P.M. **8.** 11:30 A.M. **9.** 7 min. 20 sec. **10.** Tokyo, 11 P.M. Jan. 2; St. P., 4 P.M. Jan. 2; London, 2 P.M. Jan. 2; N. Y., 9 A.M. Jan 2. **11.** 12 da. 4 hr. 3 min. **12.** 8:03 A.M. **13.** 50 min. 39 sec. after 9 A.M. **14.** In order: Dec. 31, '04, 11 P.M.; Dec. 31, '04, 10 P.M.; Dec. 31, '04, 9 P.M.; Jan. 1, '05, 1 P.M.; Jan. 1, '05, 2 P.M.; Jan. 1, '05, 3 P.M.; Jan. 1, '05, 10 A.M.; Jan. 1, '05, 6 A.M.; Jan 1, '05, 5 A.M.

Page 335. — **1.** 551 cu. cm. **2.** 96 steres. **3.** 1000 cu. cm. **4.** 1600 cu. m. **5.** $\frac{1}{60}$ mm. **6.** $175. **7.** 320,000,000 cu. m. **8.** 1,134,905 cu. cm.

Page 336. — **1.** 120 l. **2.** $144. **3.** $60. **4.** 17 Hl.

Page 337. — **5.** 225 Hl. **6.** 5040 cu. m.

Page 339. — **1.** 465.878 ft. **2.** 457.676 ft. **3.** 178.805 ft. **4.** 666.009 ft. **5.** 9.258 mi. **6.** 48.467 mi. **7.** 3305.005 mi. **8.** 9.055 in. **9.** .276 in. **10.** 59.055 ft.; 65.617 ft. **11.** 2.794 m. **12.** Portugal, 6000 sq. Km.; Spain, 3000 sq. Km.; Italy, 800 sq. Km. **13.** 12.5 M. T. **14.** 13,994.1 cu. m.

Page 340. — **15.** 1620 marks. **16.** $1526.25. **17.** 45¢. **18.** 640 g. **19.** 81,706.181 oz. **21.** .915. **22.** 13.598. **23.** .24. **24.** 21.2 Kg. **25.** 434 g. **26.** 68,342,600 lb. fresh codfish; 37,478,200 lb. dried codfish.

Page 341. — **27.** 336,000 l. **28.** 22.4 boxes. **29.** 28.65+ cu. cm. **30.** 255 francs. **31.** 11 ft. 10 in. **32.** 1000 francs. **33.** 156,600 M. T. **34.** Gt. Br., 70,547.2 T.; H., 35,273.6 T.; U. S., 22,046 T. **35.** $20.63. **36.** $1094.80.

Page 342. — **1.** 20,250 gal. **2.** 60,000 bu. **3.** 16 ft. **6.** 155° F.

Page 343. — **7.** $36\frac{2}{3}$° C. **8.** $39\frac{1}{5}$° F. **9.** Olive oil, – $1\frac{1}{9}$° C.; mercury, – $38\frac{8}{9}$° C. **10.** 6120 pieces. **11.** 10,260 lb. **12.** $39.90. **13.** $131.10. **14.** $173.75. **15.** 103 bunches. **16.** 20 ft. **17.** $15.29. **18.** $1149\frac{1}{2}$ ft.

Page 344. — **19.** 4620 laths. **20.** \$18.48. **21.** $9\frac{3}{5}$ days. **22.** \$101.64. **23.** $43\frac{1}{5}$ days. **24.** \$2678.40. **25.** \$51.16. **26.** 42 strips. **27.** \$7. **28.** \$5.46. **29.** \$4.20. **30.** \$1175.

Page 345. — **31.** $33\frac{1}{4}$ yd. **32.** 20,000 sq. yd. **33.** \$1160. **34.** \$126. **35.** 5′ 10″. **36.** \$161.12; \$.38. **37.** \$4860. **38.** \$10,401.60. **39.** \$24,921.60. **40.** \$21,410.12.

Page 346. — **41.** 20,250 bricks. **42.** \$151.88. **43.** \$243. **44.** 14.175 cu. yd. **45.** $56\frac{1}{4}$ T. **46.** \$120. **47.** \$1848. **48.** \$10.71. **49.** 20 cu. yd. **50.** 36 bbl. cement; 10 cu. yd. sand; 20 cu. yd. stone. **51.** \$116. **52.** 2025 lb. **53.** \$31.92+.

Page 347. — **54.** 156.25 lb.; 160.94− lb. **55.** 625 lb. **56.** Weight, 937.5 lb.; sp. gr., .75. **57.** 25.3 mi.; $37\frac{8}{75}$ ft. **58.** $28\frac{7}{40}$ mi.; 676.2 mi. **59.** 25.61 mi. **60.** 16 knots.

Page 348. — **1.** 390 acres. **2.** 10 da. 4 hr. **3.** \$321.75. **4.** 24 bu.; \$162. **5.** 9360 bu. **6.** $10\frac{2}{3}$ days.

Page 349. — **7.** \$140.40. **8.** 31,200 lb. **9.** 18¢. **10.** \$175.50. **11.** \$3198.25. **12.** \$453.96. **13.** .72. **14.** 4.54 bu. **15.** 1323 loaves. **16.** Great Britain, 795 bu.; France, 650 bu.; Germany, 485 bu.; Austria, 410 bu.; United States, 335 bu.; Russia, 225 bu. **17.** 550,000,000 bu.

Page 350. — **18.** \$288.60. **19.** \$11,934. **20.** \$1.90. **21.** \$720. **22.** \$2484. **23.** \$5.25. **24.** \$22,403.55. **25.** \$1849.20.

Page 351. — **26.** Sawyer, \$690; assistant, \$368. **27.** \$1242. **28.** \$1416.80. **29.** Man, \$2.25; boy, \$1.50. **30.** \$1058. **31.** \$6752.80. **32.** \$14,066.80. **33.** 17,480,000 ft. **34.** \$166.25; \$30,590. **35.** \$2104.25. **36.** \$387,182.

Page 352. — **37.** 5760 cu. yd. **38.** \$21,600. **39.** 900 days. **40.** \$850.50. **41.** 24,720 lb. **42.** \$74.25. **43.** 30 sec. **44.** Miner, \$2.53; laborer, \$1.26½. **45.** \$60.72. **46.** \$5.28.

Page 353. — **47.** \$7.23. **48.** \$53.49. **49.** \$128.40. **50.** \$390. **51.** \$261.60. **52.** \$.85. **53.** 315 L. T. **54.** \$306. **55.** \$1700. **56.** 60,242,460 T.

Page 354. — **1.** 2455.2 ft. **2.** 6780 gal. **3.** 13,993. **4.** 95,004. **5.** $114\frac{2}{7}$%. **6.** $87\frac{1}{2}$%. **7.** $16\frac{2}{3}$%. **8.** $33\frac{1}{3}$%. **9.** $62\frac{1}{2}$%. **10.** $16\frac{2}{3}$%. **11.** \$38,400. **12.** 8800. **13.** \$45. **14.** \$17.19. **15.** 83,200. **16.** 7240. **17.** 120. **18.** 292. **19.** 2680. **20.** 2480. **21.** \$60. **22.** \$18. **23.** $16\frac{2}{3}$%. **24.** $\frac{3}{10}$. **25.** 5.91. **26.** 20%. **27.** 50. **28.** $16\frac{2}{3}$%. **29.** 8938.3. **30.** \$5000. **31.** 1560 mi. **32.** 1229.76 lb. **33.** \$1642. **34.** 1006. **35.** \$14,401.20.

Page 355. — **36.** \$2800. **37.** \$26.60. **38.** Graphite, 59%; clay, 41%. **39.** 8052 ft. red fir; 2196 ft. pine; 610 ft. spruce; 610 ft. hemlock; 244 ft. cedar; 488 ft. other kinds. **40.** 25%. **41.** 280 oranges. **42.** 52%. **43.** 815,000 immigrants. **44.** 220,000,000 bu. **45.** 1,100,000,000 T.; 330,000,000 T.; 220,000,000 T.

Page 356. — **46.** Direct discount; \$33.37½. **47.** 26%. **48.** 50%. **49.** $23\frac{1}{3}$%. **50.** 31,200 sq. mi. **51.** 69%; 22%. **52.** 25,000,000 lb. from U. S.; 210,000,000 lb. from Canada. **53.** U. S., 2,875,000 T.; Eng., 900,000 T. **54.** \$2752; 34.4¢.

Page 357. — **55.** 50%. **56.** 25%. **57.** $2.40. **58.** $5.25. **59.** Lost; $100. **60.** In order: $4,051,500,000; $2,518,500,000; $2,409,000,000; $1,642,500,000; $1,533,000,000; $876,000,000; $766,500,000. **61.** Each, of Great Britain and Ireland in order: 62.2%; 59.5%; 40.5%; 37.8%; 21.6%; 18.9%. Each, of United States in order: 168.2%; 104.5%; 68.2%; 63.6%; 36.4%; 31.8%.

Page 360. — **2.** 6.72 mills. **3.** $84.84. **4.** $1\frac{3}{4}$ mills. **5.** $8\frac{3}{4}$ mills. **6.** $549.74. **7.** $54,000. **8.** $1,926,960. **9.** In order: $56,995,285.65; $3,699,980.18; $14,202,003.16; $1,975,889.69; $735,375.96.

Page 361. — **10.** $154.80. **11.** $1,191,677.51. **13.** $5.36. **14.** $6.19. **15.** $6.98. **16.** $122.41. **17.** $155.93. **18.** $1119.17. **19.** $906.54. **20.** $1305. **21.** $1115.34.

Page 362. — **1.** $144. **2.** $1650. **3.** $150. **4.** $535.50. **5.** $1286. **6.** $282. **7.** $3750. **8.** $2500.

Page 363. — **9.** $45. **10.** $330. **11.** $354. **12.** $1867.20. **13.** 75¢. **14.** $547.49. **15.** $1793.53; $1956.82.

Page 364. — **16.** $209.33. **17.** $29,127.14. **18.** $137.10. **19.** $1512.21. **20.** $4556.16. **21.** $4.56.

Page 365. — **1.** $24.50. **2.** $35. **3.** $12.50. **4.** $50. **5.** $1600. **6.** $3900. **7.** 40¢; $\frac{2}{5}$%. **8.** $500; $4500. **9.** Phœnix, $2000; Firemen's, $1200; Protective, $800. **10.** $42.

Page 366. — **11.** $444.99. **12.** $382.

Page 367. — **1.** $41.40. **2.** $147. **3.** $120.25. **4.** 36 premiums; $638.64.

Page 368. — **5.** $1764. **6.** $1585.02. **7.** $408. **8.** Lost; $1363.33. **9.** $454.44+. **10.** $630. **11.** $660. **12.** $38.25. **13.** $23,000.

Page 369. — **1.** 7200 ft. **2.** 132 rows. **3.** 690 T. **4.** $19\frac{1}{2}$ T. **5.** $1.90. **6.** $6552. **7.** $2106; $17.55. **8.** 1755 T. **9.** 234 T. **10.** $\frac{9}{10}$. **11.** 9%.

Page 370. — **12.** 180 lb.; 3510 lb. **13.** 117 T.; $8190. **14.** 585 T. **15.** 65¢. **16.** 237,500 lb. **17.** $17\frac{1}{2}$¢. **18.** 200 lb. **19.** 25 lb.; 5¢. **20.** $12\frac{1}{2}$¢. **21.** $5937.50. **22.** La., 1.44 T.; Cuba, 4.08 T.; Hawaii, 6.6 T.

Page 371. — **23.** $72. **24.** $17.05; $409.20. **25.** $744; $31. **26.** 120 loads. **27.** 276 T. **28.** $1242. **29.** $498.

Page 372. — **30.** 80%. **31.** $2859.36. **32.** $1.40. **33.** 431.8 T. **34.** 508 T. **35.** $24,150. **36.** $6118. **37.** $472.50. **38.** $1252.50. **39.** $1737. **40.** $6355.55. **41.** 4,050,000 T. **42.** 972,000 T. **43.** 1,750,000 T. **44.** 197,200 T. **45.** 57%.

Page 373. — **46.** $31\frac{1}{4}$ cu. ft. **47.** 10 compressed bales. **48.** 475 lb. **49.** India, 80%; Brazil, 46%; Peru, $36\frac{2}{3}$%; Egypt, 147%.

Page 374. — **50.** 32%; 68%. **51.** 9 bales. **52.** 45%. **53.** 12 days. **54.** $7\frac{1}{2}$ sec. **55.** 1 hr. **56.** 8640 lb. **57.** 45¢. **58.** 192 bu.; 2880 lb. **59.** 6 bales. **60.** $218.04. **61.** $54.51.

Page 375. — **62.** $67.19. **63.** 50¢. **64.** $318.75. **65.** 27,500 lb. **66.** 68,500 lb. **67.** Egypt, 1,200,000 bales; Brazil, 300,000 bales; East Indies, 3,000,000 bales; U. S., 13,500,000 bales. **68.** In order: 20%; $28\frac{1}{3}$%; 24%.

Page 377. — **2.** $25. **3.** $75. **4.** $57.75. **5.** $450. **6.** $1500. **7.** $18.125. **9.** $225. **10.** $350. **11.** $161.25. **12.** $511.44.

Page 378. — **14.** $377.50; $5377.50. **15.** $379.17; $5379.17. **16.** $284.37½; $2784.37½. **17.** $421.27; $4071.27. **18.** $965; $8465. **19.** $129.25; $1004.50. **20.** $23.43; $193.18. **21.** $112.50; $15,112.50. **22.** $2500; $502,500. **23.** $28.56; $3292.74. **24.** $97.40; $4967.61. **25.** $43.75; $3043.75. **26.** $414.17; $5414.17. **27.** $157.61; $623.91. **28.** $18.98; $193.63. **29.** $618.06; $50,618.06. **30.** $4984.37½; $129,984.37½. **31.** $50,177.78; $370,177.78.

Page 379. — **1.** $2.25. **2.** $11.25. **3.** $5. **4.** $8.66. **5.** $9. **6.** $50. **7.** $750. **8.** $1562.50.

Page 380. — **1.** $50. **2.** $41.25. **3.** $16.50. **4.** $100. **5.** $45. **6.** $111.99. **7.** $88.20. **8.** $32. **9.** $.12. **10.** $.30. **11.** $35. **12.** $133.33. **13.** $1208.33. **14.** $7333.33.

Page 381. — **2.** $18.84. **3.** $12.62. **4.** $164.37. **5.** $195.13. **6.** $260.90. **7.** $269.60. **8.** $94.75. **9.** $424.80. **10.** $1571.20. **11.** $21.95. **12.** $15,700. **13.** $9454.67. **14.** $108.55. **15.** $400.43. **16.** $18,262.50. **17.** $15,000.

Page 382. — **1.** $2.92; $252.92. **2.** $2.08; $502.08. **3.** $8; $408. **4.** $3.33; $503.33. **5.** $.84; $160.84. **6.** $2.84; $777.84. **7.** $2.10; $842.10. **8.** $.84; $450.84. **9.** $.83; $750.83. **10.** $3; $803. **11.** $3.12½; $903.12½. **12.** $7.50; $507.50. **13.** $861.11; $5861.11. **14.** $848; $4848. **15.** $717.08; $5717.08. **16.** $135.10; $1535.10. **17.** $50.83; $5050.83; $59.31; $5059.31. **18.** $67.71; $7567.71; $74.48; $7574.48. **19.** $121.19; $2561.19. **20.** $256.93; $3941.93. **21.** $2.47; $77.97. **22.** $344.71; $1169.96. **23.** $933.33; $25,933.33.

Page 383. — **2.** $6. **3.** $4.44. **4.** $9.37½. **5.** $36.37½. **6.** $8.89. **7.** $72. **8.** $23.94. **9.** $28.12½. **10.** $65.10. **11.** $30. **12.** $42. **13.** $59.80. **14.** $68.85. **15.** $120. **16.** $48.82½. **17.** $1.42. **18.** $.32. **19.** $1.50.

Page 385. — **2.** $5. **3.** $7.47. **4.** $8.25. **5.** $5. **6.** $23.20. **7.** $36.25. **8.** $92.94. **9.** $30.84. **10.** $237.68. **11.** $5.33. **12.** $6.78. **13.** $5.56. **14.** $222.78. **15.** $1080.04. **16.** $751.34. **17.** $1443.04. **18.** $15.19.

Page 386. — **1.** $67.62; $45.08; $50.71½; $33.81; $56.35. **2.** $40; $26.67; $30; $20; $33.33. **3.** $1.67; $1.11; $1.25; $.83; $1.39. **4.** $4.17; $2.78; $3.12½; $2.08; $3.47. **5.** $8.33; $5.56; $6.25; $4.17; $6.94. **6.** $25; $16.67; $18.75; $12.50; $20.83. **7.** $50; $33.33; $37.50; $25; $41.67. **8.** $75; $50; $56.25; $37.50; $62.50. **9.** $100; $66.67; $75; $50; $83.33. **10.** $104.17; $69.44; $78.12½; $52.08; $86.81. **11.** $.47; $.31; $.35; $.23; $.39. **12.** $3.54; $2.36; $2.65; $1.77; $2.95. **13.** $.79; $.53; $.59; $.40; $.66. **14.** $6.04; $4.03; $4.53; $3.02; $5.03. **15.** $11.66; $7.77; $8.74; $5.83; $9.72. **16.** $10.14; $6.76; $7.60; $5.07; $8.45. **17.** $7.75; $5.17; $5.81; $3.88; $6.46. **18.** $71.50; $47.67; $53.62½; $35.75; $59.58. **19.** $186.60; $124.40; $139.95; $93.30; $155.50. **20.** $72; $48; $54; $36; $60. **21.** $75; $50; $56.25; $37.50; $62.50. **22.** $2000; $1333.33; $1500; $1000; $1666.67. **23.** $25,000; $16,666.67; $18,750; $12,500; $20,833.33. **24.** $66,000; $44,000; $49,500; $33,000; $55,000. **25.** $2750. **26.** $2250. **27.** $3.28. **28.** $3200 better off.

Page 387. — **1.** $2.96. **2.** $2.71. **3.** $6.16. **4.** $2.37. **5.** $49.32. **6.** $77.85. **7.** $211.07. **8.** $178.75. **9.** $3.33. **10.** $8.90. **11.** $23.14. **12.** $14.44. **13.** $150.41. **14.** $151.23. **15.** $118.99. **16.** $99.37. **17.** $1.05. **18.** $522.26.

Page 388. — **2.** 5%. **3.** 6%. **4.** 4%. **5.** 4%. **6.** 6%.

Page 389. — **7.** 5%. **8.** 6%. **9.** 4%. **10.** $4\frac{1}{2}$%. **11.** $3\frac{1}{3}$%. **12.** $50; 5%. **13.** 5%. **14.** $418.50; $7\frac{1}{2}$%. **15.** $32\frac{1}{2}$%.

Page 390. — **2.** 8 yr. 4 mo. **3.** 5 yr. **4.** 2 yr. 9 mo. 10 da. **5.** 3 yr. 2 mo. 3 da. **6.** 13 yr. 4 mo. **7.** 4 yr. 1 mo. **8.** 25 yr.; 20 yr.; 50 yr.; $16\frac{2}{3}$ yr.; $22\frac{2}{9}$ yr. **9.** 40 yr.
2. $2500. **3.** $12,500. **4.** $12,500. **5.** $20,000. **6.** $50,000. **7.** $40,000.

Page 391. — **8.** $1875. **9.** $7500. **10.** $4000. **11.** $5000. **12.** $2400. **13.** $120,000. **14.** $75,000. **15.** $100,000.

Page 392. — **2.** $792.08. **3.** $869.57. **4.** $121.95. **5.** $1600. **6.** $4761.90. **7.** $4761.90.

Page 393. — **2.** $7174.64. **3.** $6910.02. **4.** $10,475.50. **5.** $10,736.20. **6.** $28,171. **7.** $40,160.22.

Page 395. — **2.** $441; $41. **3.** $1124.86; $124.86. **4.** $1950.53; $450.53. **5.** $5627.54; $627.54. **6.** $5632.46; $632.46. **7.** $8548.78; $548.78. **9.** $1628.90. **10.** 9948.94\frac{1}{2}$. **11.** $5511.61. **12.** $1842.57. **13.** $5529.02. **14.** $7074.13. **15.** $10,826.36. **16.** $259.80. **17.** $138.74. **18.** $8023.53. **19.** $7732.19.

Page 397. — **1.** 236 pt. **2.** 140,809 oz. **3.** 68,880 in. **4.** 35,307,396 sq. in. **5.** 20° 21′ 26″. **6.** 2 da. 9 hr. 58 min. 58 sec. **7.** 21 cu. ft. 606 cu. in. **8.** 8 lb. 5 oz. 8 pwt. 10 gr. **9.** 9 days. **10.** $3093\frac{3}{4}$ lb. **11.** 100,800 gal. **12.** 1,800,000 bu. **13.** 1,242,718.447− cu. ft. **14.** 835.576 gr.

Page 398. — **16.** 25° F.; $13\frac{8}{9}$° C. **17.** $.50. **18.** $\frac{1}{4}$ gal. **19.** 847 shingles; 47 bunches. **20.** $55.42+. **21.** 45%. **22.** 4,500,000 T. **23.** $200; $900.

Page 399. — **24.** 16%. **25.** $66\frac{2}{3}$%. **26.** $248. **27.** $216. **28.** $3.60 less. **29.** $18,000. **30.** $7600; $\frac{1}{4}$%. **31.** $440; $\frac{4}{9}$; $\frac{5}{9}$; $3150; $5250. **32.** $9392.

Page 400. — **33.** $13,048,000. **34.** $452.43. **35.** $7.54. **36.** 20¢. **37.** $603. **38.** $450. **39.** $510.30. **40.** $186. **41.** $178.92.

Page 401. — **42.** $2.01. **43.** $243.88. **44.** $48.24. **45.** $101.84. **46.** 11.811 ft. **47.** 1118.466 mi. **48.** 26.13 mi. **49.** 12 min. 49 sec. past 6 A.M. **50.** 79° 52′ 50″ W. **51.** $10; $11.67; $5.83; $8.33. **52.** $52.50; $61.25; 30.62\frac{1}{2}$; $43.75. **53.** $11.25; 13.12\frac{1}{2}$; $6.56; 9.37\frac{1}{2}$. **54.** $120; $140; $70; $100. **55.** $5.93; $6.92; $3.46; $4.94. **56.** $453.96; $529.62; $264.81; $378.30. **57.** $753.01; $878.51; $439.26; $627.51. **58.** $100; $116.67; $58.33; $83.33. **59.** $135. **60.** $12,090; $12,250.20; $12,255.55. **61.** $2260; 40 yr.

Page 402. — **62.** $3\frac{1}{3}$%. **63.** Mr. White's by $30.19; Mr. Brown's by $22.22.
1. $x = 12$. **2.** $x = 35$. **3.** $x = 4$. **4.** $x = 10$. **5.** $x = 4$. **6.** $x = 10$. **7.** $x = 3$, $y = 2$. **8.** $x = 1$, $y = 2$. **9.** $x = 8$, $y = 16$. **10.** $x = 15$, $y = 10$. **11.** 22 shots. **12.** Shell, 1000 lb.; powder, 265 lb. **13.** 6 hr. 25 min. **14.** $30.

Page 403. — **15.** 135 ft. **16.** Copper, 4250 lb.; tin, 500 lb.; zinc, 250 lb. **17.** $9,028,500. **18.** Shingles, 30¢; laths, 25¢. **19.** $6; $2.50. **20.** 1200 tons; 4200 tons. **21.** 14¢ per basket; 16¢ per box. **22.** "Clear," 121 days; "cloudy," 127 days; "partly cloudy," 117 days. **23.** 85,014,000.

Page 410. — **2.** $529.92. **3.** $258.68. **4.** $1246.56. **5.** $3798.19. **6.** $546.86. **7.** $34.44.

Page 411. — **1.** $900.65. **2.** $192.51. **3.** $567.31. **4.** $76.90. **5.** $1274.52; $1276.39.

Page 414. — 1. $406.98. 2. $365.99.

Page 415. — 3. $3330.92. 4. $715.07. 5. Check for $1757.50. 6. Total deposit, $1849.13.

Page 417. — 2. May 3; 57 da.; $47.50; $4952.50.

Page 418. — 3. Sept. 26; 30 da.; $25; $4975. 4. May 16; 15 da.; $20.83; $9979.17. 5. Feb. 16; 89 da.; $111.25; $7388.75. 6. Aug. 7; 10 da.; $25; $17,975. 7. Jan. 28; 120 da.; $74.67; $3125.33. 8. Dec. 11; 39 da.; $5.51; $842.89. 9. Aug. 2; 28 da.; $14.06; $2998.44. 10. Feb. 11; 40 da.; $335; $59,965. 11. May 2; 90 da.; $145.65; $8177.35. 12. Apr. 1; 31 da.; $86.97; $25,163.03. 13. May 1; 111 da.; $75.48; $4004.52. 14. June 6; 60 da.; $375; $74,625. 15. Oct. 17; 46 da.; $37.35; $7270.65. 16. 10 da. after date; 10 da.; $.97; $999.03.

Page 419. — 17. $42,851.89. 18. $10,000. 19. $3000. 20. $20,000.

Page 421. — 1. $487.59. 2. $482.68. 3. $272.56. 4. $268.97. 5. $269.63. 6. $638.17. 7. $221.90. 8. $1663.61.

Page 427. — 1. $750.75. 2. $125,256.44. 3. $3495.62. 4. $1988. 6. $1862.70. 7. Draft for bal., $757.50. 8. $1024.31.

Page 428. — 9. $4001.65. 10. $862.84. 11. $938.83. 12. $2571.01. 13. $8680.50.

Page 431. — 1. $51.27−. 2. $59.06+. 3. $365.47+. 4. $5803.80. 5. $7384.05. 6. $465.81. 7. $1090.61. 8. $1930.50. 9. $5941.41. 10. $235.59. 11. $1070.83. 12. $5201.66. 13. $385.36. 14. $24.65. 15. $4020. 16. £142 10*d.*; 4612.50 fr. (French); 1650 M.; 733.9 crowns; 2562.50 fr. (Belgian); 490.2 florins; 3075 lire; $500.

Page 432. — 17. $72,975. 18. $7.50. 19. $6047.05. 20. $120,956.25. 21. $1945.63. 22. $400. 23. $700. 24. Face, 187,290 fr.

Page 437. — 2. $4368.75. 3. $25,718.75. 4. $10,650. 5. $38,500. 6. $68,812.50. 7. $32,000. 8. $212.50. 9. $125; $250. 10. $27,775.

Page 438. — 12. 50 shares. 13. 80 shares. 14. 440 shares. 16. $531.25. 17. Increased, 4.96\frac{1}{2}$.

Page 439. — 18. $536.25. 20. $39,900. 21. $10,550. 22. $20,000. 23. 5%. 24. 8%. 25. $80 per share. 26. $120 per share.

Page 440. — 27. 37\frac{1}{2}$ per share. 28. 2%. 29. $80. 30. $1000. 31. $252. 32. $81.25. 33. $33,406.25. 34. 6.42%. 35. $4\frac{684}{825}$%. 36. $100.

Page 442. — 1. 2. 2. $1\frac{1}{2}$. 3. $2\frac{1}{2}$. 4. $\frac{7}{8}$. 5. $\frac{1}{25}$. 6. $\frac{5}{8}$. 7. $2\frac{2}{9}$. 8. $8\frac{5}{17}$. 9. $\frac{1}{3}$. 10. 3. 11. 4. 12. 16. 13. 1 : 5; 2 : 5; 2 : 25; 20%; 40%; 8%.

Page 443. — 14. 1700 : 10,000; 1330 : 10,000; 1070 : 10,000.

Page 444. — 3. $x = 2$. 4. $x = 14$. 5. $x = 30$. 6. $x = 5$. 7. $x = 3$. 8. $x = 8$. 9. $x = \frac{2}{3}$. 10. $x = 1\frac{1}{3}$. 11. $x = \frac{3}{4}$. 12. $x = 2\frac{1}{12}$. 13. $x = 2$. 14. $x = 9$. 15. 8 tons. 16. $2.70. 17. 40 bu. 18. 15 days. 19. 20 ft.

Page 445. — 2. $1833.34. 3. 3432 bu. 4. 10 sec. 5. 75¢. 6. 5 hr. 12 min. 7. $1.24. 8. 25,000 blocks. 9. 6300 photographs.

Page 446. — 11. 12 days. 12. 24 men. 13. 16 days. 14. 3 extra men. 15. 8 men. 16. $44. 17. 57$\frac{1}{7}$ mi.

Page 447. — 18. 387$\frac{1}{2}$ doz. 19. $8400. 20. $511.50. 21. 45 days. 22. 5 cars.

Page 448. — **2.** $6000; $8000; $10,000. **3.** $6000; $10,000; $20,000. **4.** $14,495,000; $2,230,000. **5.** Bread, 750 g.; meat, 375 g.; rice, 125 g.; salt, 25 g.; coffee, 25 g. **6.** 1st, $960; 2d, $1200. **7.** In order: $48,000,000; $24,000,000; $6,000,000. **8.** In order: $675; $337.50; $540; $405; $1080; $810; $945; $1620.

Page 449. — **2.** A, $7000; B, $4200. **3.** In order: $825; $1125; $450. **4.** In order: $9600; $4200; $3000. **5.** In order: $2000; $600; $1300; $700.

Page 450. — **6.** $3850; 1st, $1823.68; 2d, $2026.32. **7.** A, $1600; $10,600; B, $1600; $10,600; C, $1600; $10,600; D, $2880; $19,080. **9.** A, $1500; B, $1575. **10.** A, $2640; B, $1760; C, $1100. **11.** A, $14,400; B, $9600; C, $17,400.

Page 452. — **1.** 400. **2.** 625. **3.** 2025. **4.** 2704. **5.** 1728. **6.** 9261. **7.** 3375. **8.** 14,641. **9.** $\frac{343}{512}$. **10.** $\frac{125}{1728}$. **11.** $\frac{225}{784}$. **12.** $\frac{625}{1024}$. **13.** 2.25. **14.** 56.25. **15.** 1.5625. **16.** .012321. **17.** 441 sq. in. **18.** 289 sq. yd. **19.** 1024 sq. rd. **20.** 1998.09 sq. ft. **21.** 2540.16 sq. ft. **22.** .0361 sq. ft. **23.** .0625 sq. ft. **24.** 57.1536 sq. ft. **25.** .708964 sq. mi. **26.** 1331 cu. in. **27.** 15.625 cu. in. **28.** 10,648 cu. cm. **29.** 3.375 cu. m. **30.** 39.304 cu. ft. **31.** 1953.125 cu. Dm. **32.** 257.28 ft.; 3618 ft.

Page 455. — **2.** 27. **3.** 28. **4.** 8. **5.** 9. **6.** 32. **7.** 45. **8.** 15. **9.** 4. **10.** 108. **11.** 36. **12.** 12. **13.** 5. **14.** 3. **15.** 7. **16.** 2. **17.** 5. **18.** 18 in. **19.** 20 in.

Page 457. — **3.** 53. **4.** 51. **5.** 82. **6.** 44. **7.** 91. **8.** 72. **9.** 34. **10.** 83. **11.** 33. **12.** 94. **13.** 61. **14.** 84. **15.** 46. **16.** 95. **17.** 76. **18.** 57. **19.** 49. **20.** 98. **21.** 79. **22.** 99. **23.** 85 in. **24.** 38 in. **25.** 47 in. **26.** 29 ft. **27.** 65 cm. **28.** 68 cm. **29.** 88 m. **30.** 69 Dm. **31.** 35 Hm. **32.** 176 rd. **33.** 9216 sq. ft.; 96 ft. **34.** 75 men.

Page 459. — **2.** 123. **3.** 156. **4.** 117. **5.** 176. **6.** 211. **7.** 30.5. **8.** 25.6. **9.** 16.8. **10.** 3.32. **11.** 3.82. **12.** .745. **13.** .796. **14.** .997. **15.** .0822. **16.** .0876. **17.** $\frac{144}{175}$. **18.** $\frac{161}{225}$. **19.** $5\frac{17}{25}$. **20.** $295\frac{29}{32}$. **24.** 2.828+. **25.** 2.449+. **26.** 2.646−. **27.** .577+. **28.** .707+. **29.** .447+. **30.** .316+. **31.** 1.581+. **32.** 1.897+. **33.** .886+. **34.** .642+. **35.** .860+. **37.** 255.6+ ft. **38.** 109.5+ ft. **39.** 612.4− ft.

Page 460. — **40.** 229.8− ft. **41.** 80 rd. long; 40 rd. wide. **42.** 240 rd. long; 60 rd. wide. **43.** 120 rd.
2. 13 in.

Page 461. — **3.** In order: $x = 17$; $x = 12$; $x = 51$; $x = 24$. **4.** 1.414+ in. **5.** 72.25 sq. in. **6.** As 2 : 1. **9.** 39.55− ft. **10.** 150 knots. **11.** 339.41+ rd.

Page 463. — **1.** 87 sq. in. **2.** 8 A. **3.** 1800 sq. m. **4.** 5 sq. Km. **5.** $44\frac{4}{9}$ sq. ft. **6.** 104 sq. rd. **7.** 6 in. **8.** 144 sq. ft. **9.** $2\frac{1}{4}$ A. **10.** 572 sq. m. **11.** 208 sq. m. **12.** $10\frac{1}{2}$ ft. **13.** 21 in.

Page 464. — **1.** 28 sq. in. **2.** 80 sq. cm. **3.** 35 sq. ft.; 70 board feet. **4.** 8 in. **5.** 60 A. **6.** 40 A. 128 sq. rd. **7.** 39 A. 32 sq. rd. **8.** 48 A. $23\frac{1}{8}$ sq. rd. **9.** 21 A. $61\frac{7}{8}$ sq. rd. **10.** 20 A. 75 sq. rd. **11.** 10 A. **12.** 240 A. **13.** 329.85 rd.

Page 466. — **1.** 23,040 A.; 640 A.; 40 A. **2.** 160 A. **3.** 80 A. **4.** 40 A. **5.** 40 A. **6.** 400 A. **7.** $2040. **8.** 320 rd. **9.** $4500.

Page 468. — **1.** 93.6 sq. in. **2.** 8 in.; 256 sq. in. **3.** 390.96 sq. in. **4.** 6 in.

Page 469. — **1.** 110 in. **2.** 242 ft. **3.** 198 yd. **4.** 154 rd. **5.** 264 m. **6.** 176 cm. **7.** 396 m. **8.** 528 mm. **9.** 176 ft. **10.** 264 ft. **11.** 572 rd. **12.** 74.8 m. **13.** 83.6 Hm. **14.** 36.96 cm. **15.** 154 ft. **16.** 228.8 in. **17.** 28 yd. **18.** $17\frac{1}{2}$ ft. **19.** 3.85 in. **20.** 1.19 cm. **21.** 8.05 m. **22.** 1.54 Dm. **23.** 315 rd. **24.** $206\frac{1}{2}$ yd. **25.** 37.6992 ft. **26.** 62.832 m. **27.** 48.6948 yd. **28.** 71.4714 rd. **29.** 1100 ft. **30.** 22 ft.

Page 470. — **1.** 616 sq. in. **2.** 3850 sq. ft. **3.** 24.64 sq. rd. **4.** 55.44 sq. m. **5.** 124.74 sq. cm. **6.** 2.6026 sq. dm. **7.** .0154 sq. ft. **8.** 346.5 sq. m. **9.** 5544 sq. ft. **10.** 346.5 sq. ft. **11.** 38.5 sq. ft. **12.** $962\frac{1}{2}$ sq. ft. **13.** 5.05 rd.

Page 473. — **1.** $31\frac{3}{7}$ sq. ft. **2.** 24 sq. cm. **3.** 60 sq. ft. **4.** $89\frac{4}{7}$ sq. ft.

Page 474. — **1.** 60 sq. ft. **2.** $235\frac{5}{7}$ sq. cm. **3.** $628\frac{4}{7}$ sq. ft. **4.** $64. **5.** $89\frac{4}{7}$ sq. ft. **6.** 400 sq. ft. **7.** $75\frac{3}{7}$ sq. ft.

Page 475. — **1.** $28\frac{2}{7}$ sq. in. **2.** $1810\frac{2}{7}$ sq. cm. **3.** $201\frac{1}{7}$ sq. in. **4.** $2.06.

Page 476. — **1.** 75 cu. in. **2.** $1005\frac{5}{7}$ cu. dm. **3.** 460.8 bu. **4.** 2310 gal. **5.** $1360.80.

Page 477. — **1.** $84\frac{6}{7}$ cu. ft. **2.** $666\frac{2}{3}$ cu. m. **3.** $12{,}445\frac{5}{7}$ lb. **4.** 13,860 cu. ft.

Page 478. — **5.** 7296 lb. **6.** 89,042,560 cu. ft.
1. $14\frac{1}{7}$ cu. ft. **2.** 4851 cu. cm. **3.** $179\frac{2}{3}$ cu. in. **4.** $1437\frac{1}{3}$ cu. in.; 51.9– lb. **5.** 29.505+ lb.

Page 480. — **1.** 6 in. **2.** 20 sq. in. **3.** 10 in. **4.** 1 : 9. **5.** 9 sq. ft.; 36 sq. ft. **6.** As 1 : 16. **7.** 5 cm., 10 cm., $12\frac{1}{2}$ cm. **8.** 4. **9.** 20 ft. **10.** 150 ft.

Page 481. — **12.** 1000 ft.

Page 482. — **1.** 1 : 2; 972 cu. ft.; 8; 7776 cu. ft. **2.** 1 : 3; by multiplying by 27; 1st, $134\frac{2}{21}$ cu. ft.; 2d, $3620\frac{4}{7}$ cu. ft. **3.** 240 cu. in. **4.** 12 cu. ft. **5.** 6 ft. **6.** $3\frac{3}{8}$ times. **7.** 270 lb. **8.** 91 cu. dm. **9.** 109,725 gal.

Page 483. — **1.** $6412.50. **2.** 8 %. **3.** 1088 doz. pairs. **4.** $832\frac{1}{2}$ lb. **5.** $1704.96. **6.** Second; $29.20. **7.** $231.14. **8.** 320 doz. pairs.

Page 484. — **9.** 8640 doz.; 8000 doz. **10.** 4 or $4\frac{1}{2}$, $11\frac{1}{4}$ doz.; 5 or $5\frac{1}{2}$, $9\frac{3}{8}$ doz.; 6 or $6\frac{1}{2}$, $8\frac{1}{2}$ doz. **11.** 56 lb. **12.** $1.40. **13.** $7\frac{1}{3}$ doz.; 18 ¢ per doz.; $1.32. **14.** $429. **15.** $630. **16.** 4's, 7472 doz.; $4\frac{1}{2}$'s, 12,160 doz.; 5's, 17,648 doz.; $5\frac{1}{2}$'s, 16,208 doz.; 6's, 11,688 doz.; $6\frac{1}{2}$'s, 5632 doz.; white, 20,784 doz.; black, 36,408 doz.; colored, 13,616 doz.; total, 70,808 doz.

Page 485. — **17.** 14,062 lb. **18.** 14,240 lb. **19.** $17,800. **20.** 37,938 lb. **21.** $48,370.95. **22.** $66,170.95. **23.** $26,907.04; $93,077.99. **24.** $108,426.80. **25.** $127,808.44. **26.** $265.53. **27.** $19,647.17; 18.1 %.

Page 486. — **28.** 24 %. **29.** $1440. **30.** 22.5 %. **31.** $8\frac{1}{2}$ M.; 85 pf. **32.** 4*s.* 2*d.* **33.** Am.; $4.136–. **34.** Am.; $.486+; Eng., $\frac{14}{31}$; Am., $\frac{1}{3}$. **35.** $226.70–. **36.** $86.40. **37.** 108 pencils.

Page 487. — **38.** 239,580 sq. ft. **39.** 69,750 cords; 62,000 T. **40.** 312,500 sq. ft. **41.** 4000 trees. **42.** In order: 956,924,400 lb.; 202,236,600 lb.; 73,989,000 lb. **43.** $126,663,622.42. **44.** 45 %. **45.** $937,500. **46.** $63.40. **47.** $6,299,102.

Page 488.—**48.** $ 171. **49.** $ 2940; $5\frac{1}{4}$ ¢. **50.** $ 30.60. **51.** $ 17.28; 30 %; 12 ¢. **52.** $ 22.40; $ 40.32; $ 17.92; 80 %; $\frac{1}{7}$. **53.** 720 tons. **54.** 166,100 images. **55.** 190 beats per sec.; 330 beats per sec.

Page 489.—**56.** 3000 m.; 30 cm. **57.** $ 400. **58.** $ 2.37+. **59.** $ 7044.26−. **60.** In order: 136 ft. $\frac{1}{4}$ in.; 65 ft. $4\frac{1}{2}$ in.; 23 ft. $7\frac{1}{2}$ in.; 169 lb.; 9 yd. 1 ft. 1 in.; 358.75 ft.; $7\frac{2}{3}$ ft. less.

Page 490.—**61.** 55 gal. **62.** $ 3.81−. **63.** 1st class, 10−pf. or 2.3− ¢; 2d class, 6+ pf. or 1.5+ ¢; 3d class, 4+ pf. or 1− ¢. **64.** 4 M. 16− pf. **65.** 8816.26 mi. **66.** 1950 mi. **67.** 33 min. $6\frac{3}{4}$ sec. after 7 P.M. **68.** 70.104 m. **69.** $ 410.13. **70.** In order: 131 ft. 2.8 in.; 2 ft. .803+ in.; 22 ft. 11.59 in. **71.** $82\frac{2}{9}$° F.

Page 491.—**72.** £ $226,416\frac{2}{3}$. **73.** £ 35,750. **74.** 3672.8 mi. **75.** $ 61,565.28. **76.** 569,477.252− Kg. **77.** $ 130. **78.** $ 228; $28\frac{1}{2}$ %. **79.** 9.2 T.; $ .025.

Page 492.—**80.** 43.19 %. **81.** $ 810. **82.** $ 5.82. **83.** $ 1000. **84.** 1 %. **85.** $ 157.50. **86.** $ 4293.75. **87.** $ 119.77. **88.** $ 32,928.

Page 493.—**89.** $ 252.45. **90.** 19.04 mills. **91.** $ 1904. **92.** 3640 M.; $ 866.32. **93.** $ 3263.60. **94.** 35 : 44. **95.** $ 1339.535−. **96.** $2\frac{1}{3}$ oz. **97.** 138,686.77+ lb. **98.** $ 160.442+. **99.** $412\frac{1}{2}$ gr.

Page 494.—**100.** 1 hr. 2 min. **101.** $ 1.75. **102.** $ $1.78\frac{1}{3}$. **103.** $ 16,921,479.76+. **104.** $ 1050. **105.** $ 27,525. **106.** $ 1014.68. **107.** $ $3,986,678.57\frac{1}{7}$. **108.** 72 ft. **109.** 405 ft. **110.** $32,812\frac{1}{2}$ cu. ft.

Page 495.—**2.** 24. **3.** 24. **4.** 30. **5.** 11. **6.** 42. **7.** 70. **8.** 45. **9.** 165. **10.** 32. **11.** 8. **12.** 8. **13.** 12. **14.** 15. **15.** 72. **16.** 42. **17.** 132 boards.

Page 496.—**2.** 26. **3.** 23. **4.** 29. **5.** 31. **6.** 39. **7.** 51. **8.** 76. **9.** 37. **10.** 17. **11.** 73. **12.** 43. **13.** 19.

Page 497.—**2.** 240. **3.** 756. **4.** 2520. **5.** 5040. **6.** 2700. **7.** 11,088. **8.** 1680. **9.** 263,340. **10.** 27,720 **11.** 28,512.

Page 498.—**12.** 60 qt. **13.** 270 in. **14.** $7\frac{3}{11}$ mi.

2. 16,638. **3.** 3828. **4.** 7225. **5.** 9522. **6.** 33,228. **7.** 246,963. **8.** 9690. **9.** 5320. **10.** 156,240. **11.** 12,194. **12.** 7770. **13.** 35,245. **14.** 218,736. **15.** 48,450. **16.** 16,544.

Page 504.—**5.** 38. **6.** 56. **7.** 74. **8.** 95. **9.** 145. **10.** 352. **11.** 507. **12.** 1231. **13.** 1596. **14.** 4567. **15.** 7894. **16.** 23.7. **17.** 3.04. **18.** .055. **19.** .0125. **20.** 1.2599+. **21.** 1.81712+. **22.** .7631+; $\frac{19}{27}$; $\frac{55}{327}$.

Page 505.—**1.** 38 in. **2.** 14 ft. **3.** 3249 sq. in. **4.** 4 ft. **5.** 11.4+ ft. **6.** 11.9− ft. **7.** 10 ft. **8.** 10 in. **9.** 24 ft. **10.** Diameter, 11.65+ in.; depth, 5.04− in.

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